MODULE 1

BASIC CONCEPTS

NETWORK :

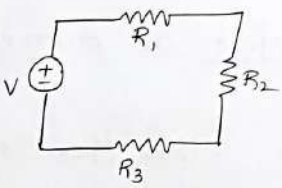
Interconnection of electrical components is called a network

CIRCUIT :

An electrical ckt. is an interconnection of electrical elements linked together in a closed path so that an electric current may continuously flow.

(or)

An electric circuit is a pipe line that facilitates the transfer of charge from one point to another.



* **NOTE :-** All circuits are network but all networks are not circuit.

ELECTRIC CURRENT :

Charge is the quantity of electricity responsible for electric phenomena.

Charge is denoted by 'q' & is -1.602×10^{-19} coulombs

∴ -1 coulomb is charge on 6.24×10^{18} electrons.

The net displacement of charge carriers through the cross-sectional area of a conductor, such as copper wire is called <u>electric current</u> & is denoted by 'I' (or) 'i'

(or)

"Current is the rate of flow of charge"

i.e $\boxed{I = \frac{Q}{t}}$

If the charge flowing in the conductor is varying with time, then

$$\boxed{i = \frac{dq}{dt}}$$

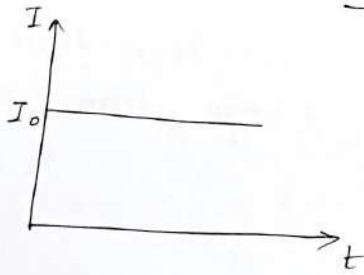


fig1: dc current

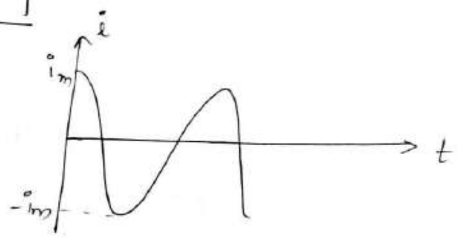


fig2: ac current

When the magnitude & direction of current flowing in an element does not change with respect to time is called <u>direct current</u> [fig. 1]

If the current in an element has a continuously varying magnitude with time & changes in its direction of flow is called <u>alternating current</u> [fig. 2]

VOLTAGE :

The voltage or potential energy difference between two points in an electric circuit is the amount of energy to move a unit charge

$$\boxed{V = \frac{W}{Q} = \frac{\text{Work (or) energy}}{\text{charge}}}$$

POWER :

Power is the rate of doing work.

$$\boxed{P = \frac{W}{t}}$$

DC : $P = VI$
AC : $P = VI \cos\phi$
$\cos\phi \rightarrow$ power factor

(or) $P = \frac{W}{t} = \frac{W}{Q} \cdot \frac{Q}{t} = V.I$

If the sign of power is positive, power is being absorbed by the element; if the sign is negative, power is being supplied by the element.

CIRCUIT ELEMENTS :

Any individual electrical component like resistor, inductor, capacitor, voltage source, current source with terminals is an electric circuit elements.

· BRANCH :-

A branch is a single ckt element (or) ckt elements connected in series. [AB, BC, CD, AD, CF, EF, DE]

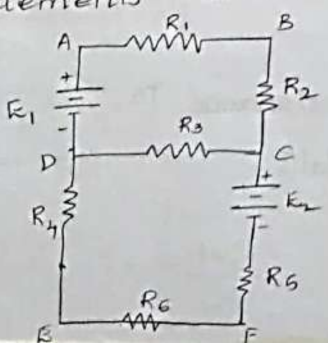


fig3: An electrical ckt.

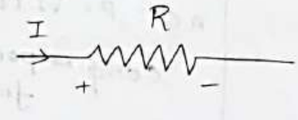
- **NODE :-**

A node is a junction of two (or) more ckt. elements (or) branches [A, B, C, D, E, F]

- **Junction point :-**

A point where three or more branches meet is called junction point [C & D]

- **Resistor :**



→ Resistor opposes the flow of current through it.

→ It is denoted by 'R' & measured interms of "ohms" (Ω)

→ Resistance of a given material is given by.

$$\boxed{R = \frac{\rho l}{a}}$$

Where,
ρ → resistivity in ohm metre
l → length in metre
a → Cross-sectional area in square metre
R → Resistance in Ω

→ Relation b/w. voltage, current & resistance is given by ohm's law.

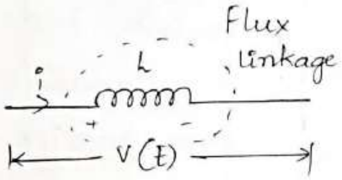
i.e. $\boxed{V = IR}$

or $R = \frac{V}{I}$

→ Power absorbed by a resistance is,

$$\boxed{P = VI = \frac{V^2}{R} = I^2 R} \text{ Watts}$$

- Inductance :



→ Energy is stored in the form of electromagnetic field, it is denoted by 'L' & the unit is henry

→ Voltage across inductor is given by,

$$\boxed{V = L\frac{di}{dt}}$$

and, $\boxed{i = \frac{1}{L}\int_{-\infty}^{t} v\,dt}$

∴ Power is, $P = vi = L\frac{di}{dt}\cdot i$

→ If inductor has 'N' turns & flux 'φ' is produced which is produced by current $i(t)$, then

$$\boxed{V(t) = N\frac{d\phi}{dt}}$$

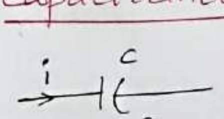
∴ Total flux linkage is proportional to current through the coil,

$$N\phi = Li$$

$$L = \frac{N\phi}{i}$$

And energy stored is, $W = \frac{1}{2}Li^2(t)$

- Capacitance :

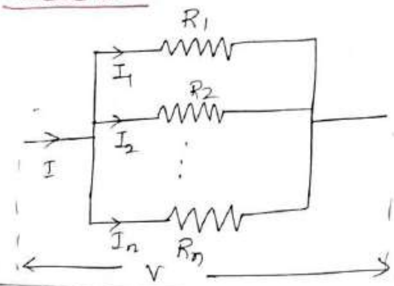
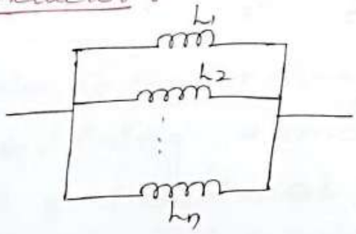
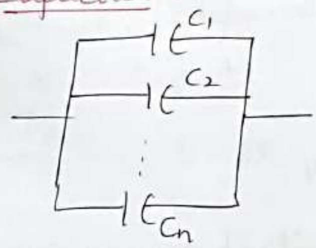
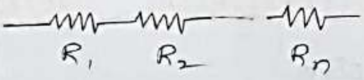
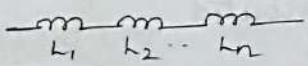


→ Energy is stored in the form of electrostatic field.

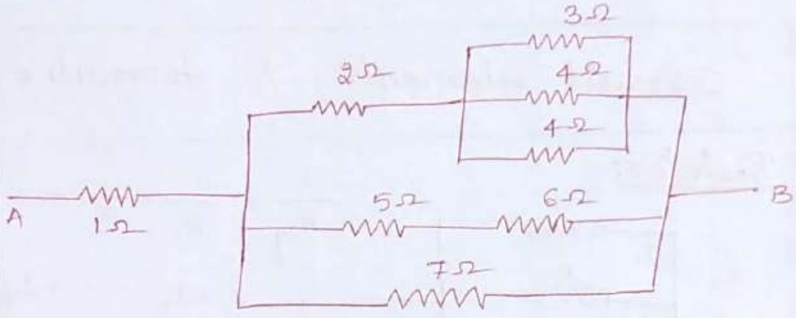
→ It is denoted by 'C' & the unit is farad. (F)

→ ∴ $q = C\frac{dv(t)}{dt}$ & $C = \frac{q}{V}$ (or) $v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt$

* Series & parallel combinations:-

	Circuit elements & formulae
Parallel - Current gets divided - voltage remains same across all the element	**Resistor:-**  $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$ $I = I_1 + I_2 + \cdots + I_n$ $I = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_n}$ $I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right]$
	Inductor:-  $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \cdots + \frac{1}{L_n}$
	Capacitor:-  $C_{eq} = C_1 + C_2 + \cdots + C_n$
Series - voltage gets divided - Current remains same.	**Resistor:-**  $R_{eq} = R_1 + R_2 + \cdots + R_n$
	Inductor:-  $L_{eq} = L_1 + L_2 + \cdots + L_n$
	Capacitor:-  $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

1. Find the equivalent resistance b/w 'A' & 'B'.



→ Soln. $R_{eq}' = R_1 + R_2 = 5+6 = 11\Omega$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = \frac{10}{12}$$

$\therefore R_{eq}'' = 1.2\Omega$

$R_{eq}'' = 2 + 1.2 = 3.2\Omega$

$$\frac{1}{R_{eq}''} = \frac{1}{3.2} + \frac{1}{11} + \frac{1}{7}.$$

$R_{eq}'' = 1.8304\Omega$.

$\therefore R_{AB} = 1 + 1.8304$

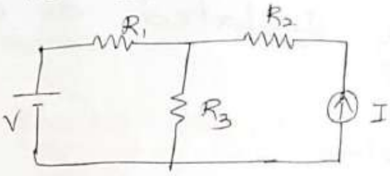
$$\boxed{R_{AB} = 2.8304\ \Omega}$$

* CLASSIFICATION OF ELECTRICAL NETWORK :-

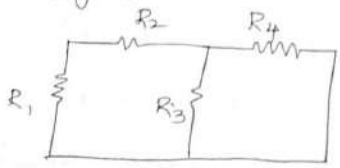
1. Active & Passive
2. Linear & Non-linear
3. Unilateral & bilateral
4. Lumped & distributed

1 Active & Passive n/ws :-

If the n/w contain a voltage source or current source in addition to passive element is called active n/w [fig. a]



fig(a)



fig(b).

A n/w. Containing only passive elements is Called passive n/w. [fig(b)]

* NOTE:-

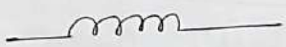
Voltage sources & current sources are "active sources."

2. Linear & Non-linear network :-

A ckt containing elements like resistance, capacitance & inductance whose parameters are always constant irrespective of the change in time, voltage, temperature is known as linear network.

R

$V = iR \; ; \; i = V/R$



L

$V = L\frac{di}{dt} \; ; \; i = \frac{1}{L}\int v\,dt$

C

$V = \frac{1}{C}\int i\,dt \; ; \; i = C\frac{dv}{dt}$.

Nonlinear n/w. is a ckt. whose para-meters change their values with change in time, temperature, voltage etc.,

3. Lumped & distributed n/w. :-

A n/w. in which physically separate R, L (or) C can be represented is called Lumped n/w

In this n/w R, L & C cannot be electrically separated & individually isolated as a separate ckt. element

Eg:- A transmission line

4. Unilateral ckt. & bilateral ckt:-

In a unilateral ckt, the relation b/w voltage & current changes with the change in direction of current.

Eg: Diode

In a bilateral ckt., the relation b/w. voltage & current remains the same for either directions of current flow.

Eg: R, L & C are bilateral ckt.

• NOTE:
A n/w containing R, L & C is called a Linear bilateral n/w.

* Regulation & loading of sources:-

Regulation is given by,

$$\% \text{ Regulation} = \frac{\text{No load voltage} - \text{Full load voltage}}{\text{Full load voltage}} \times 100$$

$$\boxed{\% \text{ Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100}$$

→ If the source is loaded in such a way that the load voltage falls below specified full load value & the regulation is higher than that specified for the source, then the source is said to be "loaded"

* ENERGY SOURCES:-

The two basic energy sources are:

- Voltage source
- Current source

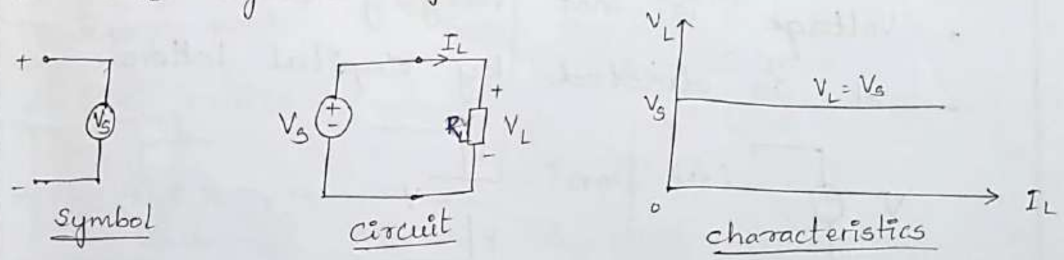
Independent sources:-

i) Voltage source :-

→ A V.S. can be ideal (or) practical.

→ Ideal voltage source is defined as the constant voltage energy source which gives 'V_L' independent of terminal current 'I_L'.

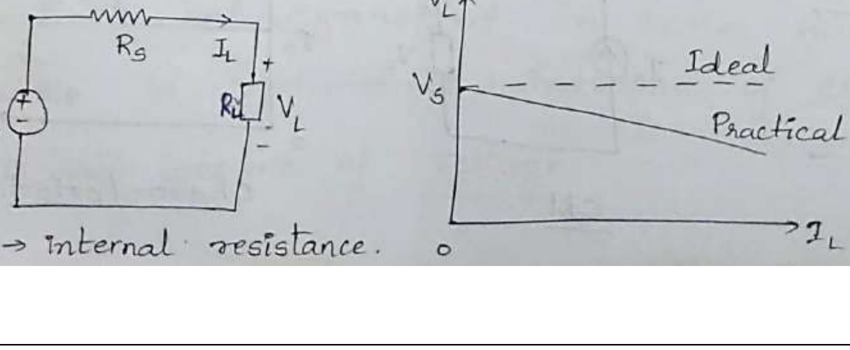
→ The symbol of ideal V.S. is shown below.



i) Ideal Voltage source

→ All practical voltage sources will have internal resistance called source resistance

→ Due to the presence of source resistance, 'V_L' depends on 'I_L' as shown below,



R_S → internal resistance.

$\therefore$ from fig, $I_L = \dfrac{V_S}{R_S + R_L}$

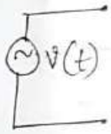
$$V_L = V_S - I_L R_S.$$

$\therefore$ For ideal V.S., $R_S = 0$, $\therefore$ $\boxed{V_L = V_S}$

→ V.S cane be classified as,

- Time invariant sources.
- Time variant sources.

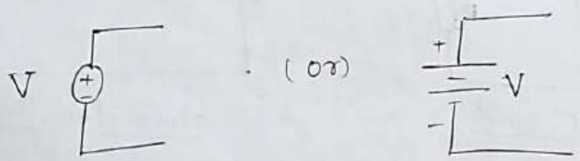
- **Time Variant source [AC Source] :-**



* Voltage varies w.r.t. time
* It is denoted by small letters

- **Time invariant source [DC source] :-**

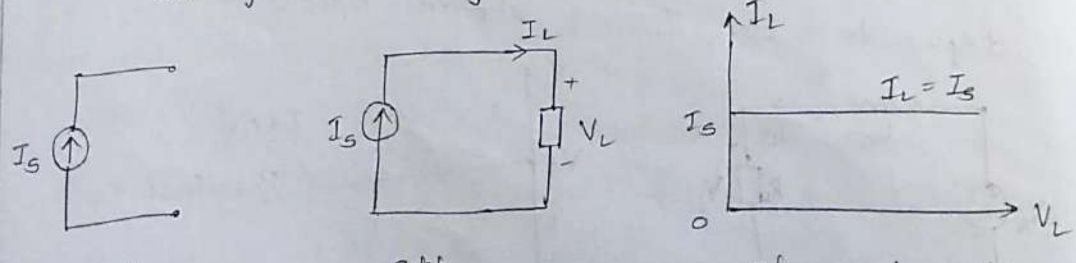
* Voltage is not varying w.r.t. time
* It is dinoted by capital letters



ii) Current source :-

→ A current source can be ideal/practical

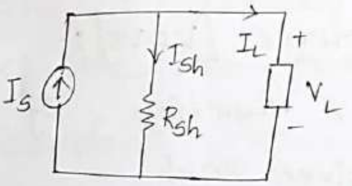
→ In an ideal c.s. 'I_L' remains constant independent of 'V_L' & it is as shown below,

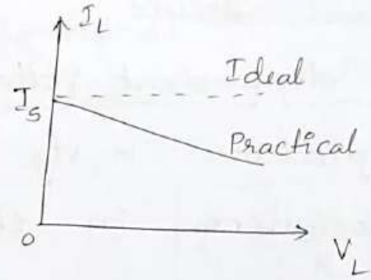


Symbol Ckt Characteristic

→ In a practical C.S. an interal resistance is presence known as source resistance

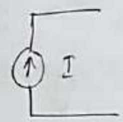
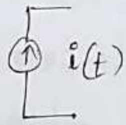
→ Due to the presence of source resistance. 'I_L' depends on 'V_L'





→ C.S. can be classified as,

- Time Variant C.S
- Time invariant C.S

Time invariant C.S	Time variant C.S
* Current does not vary w.r.t time	* Current varies w.r.t time
* DC Source	* AC Source
* 	* 
* Capital letters	* Small letters.

* **NOTE :-**

→ Any element connected across [$\|^{le}$] voltage source is redundant interms of voltage but not interms of current.

→ Any element connected in series with current source is redundant interms of current but not interms of voltage.

Dependent source :-

The value of source that depends on voltage / current in the ckt is called dependent source.

i) Voltage dependent voltage source [VDVS] :-

It produces a vtg. as a function of vtg. elsewhere in the given ckt.

ii) current dependent current source [CDCS] :-

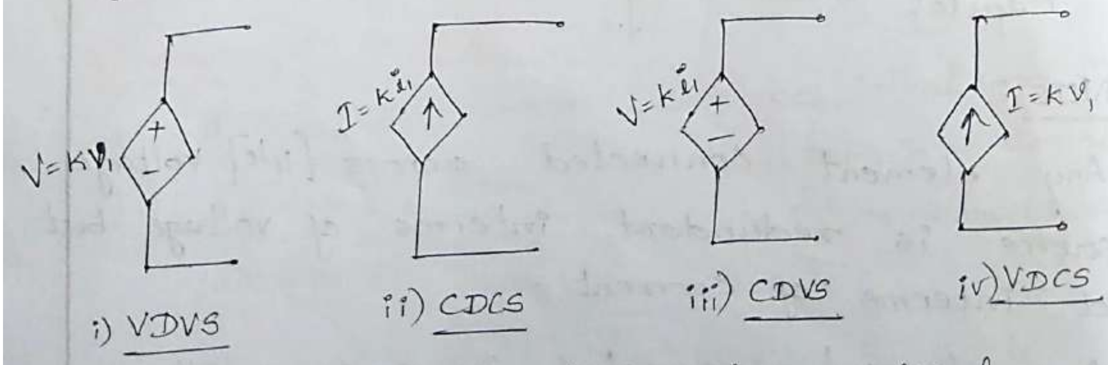
It produces a current as a function of current elsewhere in the given ckt.

iii) Current dependent voltage source [CDVS] :-

It produces a voltage as a function of current elsewhere in the given ckt.

iv) Voltage dependent current source [VDCS] :-

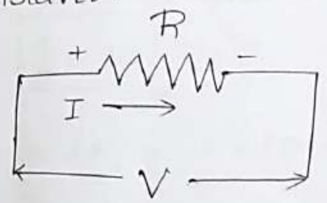
It produces a current as a function of voltage elsewhere in the given ckt.



※ Dependent sources are also called "controlled sources".

* ## Ohm's Law :- [Georg Simon ohm]

Ohm's law is stated as, "the current flowing through the electric ckt is directly proportional to the potential difference across the ckt. & inversely proportional to the resistance of the ckt, provided the temperature remains constant."



i.e. $\boxed{I \propto \frac{V}{R}}$

$I = \frac{V}{R}$

$V = IR$

$R = \frac{V}{I}$

(or)

ohm's law states that the Voltage across conducting material is directly proportional to current flowing through the material $V \propto I$; $V = IR$

The limitations of ohm's law are,

- It is not applicable to the non linear devices such as diodes, zener diodes etc.,
- It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is,

$$\boxed{V = kI^m}$$

Where,
$k, m \rightarrow$ constant

* NOTE:-

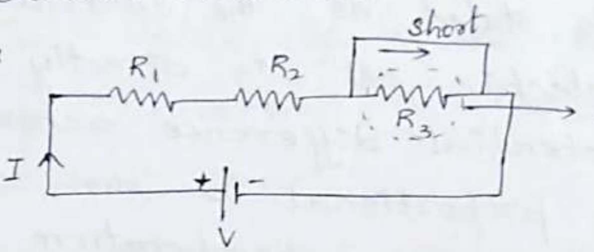
→ The vtg. & resistance across short ckt. is "zero" i.e. $V_{sc} = R_{sc} I_{sc} = 0 \cdot I_{sc} = 0$,

→ The resistance of open ckt. is infinity & current is zero.

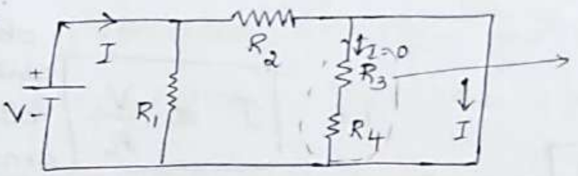
i.e. $I_{oc} = \frac{V}{R_{oc}} = \frac{V}{\infty} = 0$,

* Redundant branches :-

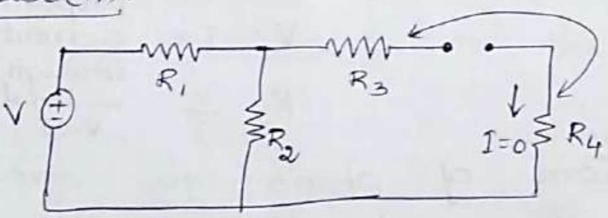
Case (i):



Case (ii):

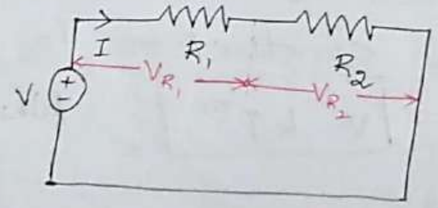


Case (iii):



* VOLTAGE DIVISION :-

Consider the ckt,



Apply KVL

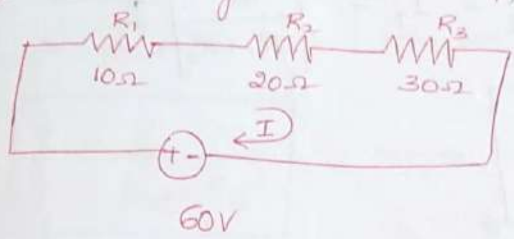
$V = IR_1 + IR_2$

$V = I(R_1 + R_2)$

$$\boxed{I = \frac{V}{R_1 + R_2}}$$

$$\therefore \boxed{\begin{aligned} V_{R_1} &= IR_1 = \frac{VR_1}{R_1 + R_2} \\ V_{R_2} &= IR_2 = \frac{VR_2}{R_1 + R_2} \end{aligned}}$$

Find the vtg. across R_1, R_2 & R_3.



→ Soln.

$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{60}{10 + 20 + 30} = \frac{60}{60}$$

$$\boxed{I = 1A}$$

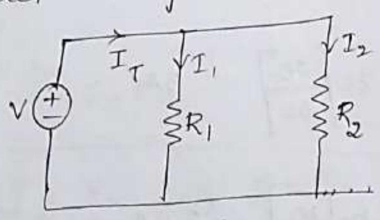
$V_{R_1} = IR_1 = 1 \times 10 = 10V$

$V_{R_2} = IR_2 = 1 \cdot 20 = 20V$

$V_{R_3} = IR_3 = 1 \cdot 30 = 30V$

* CURRENT DIVISION :-

Consider a parallel ckt.



∴ $I_T = I_1 + I_2$

But, $I_1 = \frac{V}{R_1}$ & $I_2 = \frac{V}{R_2}$

i.e.

$V = I_1 R_1 = I_2 R_2$

$I_1 = I_2 \left(\frac{R_2}{R_1}\right)$

Subs 'I_1' in I_T

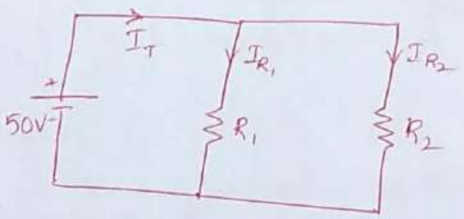
$$\therefore I_T = \left(\frac{R_2}{R_1}\right) I_2 + I_2 = I_2 \left[\frac{R_2}{R_1} + 1\right] = I_2 \left[\frac{R_2 + R_1}{R_1}\right]$$

$$\therefore \boxed{I_2 = \frac{R_1 \, I_T}{R_1 + R_2}}$$

Now, $I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1+R_2}\right] I_T$

$$\boxed{I_1 = \frac{R_2 I_T}{R_1+R_2}}$$

Find the magnitude of total current I_{R_1} & I_{R_2} if $R_1 = 10\Omega$, $R_2 = 20\Omega$ & $V = 50V$



→ Soln:- $R_{eq} = \frac{R_1 R_2}{R_1+R_2} = \frac{10 \times 20}{30} = 6.67\Omega$

$$I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = 7.5A$$

Now, $I_1 = I_T\left[\frac{R_2}{R_1+R_2}\right] = 7.5 \cdot \left[\frac{20}{30}\right] = 5A$

$$I_2 = I_T\left[\frac{R_1}{R_1+R_2}\right] = 7.5\left[\frac{10}{30}\right] = 2.5A$$

* SOURCE TRANSFORMATION :-

Consider voltage source in fig(a). having internal resistance Rse, connected to load resistance 'R_L'.

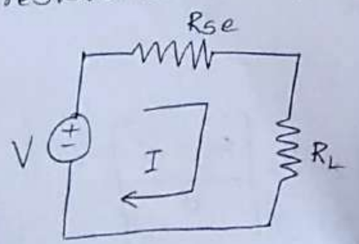


fig.(a)

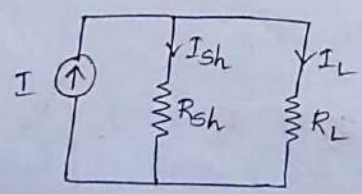


fig.(b)

The voltage source is replaced by equivalent current source as shown in fig.(b)

$$\therefore \boxed{I = \frac{V}{R_{se} + R_L}} \longrightarrow (1)$$

Load current 'I_L' is given by.

$$\boxed{I_L = I \cdot \frac{R_{sh}}{R_{sh} \cdot R_L}} \longrightarrow (2)$$

Now equating (1) & (2)

$$I = I_L$$

$$\frac{V}{R_{se} + R_L} = \frac{I \cdot R_{sh}}{R_{sh} + R_L}$$

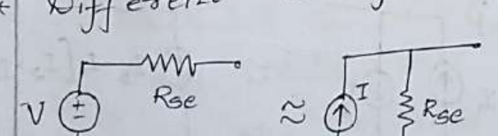
$\therefore$ (Equating denominator) ; $R_{se} + R_L = R_{sh} + R_L$

$$\therefore \boxed{R_{se} = R_{sh}}$$

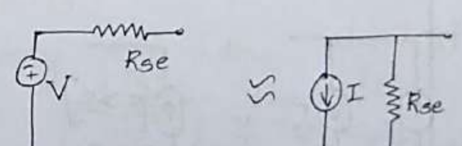
$$\therefore V = I \cdot R_{sh} = I \cdot R_{se}.$$

$$\boxed{I = \frac{V}{R_{sh}} \text{ (or) } I = \frac{V}{R_{se}}.}$$

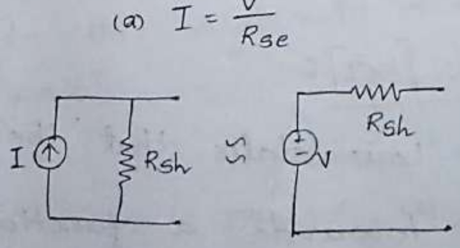
* Different transformed sources are:



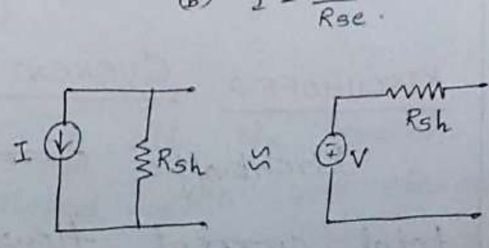
(a) $I = \frac{V}{R_{se}}$



(b) $I = \frac{V}{R_{se}}$.

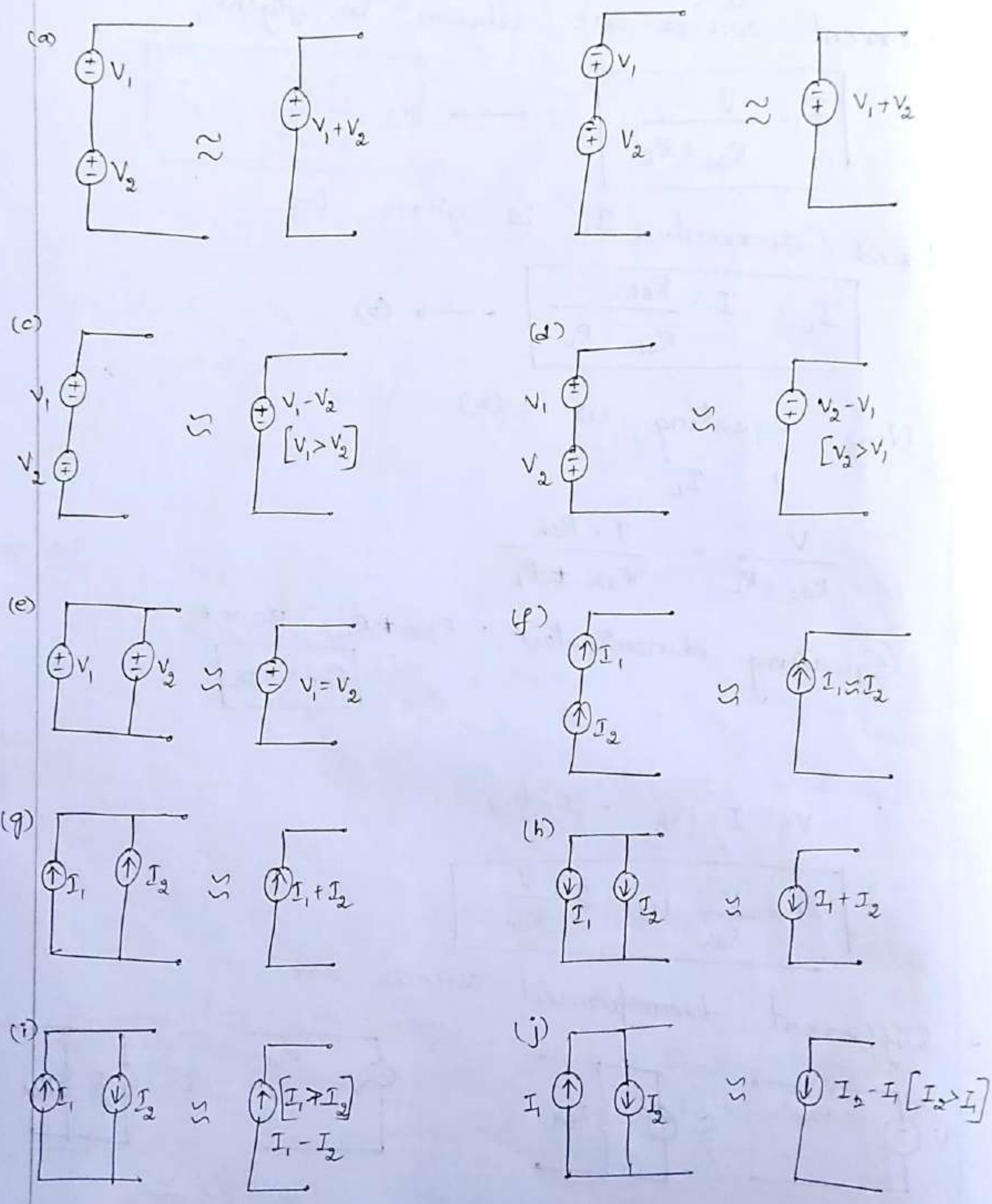


(c) $V = I \cdot R_{sh}$



(d) $V = I \cdot R_{sh}$

- Combinations:-



* KIRCHHOFF'S CURRENT LAW [KCL]:-

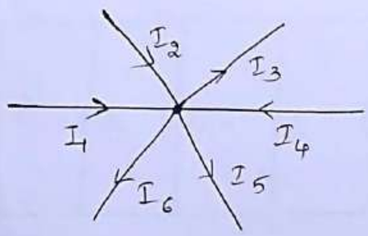
Kirchhoff's current law states that, 'the total current flowing towards a junction point is equal to the total current flowing away from that junction point."

(OR)

The algebraic sum of all the current meeting at a junction point is always zero.

i.e. $\boxed{\Sigma I \text{ at junction point} = 0}$

Consider a junction point shown in fig.(a)



$$I_1 + I_2 - I_3 + I_4 - I_5 - I_6 = 0$$

$$I_1 + I_2 + I_4 = I_3 + I_5 + I_6$$

* NOTE :-

Current flowing towards a junction point is assumed to be positive while current flowing away from a junction point is assumed to be negative

* KIRCHHOFF'S VOLTAGE LAW [KVL] :-

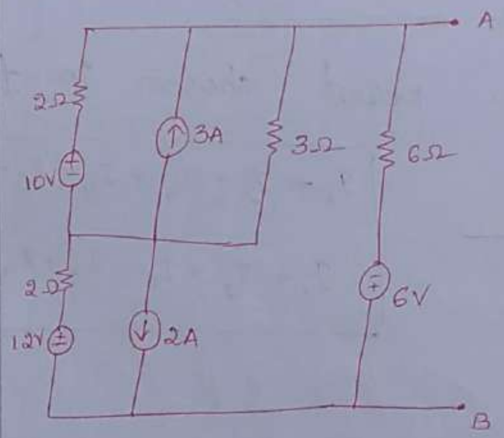
KVL states that, "In any n/w., the algebraic sum of the voltage drop across the ckt. elements of any closed path (or) loop is equal to the algebraic sum of the e.m.f's in the path"

(OR)

The algebraic sum of all the branch voltages around any closed path (or) closed loop is always zero.

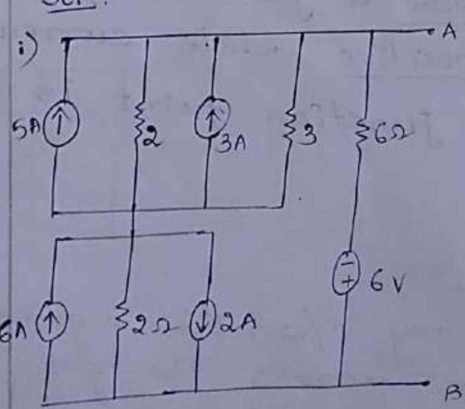
Sum of all the potential rises must be equal to the sum of all the potential drop while tracing any closed path of the ckt. The total change in potential along a closed path is always zero.

1 Reduce the n/w given below into a single voltage source in series with the resistance b/w the terminals A & B

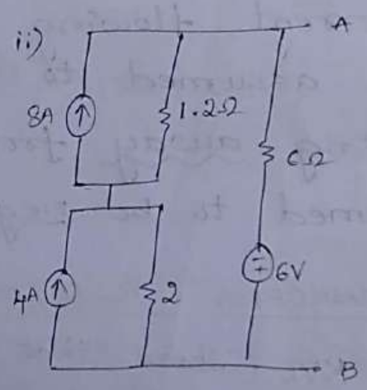


Sol^n:-

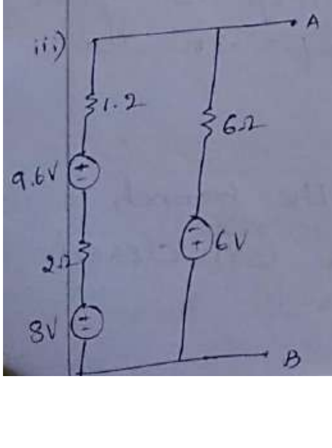
i)



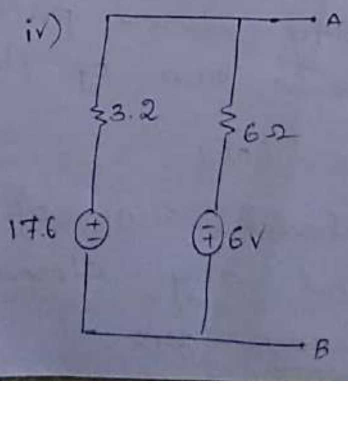
ii)



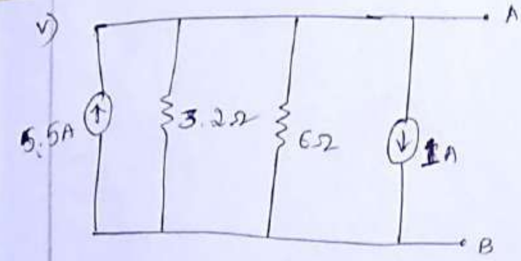
iii)



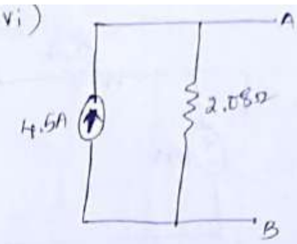
iv)



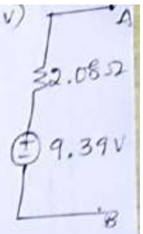
v)



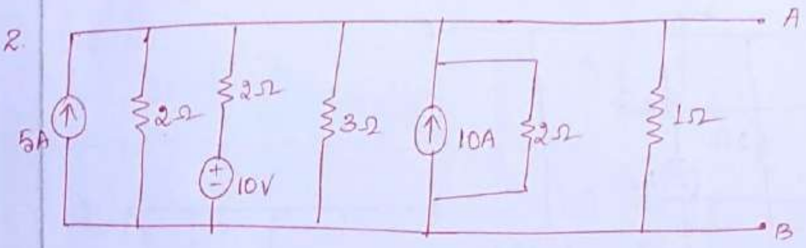
vi)



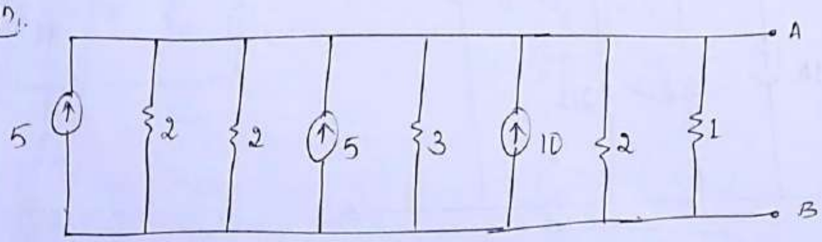
v)



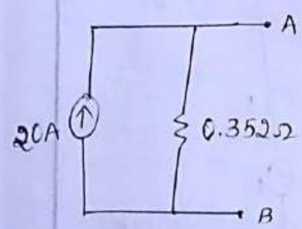
2.

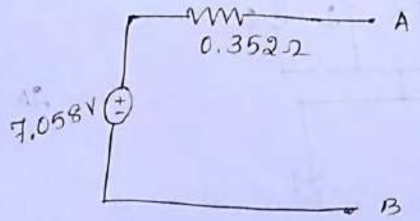


⇒ Soln.

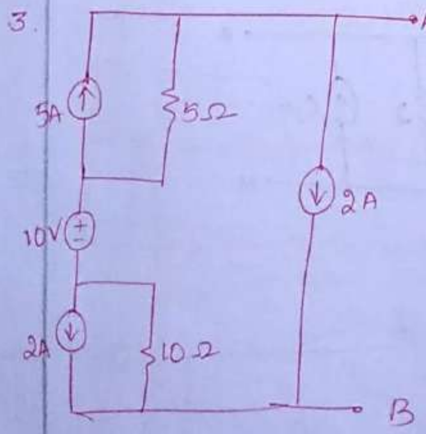


$$\frac{1}{R} = \frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{2} + 1 = \frac{17}{6} \quad \therefore R = \frac{6}{17} = 0.352\Omega$$

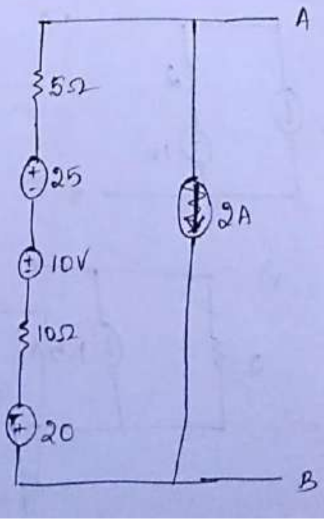


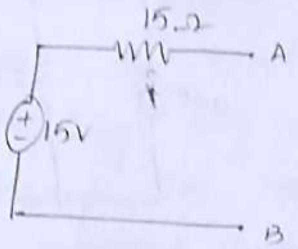


3.

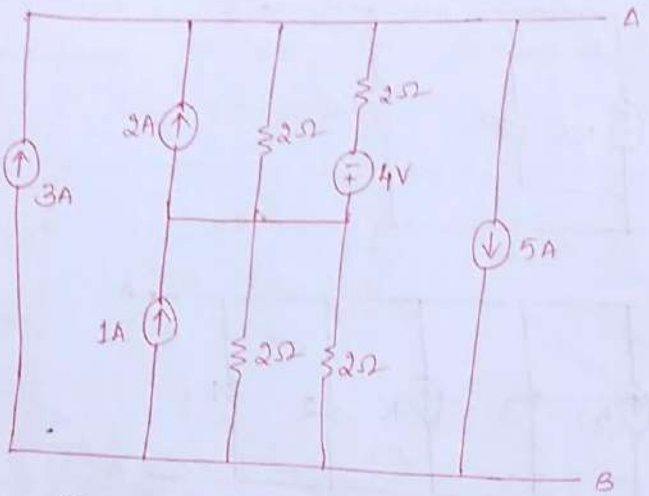


⇒ Soln:-



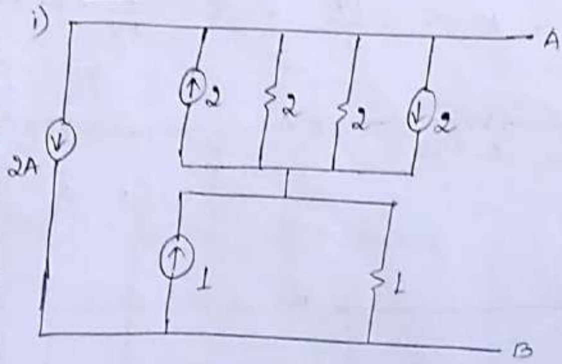


4

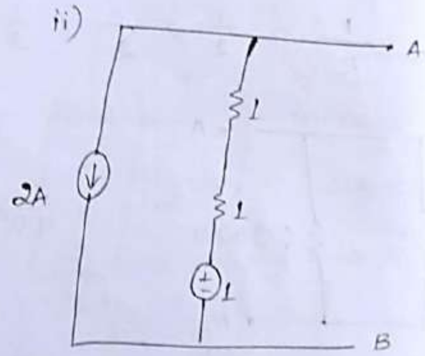


Sol^n:-

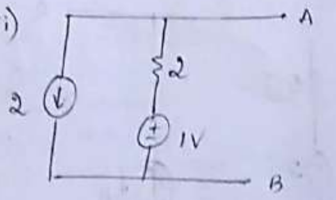
i)



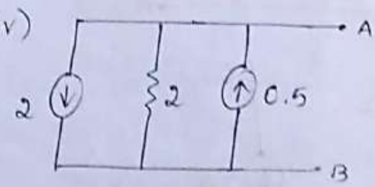
ii)



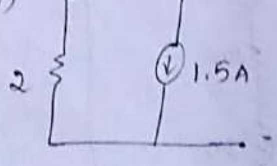
iii)



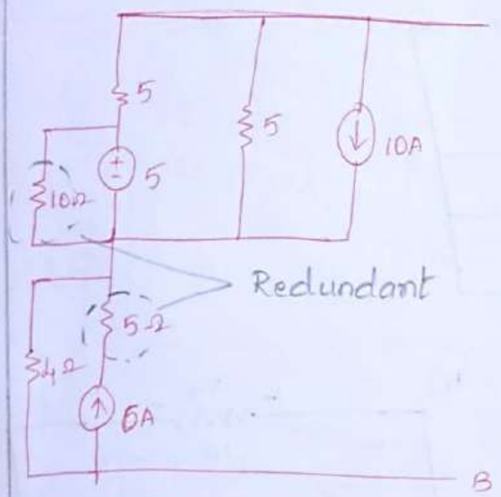
iv)



v)

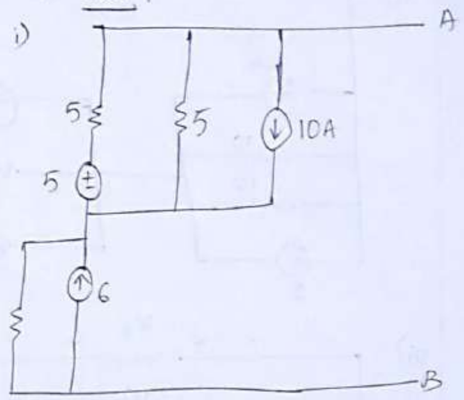


*5

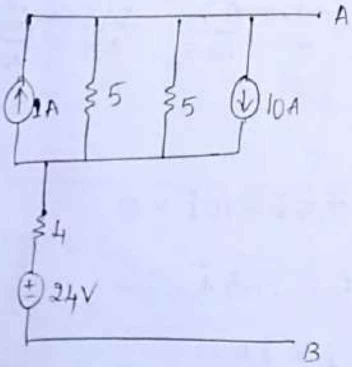


⟹ Soln:-

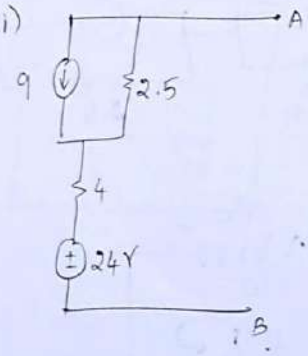
i)



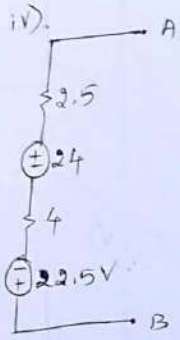
ii)



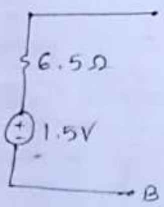
iii)



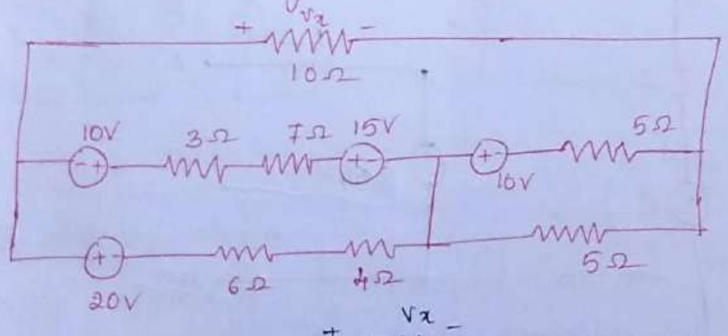
iv).



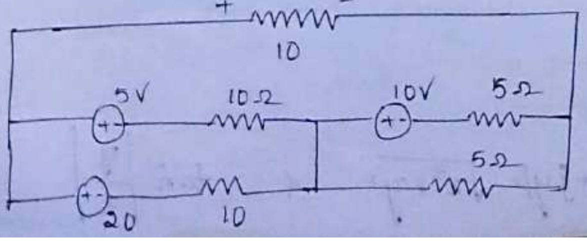
v)



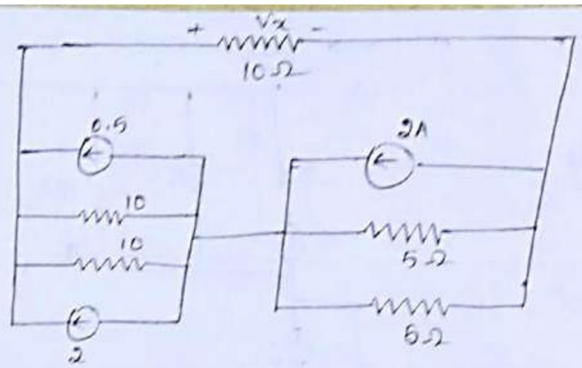
*6. Find the vtg. 'Vx' in the n/w shown below.



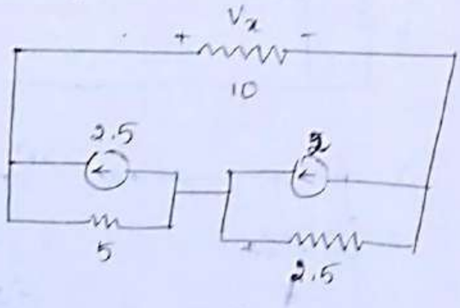
⟹ Soln:-



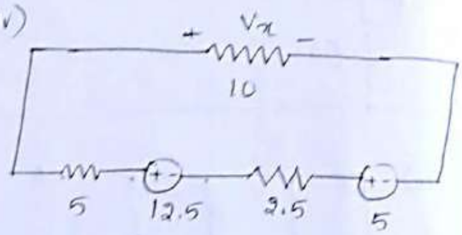
ii)



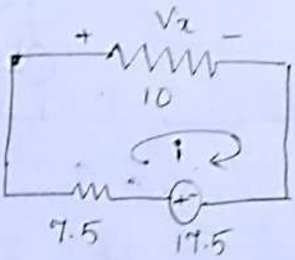
iii)



iv)



v)



$$17.5 - 7.5i - 10i = 0$$

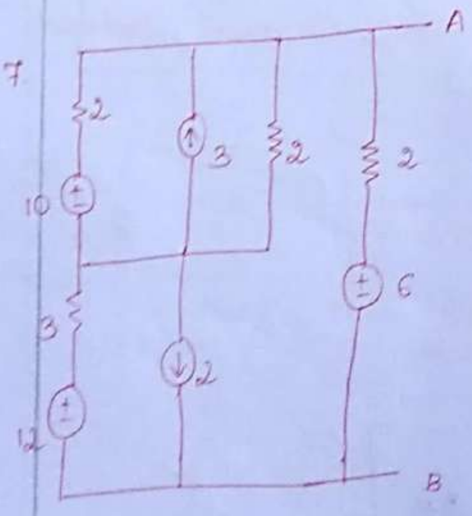
$$17.5 = 17.5i$$

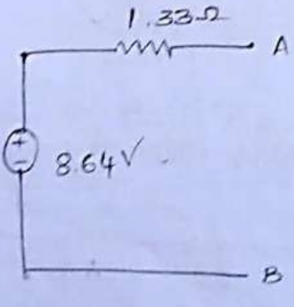
$$\therefore i = 1A$$

$$\therefore V_x = 10i = 10 \times 1$$

$$\boxed{V_x = 10V}$$

7.

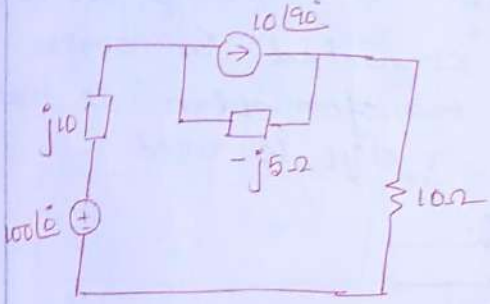




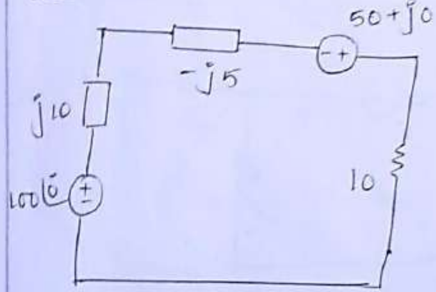
* NOTE:-

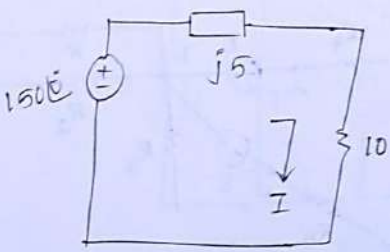
Complex nos.: $|x+jy| = \sqrt{x^2+y^2}$ & $\tan^{-1}\left|\frac{y}{x}\right|$

8. Find the current through 10Ω resistor



→ Solⁿ:



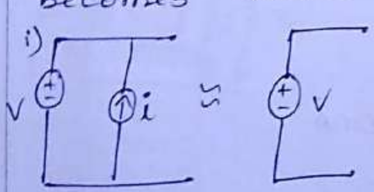


$$\therefore \quad I = \frac{150\angle 0^\circ}{10+j5} = (12-6j)\ A$$

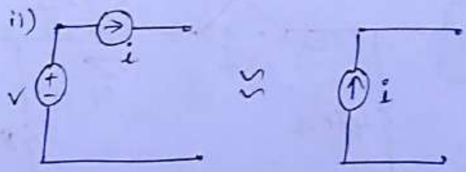
* NOTE:

Any element in parallel with ideal V.S. becomes trivial & any element in series with ideal C.S. becomes trivial.

i)

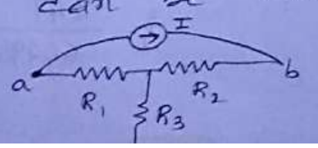


ii)

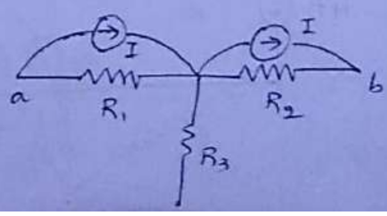


* SOURCE SHIFTING:-

I-shift:- When a n/w. contains a C.S without any element connected across it, source transformation is not possible. In such cases I-shift can be used.



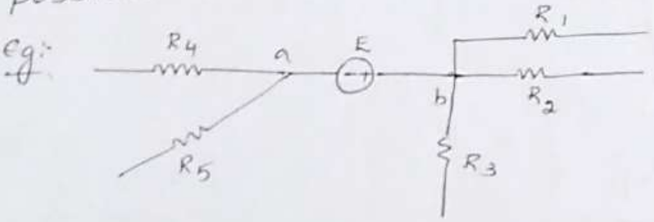
≃



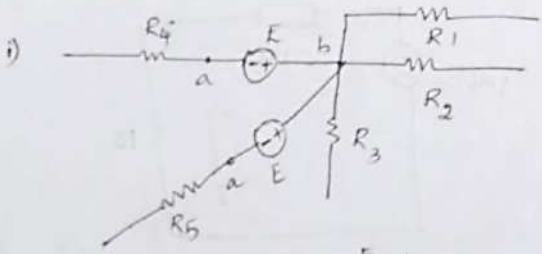
V-shift :-

When a n/w contain a voltage source without any element connected in series with it, then source transformation is not possible. In such cases v-shift is used.

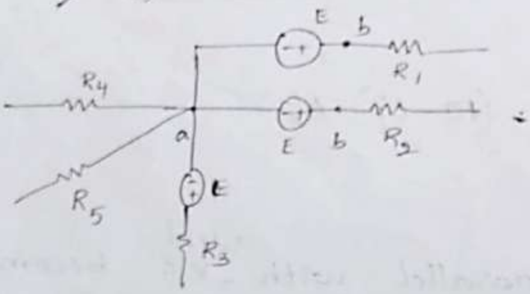
eg:-



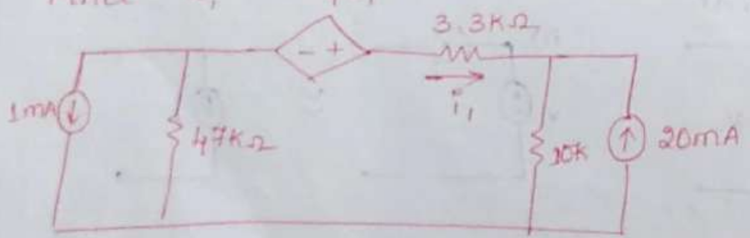
i)



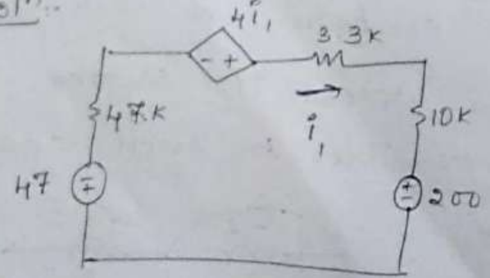
ii)



9 Find i_1



→ Soln:-

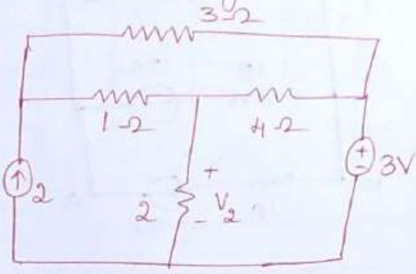


$$-47 - 47\times10^3 i_1 + 4i_1 - 3.3\times10^3 i_1 - 10\times10^3 i_1 - 200 = 0$$

$$-247 - 60296 i_1 = 0$$

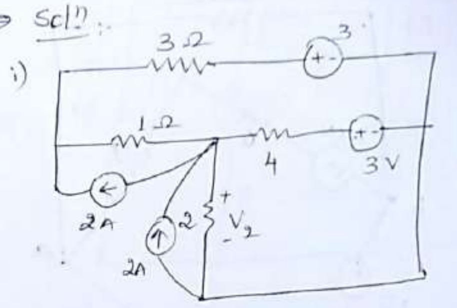
$$i_1 = \frac{-247}{60296} = -4.09\text{mA}$$

10. Dec-11 Find voltage 'V$_2$'

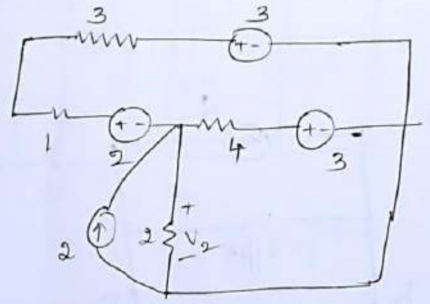


⟹ Sol^n:

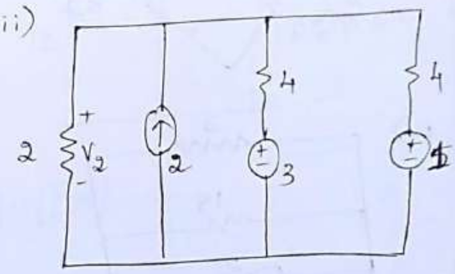
i)



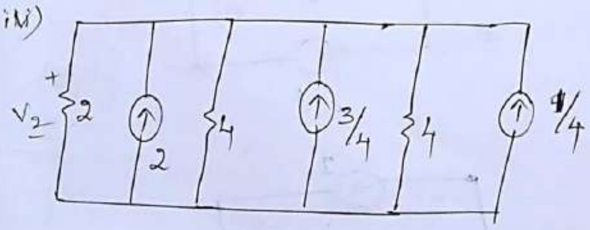
ii)



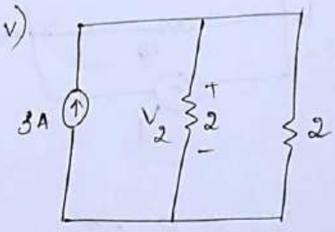
iii)



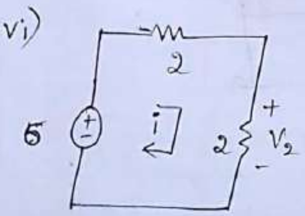
iv)



v)



vi)

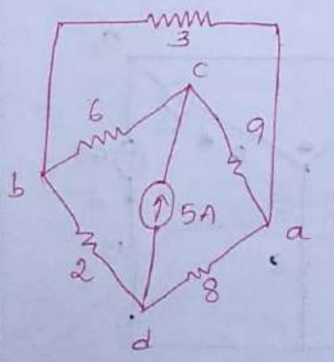


$$i = \frac{5}{4}$$

$$V_2 = 2 \times i = 2 \times \frac{5}{4} = 3V \quad \therefore \boxed{V_2 = 3V}$$

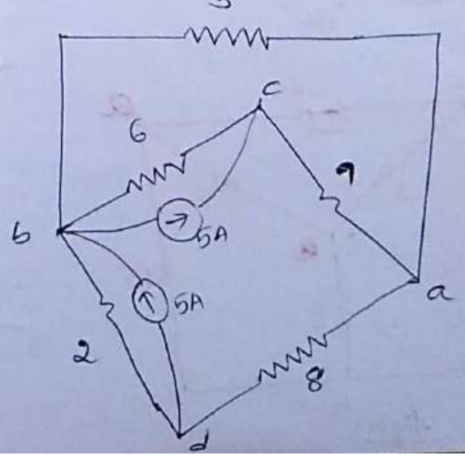
Verify pr. (10) using node analysis

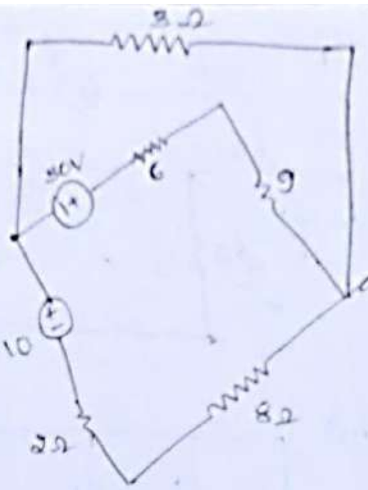
11. Find 'V$_{ab}$'

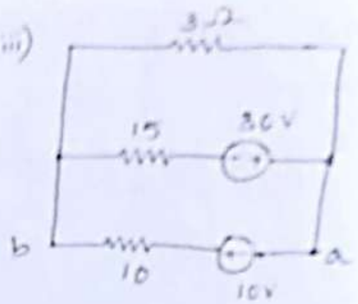


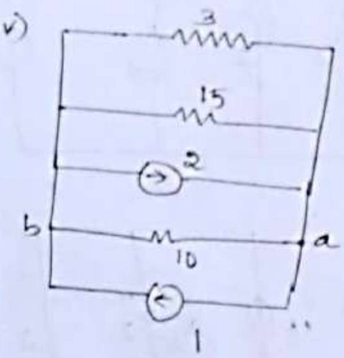
⟹ Sol^n:

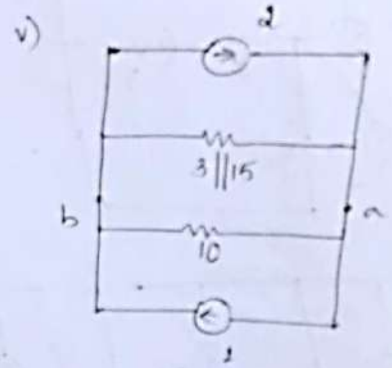
i)







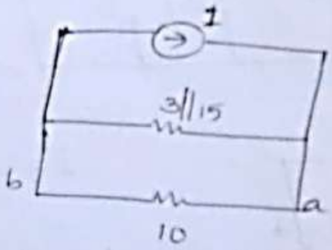




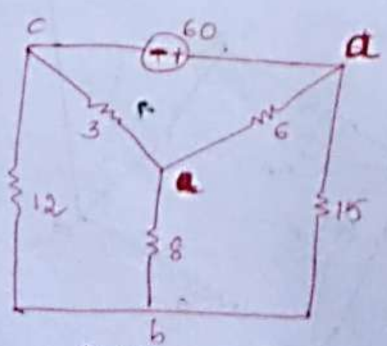
$V_{ab} = R_{eq} * I$

$V_{ab} = [3||15||10] * 1$

$\boxed{V_{ab} = 2V}$

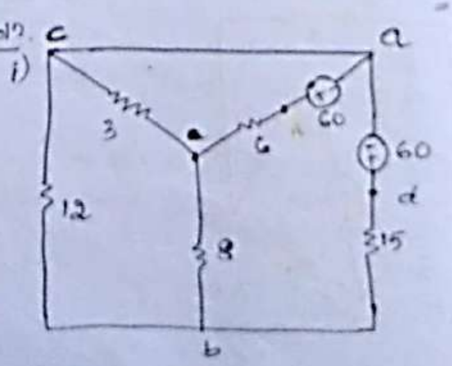


\#\#12 Find V_{db}

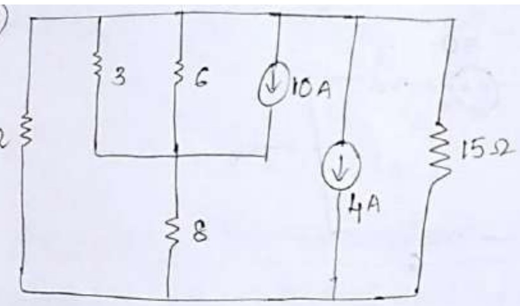


fig(a)

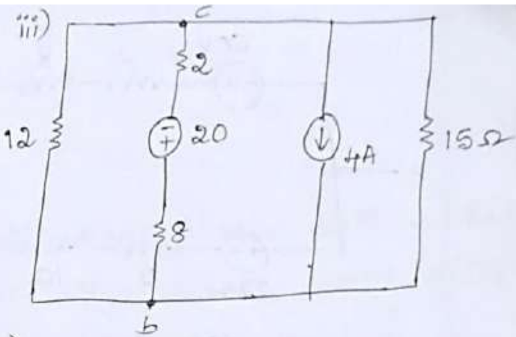
→ soln. i)



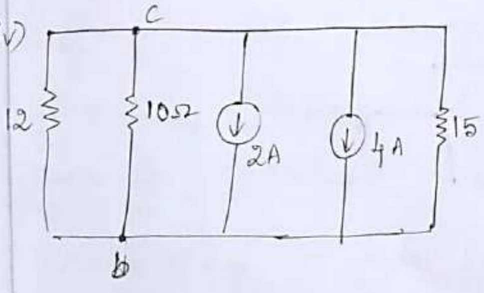
ii)



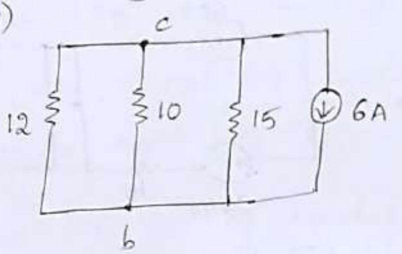
iii)



iv)



v)



$\therefore\ V_{bc} = R_{eq} \times I = [12 \| 10 \| 15] \times 6$

$V_{bc} = 24\,V$

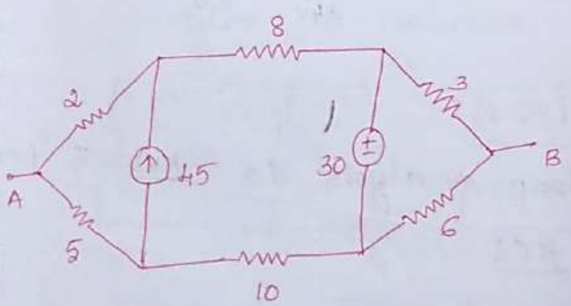
Apply KVL to fig(a) path c – a – b – c

$V_{ab} = V_{ac} - V_{bc}$

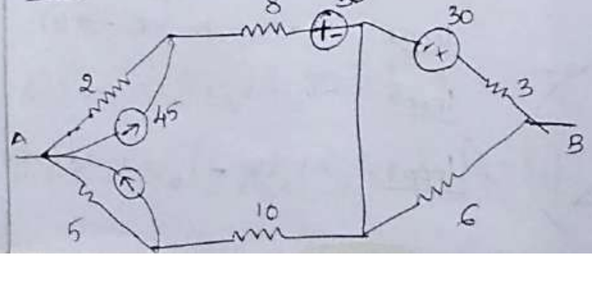
$V_{ab} = 60 - V_{bc} = 60 - 24$

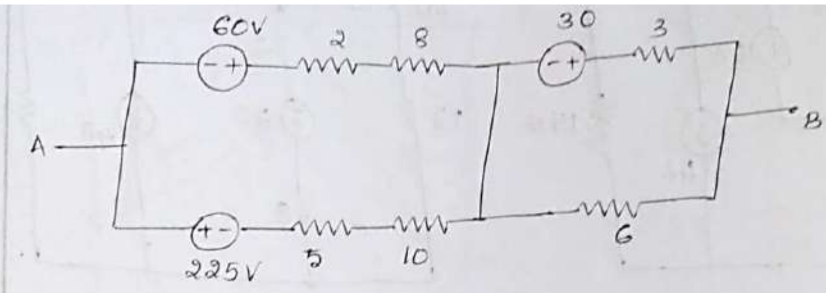
$\boxed{V_{ab} = 36\,V}$

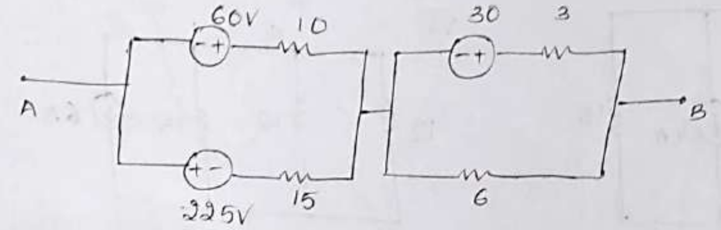
13.

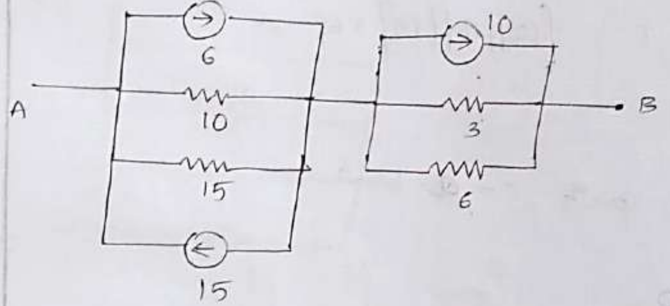


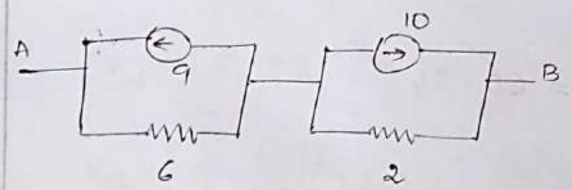
→ Soln.-

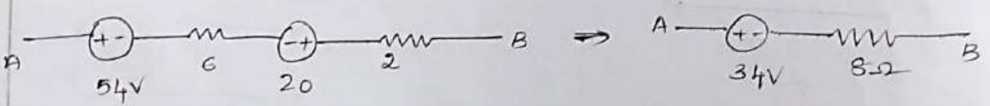








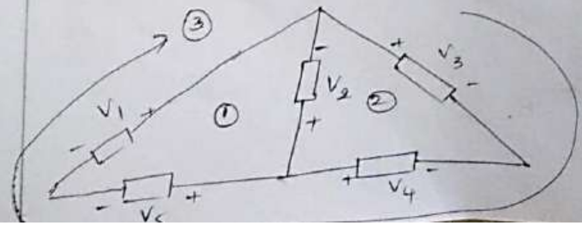




* LOOP AND NODE ANALYSIS :-

The basis for loop analysis is 'KVL' & for node analysis is KCL

Consider the linear n/w,



loop 1 : $V_1 + V_2 - V_5 = 0 \rightarrow (1)$

loop 2 : $-V_2 - V_3 + V_4 = 0 \rightarrow (2)$

loop 3 : $-V_3 + V_4 - V_5 + V_1 = 0 \rightarrow (3)$

(1) + (2), $V_1 + \cancel{V_2} - V_5 - \cancel{V_2} - V_3 + V_4 = 0$

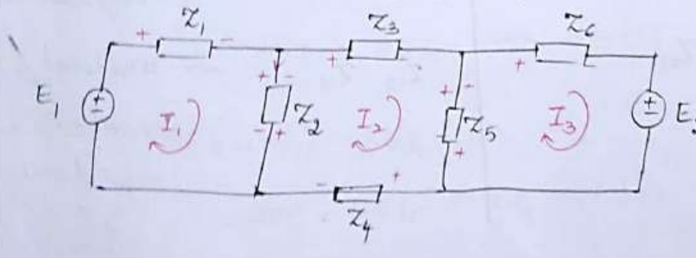
$V_1 + V_4 - V_5 - V_3 = 0.$

$\therefore$ (1) + (2) = (3)

* Linearly independent eqns are those eqns which cannot be obtained from any other eqns. after any form of manipulations [Eqn. (1) & Eqn. (2)]
* Linearly dependent eqns are those eqns. which can be obtained from other eqns. [Eqn. (3)]
* Assuming a n/w. contains 'n' no of nodes & 'b' branches, $\therefore$ minimum no of eqns. necessary to perform nodal analysis is "n-1"
* In node analysis one node is considered as reference / ground / datum node & the remaining nodes are called independent/principle nodes.
* Minimum no. of equations necessary to perform loop analysis is "b-(n-1)"

* Loop analysis :-

Consider a network,



loop1: $E_1 - Z_1 I_1 - Z_2 [I_1 - I_2] = 0$

$$(Z_1 + Z_2) I_1 - Z_2 I_2 = E_1 \longrightarrow (1)$$

loop2: $-Z_2[I_2 - I_1] - Z_3 I_2 - Z_5 [I_2 - I_3] - Z_4 I_2 = 0$

$$-Z_2 I_1 + (Z_3 + Z_4 + Z_5 + Z_2) I_2 - Z_5 I_3 = 0 \longrightarrow (2)$$

loop 3: $-Z_6 I_3 - E_2 - Z_5 (I_3 - I_2) = 0$

$$-Z_5 I_2 + (Z_5 + Z_6) I_3 = -E_2 \longrightarrow (3)$$

In matrix form,

$$\begin{bmatrix} (Z_1 + Z_2) & -Z_2 & 0 \\ -Z_2 & (Z_2 + Z_3 + Z_4 + Z_5) & -Z_5 \\ 0 & -Z_5 & (Z_5 + Z_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ -E_2 \end{bmatrix}$$

$$[Z][I] = [E]$$

where, $[Z]$ = Impedance matrix

$[I]$ = Current matrix

$[E]$ = Column matrix containing V.S.

The general form of $[Z]$ is,

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots \\ Z_{21} & Z_{22} & Z_{23} & \cdots \\ \vdots & & & \\ \vdots & & & \end{bmatrix}$$

$Z_{11}, Z_{22} \ldots \rightarrow$ self impedance.

$Z_{12}, Z_{13} \ldots \rightarrow$ mutual / transfer impedance.

* Node analysis :-

Assuming a n/w contain 3 independent source & a datum node, the 3 nodal eqns can be written in general form as,

$$\begin{bmatrix} y_{11}V_1 + y_{12}V_2 + y_{13}V_3 \\ y_{21}V_1 + y_{22}V_2 + y_{23}V_3 \\ y_{31}V_1 + y_{32}V_2 + y_{33}V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

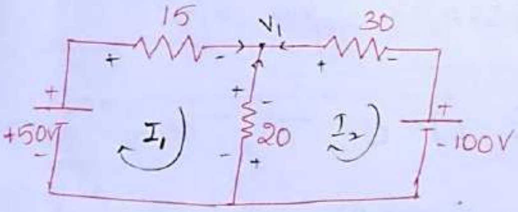
Where,

$[Y]$ = admittance matrix

$$\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$[Y][V] = [I]$$

14 Find I_1 & I_2 by using KVL & KCL



→ Sol^n:-

KVL:-

loop1: $50 - 15I_1 - 20(I_1 - I_2) = 0$

$$50 - 35I_1 + 20I_2 = 0$$

$$-35I_1 + 20I_2 = -50 \longrightarrow (1)$$

loop2: $-20(I_2 - I_1) - 30I_2 - 100 = 0$

$$20I_1 - 50I_2 = 100 \longrightarrow (2)$$

$$\therefore \boxed{I_1 = 0.37A, \; I_2 = -1.85A}$$

KCL:- $\dfrac{50 - V_1}{15} + \dfrac{0 - V_1}{20} + \dfrac{100 - V_1}{30} = 0$

$$200 - 4V_1 - 3V_1 + 200 - 2V_1 = 0$$

$$400 - 9V_1 = 0$$

$$\boxed{V_1 = 44.4V}$$

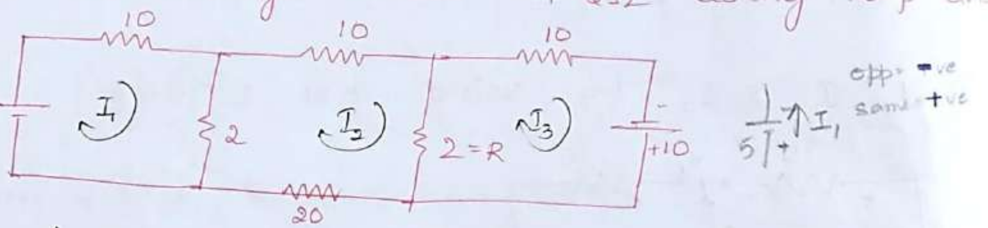
$$\therefore I_1 = \frac{50 - V_1}{15} = 0.37A$$

$$I_1 - I_2 = \frac{0 - V_1}{20}$$

$$0.37 - I_2 = \frac{-44.4}{20}$$

$$\therefore I_2 = -1.85A$$

15. Find the voltage across $R = 2\Omega$ using loop analy.



→ Soln:- loop1:- $5 - 10I_1 - 2(I_1 - I_2) = 0$

$$-12I_1 + 2I_2 = -5 \longrightarrow (1) \quad \times'y \text{ by } '-1'$$

loop2:- $-2(I_2 - I_1) - 10I_2 - 2(I_2 - I_3) - 20I_2 = 0$

$$2I_1 - 34I_2 + 2I_3 = 0 \longrightarrow (2) \quad \times'y \text{ by } '-1'$$

loop3:- $-2(I_3 - I_2) - 10I_3 + 10 = 0$

$$2I_2 - 12I_3 = -10 \longrightarrow (3) \quad \times'y \text{ by } '-1'$$

$$\begin{bmatrix} 12 & -2 & 0 \\ -2 & 34 & -2 \\ 0 & -2 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

$$V_{(R=2\Omega)} = 2(I_2 - I_3) = 2(I_3 - I_2)$$

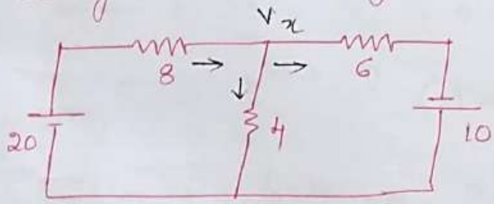
$$\Delta = \begin{vmatrix} 12 & -2 & 0 \\ -2 & 34 & -2 \\ 0 & -2 & 12 \end{vmatrix} = 12\left(34\times 12 - 4\right) + 2\left[-24\right] + 0 = 4800$$

$$I_2 = \frac{\begin{vmatrix} 12 & 5 & 0 \\ -2 & 0 & -2 \\ 0 & 10 & 12 \end{vmatrix}}{\Delta} = \frac{12[20] - 5[-24]}{4800} = 0.075$$

$$I_3 = \frac{\begin{vmatrix} 12 & -2 & 5 \\ -2 & 34 & 0 \\ 0 & -2 & 10 \end{vmatrix}}{\Delta} = \frac{12[340] + 2[-20] + 5[4]}{4800} = 0.845$$

$$\therefore\ V_{(R=2\Omega)} = 2\left[0.845 - 0.075\right] = 1.54\,V$$

16. Using node analysis find all branch currents.



⇒ Soln.

$$\frac{20 - V_x}{8} = \frac{V_x - 0}{4} + \frac{V_x - (-10)}{6}$$

$$\frac{20 - V_x}{8} = \frac{V_x}{4} + \frac{V_x + 10}{6}$$

$$\frac{20 - V_x}{8} = \frac{6V_x + 4V_x + 40}{24}$$

$$3[20 - V_x] = 10V_x + 40$$

$$60 - 3V_x = 10V_x + 40$$

$$20 = 13V_x$$

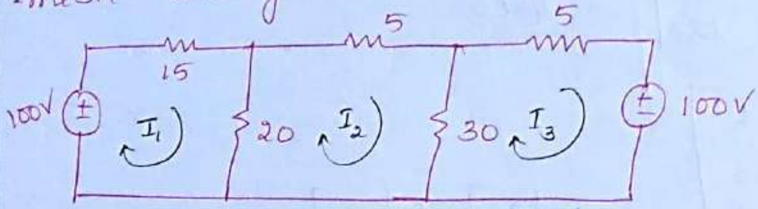
$$V_x = 20/13 = 1.5384$$

$$\therefore\ I_{8\Omega} = \frac{20 - V_x}{8} = 2.3077A$$

$$I_{4\Omega} = \frac{V_x}{4} = 0.3845A$$

$$I_{6\Omega} = \frac{V_x + 10}{6} = 1.9231A$$

17. Find current through 30Ω resistor using mesh analysis.



→ Soln:-

loop1:- $100 - 15I_1 - 20(I_1 - I_2) = 0$

$$-35I_1 + 20I_2 = -100 \rightarrow (1)$$

loop2:- $-20[I_2 - I_1] - 5I_2 - 30[I_2 - I_3] = 0$

$$20I_1 - 55I_2 + 30I_3 = 0 \longrightarrow (2)$$

loop3:- $-30(I_3 - I_2) - 5I_3 - 100 = 0$

$$30I_2 - 35I_3 = 100 \longrightarrow (3)$$

$I_1 = 1.9A$

$\therefore I_2 = -1.6A$; $I_3 = -4.228$

$\therefore \boxed{I_{30\Omega} = I_2 \sim I_3 = \pm 2.628A}$

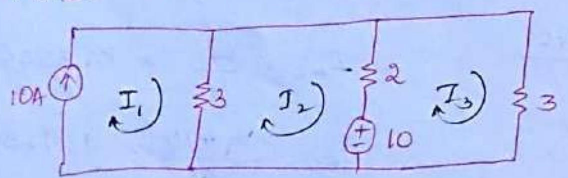
Method 2:

$\Delta = 21,875$

$\Delta_2 = -35000$

$\Delta_3 = -92500$

18. Find mesh currents.



→ Soln:- loop1: $I_1 = 10A$

Loop2:- $-3[I_2 - I_1] - 2[I_2 - I_3] - 10 = 0$

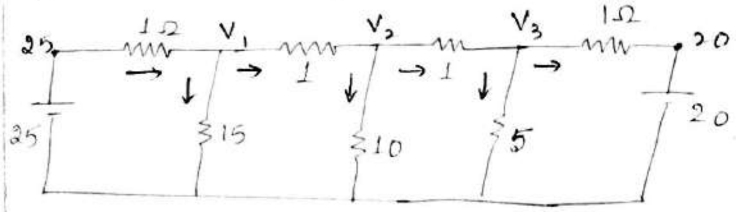
$$-5I_2 + 2I_3 = -20 \longrightarrow (1)$$

loop3:- $10 - 2[I_3 - I_2] - 3I_3 = 0$

$$2I_2 - 5I_3 = -10 \rightarrow (2)$$

$$\therefore \boxed{\begin{aligned} I_1 &= 10A \\ I_2 &= 5.7142A \\ I_3 &= 4.2857A \end{aligned}}$$

19. Find node vtg^s



→ Sol^n: Node 1:

$$\frac{25 - V_1}{1} = \frac{V_1}{15} + \frac{V_1 - V_2}{1}$$

$$31V_1 - 15V_2 = 375 \longrightarrow (1)$$

Node (2):

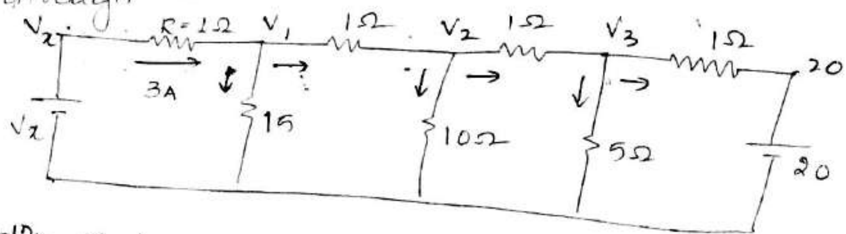
$$\frac{V_1 - V_2}{1} = \frac{V_2}{10} + \frac{V_2 - V_3}{1} \quad ; \quad 10V_1 - 21V_2 + 10V_3 = 0 \longrightarrow (2)$$

Node (3):-

$$\frac{V_2 - V_3}{1} = \frac{V_3}{5} + \frac{V_3 - 20}{1} \quad ; \quad 5V_2 - 11V_3 = -100 \longrightarrow (3)$$

$$\therefore \boxed{\begin{aligned} V_1 &= 20.92V \\ V_2 &= 18.24V \\ V_3 &= 17.38V \end{aligned}}$$

20. Find the value of voltage 'V_x' such that current through $R = 1\Omega$ is 3A



→ Sol^n: Node 1:

$$\frac{V_x - V_1}{1} = \frac{V_1 - 0}{15} + \frac{V_1 - V_2}{1}$$

$$3 = \frac{V_1 + 15V_1 - 15V_2}{15}$$

$$16V_1 - 15V_2 = 45 \longrightarrow (1)$$

Node 2:- $\dfrac{V_1 - V_2}{1} = \dfrac{V_2}{10} + \dfrac{V_2 - V_3}{1}$

$$-10V_1 + 21V_2 - 10V_3 = 0 \longrightarrow (2)$$

Node 3:- $\dfrac{V_2 - V_3}{1} = \dfrac{V_3}{5} + \dfrac{V_3 - 20}{1}$

$$-5V_2 + 11V_3 = 100 \longrightarrow (3)$$

$V_1 = 18.57V$

$V_2 = 16.81V$

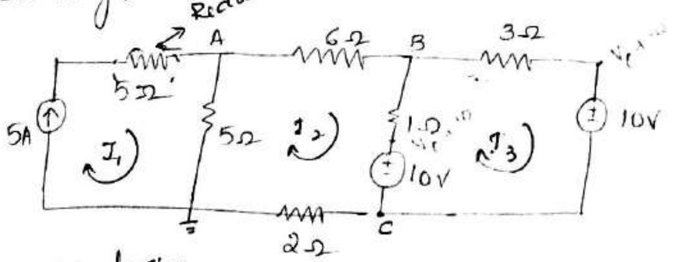
$V_3 = 16.73V$

$$\therefore \frac{V_x - V_1}{1} = 3$$

$$\therefore V_x = 3 + V_1 = 3 + 18.57$$

$$\boxed{V_x = 21.57V}$$

21. Determine voltages at 'A' & 'B' using loop & node analysis



In both loop & node analysis gnd. should be the common ground.

loop analysis:-

$I_1 = 5A$

loop 2:- $-5[I_2 - I_1] - 6I_2 - 1[I_2 - I_3] - 10 - 2I_2 = 0$

$$-5I_1 + 14I_2 - I_3 = -10 \longrightarrow (1)$$

loop3: $-3I_3 - 10 + 10 - 1[I_3 - I_2] = 0$

$$-I_2 + 4I_3 = 0 \longrightarrow (2)$$

$\therefore \; I_1 = 5A; \; I_2 = 1.09A; \; I_3 = 0.27A$

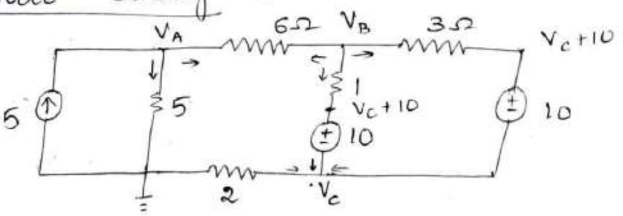
$\therefore \; V_A = 5[I_1 - I_2] = 5[5 - 1.09] = 19.55V$

$V_B = -5[I_2 - I_1] - 6I_2 = -5[-5 + 1.09] - 6 \times 1.09$

$V_B = 13.01V$

$V_C = 2I_2 = 2 \times 1.09 = 2.18V$

Node analysis :-



NodeA :- $5 = \dfrac{V_A}{5} + \dfrac{V_A - V_B}{6}$

$$11V_A - 5V_B = 150 \longrightarrow (1)$$

NodeB :- $\dfrac{V_B - V_A}{6} + \dfrac{V_B - (V_C + 10)}{1} + \dfrac{V_B - (V_C + 10)}{3} = 0$

$$-V_A + 9V_B - 8V_C = 80 \longrightarrow (2)$$

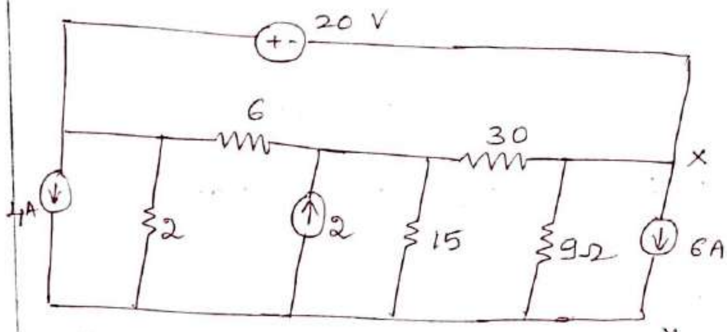
Node c :- $\dfrac{0 - V_C}{2} + \dfrac{V_B - (V_C + 10)}{1} + \dfrac{V_B - (V_C + 10)}{3} = 0$

$$-8V_B + 11V_C = -80 \longrightarrow (3)$$

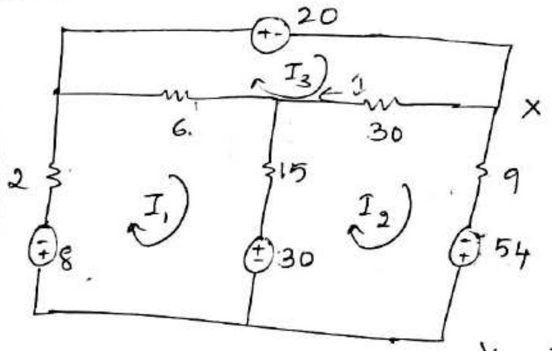
$\therefore$

$$\boxed{\begin{aligned} V_A &= 19.54V \\ V_B &= 13V \\ V_C &= 2.18V \end{aligned}}$$

22. Find the vtg. & power developed across current source in the n/w shown below. Use source transformation & mesh analysis.



→ Sol^n:-



V_{xy} → 'x' is at higher potential than 'y'.
∴ we shld. go from 'y' to 'x'

$$\therefore V_{xy} = -54 + 9I_2$$

loop1:- $-8 - 2I_1 - 6[I_1 - I_3] - 15[I_1 - I_2] - 30 = 0$

$$23I_1 - 15I_2 - 6I_3 = -38 \longrightarrow (1)$$

loop2:- $30 - 15[I_2 - I_1] - 30[I_2 - I_3] - 9I_2 + 54 = 0$

$$-15I_1 + 54I_2 - 30I_3 = 84 \longrightarrow (2)$$

loop3:- $-6[I_3 - I_1] - 20 - 30[I_3 - I_2] = 0$

$$-6I_1 - 30I_2 + 36I_3 = -20 \longrightarrow (3)$$

$\therefore I_1 = 0.62\ A$

$I_2 = 2.75\ A$

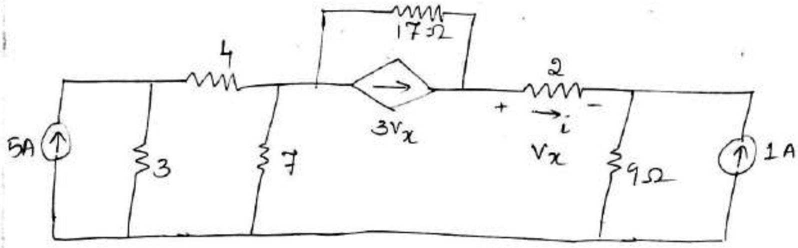
$I_3 = 1.84\ A$

$\therefore V_{xy} = -54 + 9 \times 2.75$

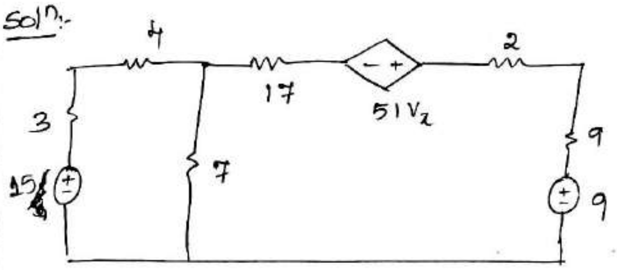
$$\boxed{V_{xy} = -29.25\,V}$$

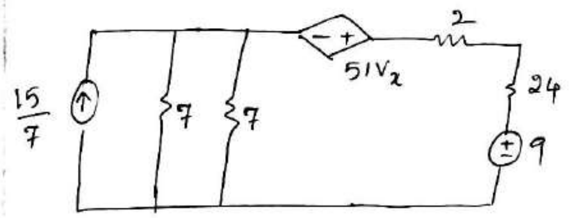
$\therefore \quad P = VI = -29.25 \times 6 = -175.5\,W.$

23 Dec-17 Calculate the current through 2Ω using S.T.

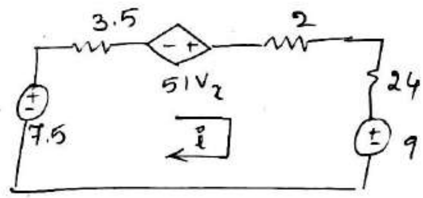


→ Soln:-





→



$$i = \frac{7.5 + 51V_x - 9}{3.5 + 2 + 24} \quad (or) \quad i = \frac{7.5 + 51V_x - V_x - 9}{3.5 + 24}$$

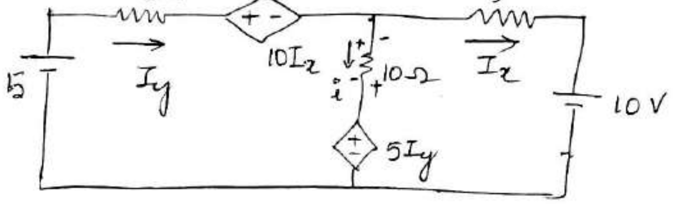
$$31.5i = 51V_x - 15$$

But $V_x = 2i$

$$\therefore \quad 31.5i = 51(2i) - 1.5 \quad ; \quad i = 0.02127A$$

$$\boxed{i = 21.27\,mA}$$

24. Dec-17 Find current through 10Ω.



⇒ Solⁿ:- $i = I_y - I_x$

loop1:- $5 - 5I_y - 10I_x - 10[I_y - I_x] - 5I_y = 0$

$$5 - 10I_y - \cancel{10I_x} - 10I_y + \cancel{10I_x} = 0$$

$$5 - 20I_y = 0$$

$$I_y = 5/20$$

$$I_y = 0.25A$$

loop2:-

$5I_y - 10[I_x - I_y] - 5I_x - 10 = 0$

$15I_y - 15I_x - 10 = 0$

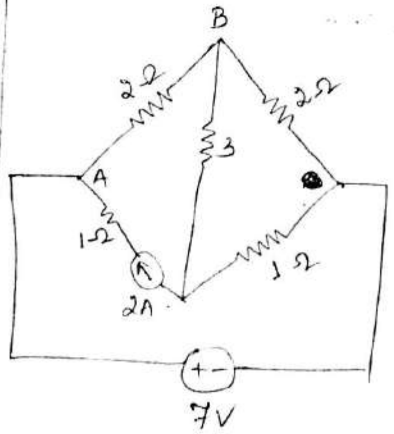
$15(0.25) - 15I_x - 10 = 0$

$I_x = -0.4166A.$

$\therefore\ i = I_y - I_x = 0.25 - (-0.4166)$

$$\boxed{i = 0.666A}$$

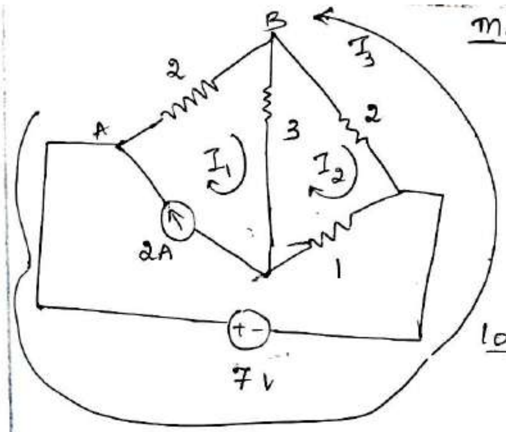
*25. Find current in branch 'AB'



When a n/w. contain a C.S. without any element connected across it KVL cannot be applied directly

Remedies:-

1. Choose loop currents such that only one current flows through C.S
2. By assuming arbitrary V.S across C.S.
3. Super mesh analysis.



Method 1:

loop1 :- $I_1 = 2A$

loop2 :- $-3[I_2 - I_1] - 2[I_2 - I_3] - I_2 = 0$

$$6I_2 - 2I_3 = 6 \rightarrow (1)$$

$$6I_2 + 2I_3 = 6$$

loop3 :-

$$-7 - 2[I_3 - I_2] - 2[I_3 - I_1] = 0$$

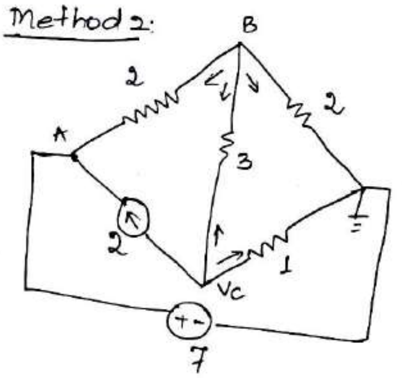
$$-2I_2 + 4I_3 = -3 \rightarrow (2)$$

$$2I_2 + 4I_3 = 3$$

$\therefore\ I_2 = 0.9A,\ I_3 = 0.3A$

$$I_{AB} = I_1 + I_3 = 2 + 0.3 = 2.3\,A$$

Method 2:



Node A :- $V_A = 7V$

Node B :- $\frac{V_B - V_A}{2} + \frac{V_B}{2} + \frac{V_B - V_C}{3} = 0$

$$8V_B - 2V_C = 21 \rightarrow (1)$$

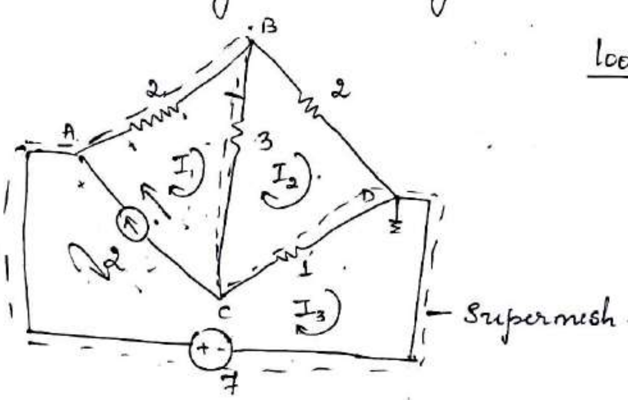
Node C :- $2 + \frac{V_C - V_B}{3} + \frac{V_C}{1} = 0$

$$-V_B + 4V_C = -6 \rightarrow (2)$$

$\therefore\ V_B = 2.4V;\ V_C = -0.9V$

$\therefore\ i_{AB} = \frac{V_A - V_B}{2} = \frac{7 - 2.4}{2} = 2.3A$

Method 3 :- By assuming arbitrary v.s.



loop2 : $-I_2 - 1(I_2 - I_3) - 3(I_2 - I_1) = 0$

$$-3I_1 + 6I_2 - I_3 = 0 \rightarrow (1)$$

$$I_1 - I_3 = 2 \rightarrow (2)$$

Let an arbitrary vtg across 2A be V_{AC}

loop1:- $-2I_1 - 3(I_1 - I_2) + V_{AC} = 0 \longrightarrow (3)$

loop3:- $-1(I_3 - I_2) + 7 - V_{AC} = 0 \longrightarrow (4)$

(3) + (4)

$-2I_1 - 3I_1 + 3I_2 + \cancel{V_{AC}} - I_3 + I_2 + 7 - \cancel{V_{AC}} = 0$

$$5I_1 - 4I_2 + I_3 = 7 \longrightarrow (5)$$

Solving (1), (2) & (5)

$I_1 = 2.3A$

$I_2 = 1.2A$

$I_3 = 0.3A$

Method 4: SUPER MESH

Supermesh is superposition of two or more meshes to avoid C.S.

∴ Mesh1:- 2Ω, 3Ω, 2A

Mesh3:- 2A, 1Ω, 7V

Super mesh contains all the elements in mesh(1) & mesh (3) except '2A' C.S.

∴ Super mesh eqn. is,

$-2I_1 - 3(I_1 - I_2) - 1(I_3 - I_2) + 7 = 0$

$5I_1 - 4I_2 + I_3 = 7 \longrightarrow (3)$

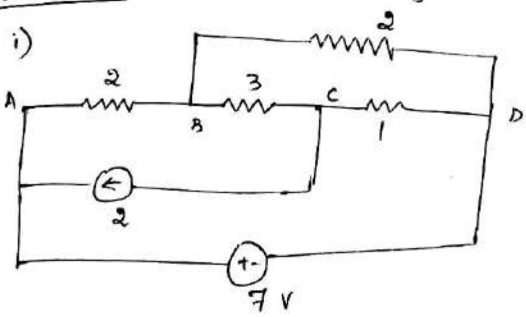
loop2: $-3I_1 + 6I_2 - I_3 = 0 \longrightarrow (1)$

$I_1 - I_3 = 2 \longrightarrow (2)$

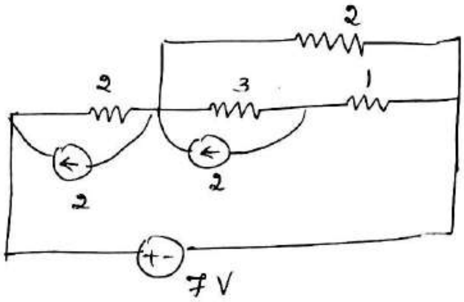
∴ $I_1 = 2.3A$, $I_2 = 1.2A$, $I_3 = 0.3A$

Method 5 : Source shift.

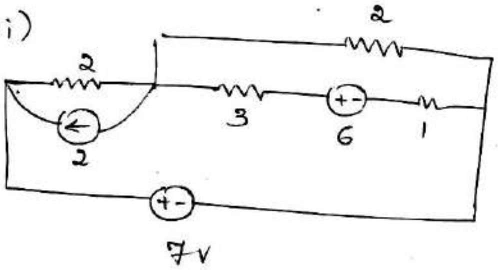
i)



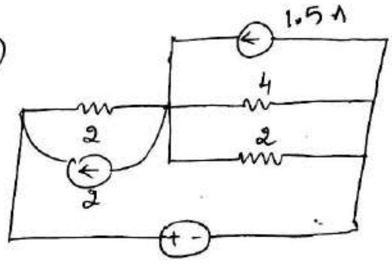
ii)



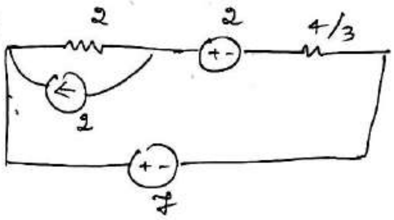
iii)



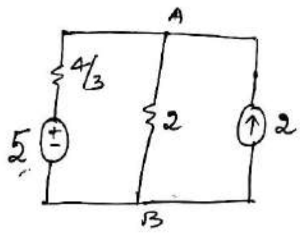
iv)



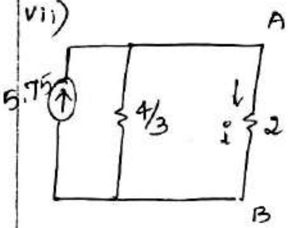
v)



vi)



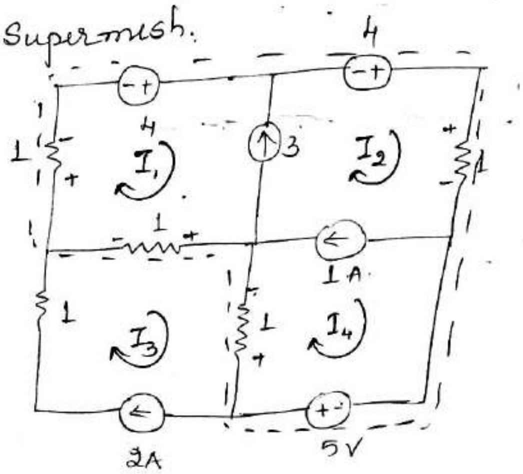
vii)



$$\therefore\ i = I_{AB} = \frac{4/3 \times 5.75}{4/3 + 2}$$

$$\boxed{i = 2.3A}$$

26. Supermesh.



$\Rightarrow$ Sol^n:

$I_3 = 2A$.

$I_2 - I_1 = 3 \rightarrow (1)$

$I_2 - I_4 = 1 \rightarrow (2)$

Mesh 1: $1\Omega, 1\Omega, 4V, 3A$

Mesh2: $4V, 1\Omega, \cancel{1A}, \cancel{3A}$

Mesh4: $\cancel{1A}, 5V, 1\Omega$

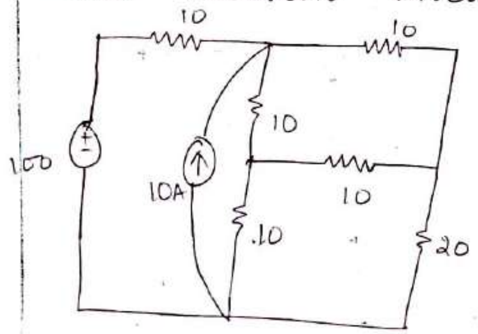
Supermesh eqn. is ,

$$-I_1 + 4 + 4 - I_2 + 5 - [I_4 - I_3] - 1[I_1 - I_3] = 0$$

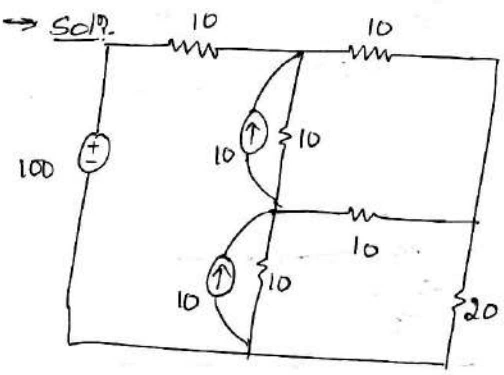
$$\therefore \quad 2I_1 + I_2 + I_4 = 17 \longrightarrow (3)$$

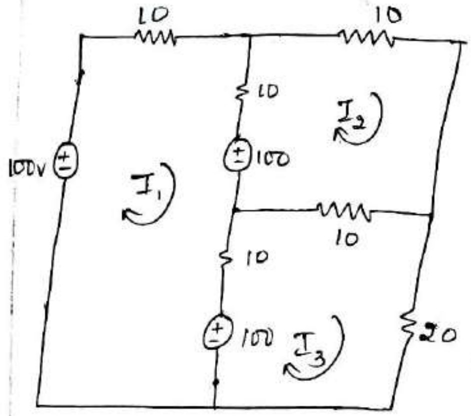
$$\therefore \quad \boxed{I_1 = 3A;\ I_2 = 6A;\ I_3 = 2A;\ I_4 = 5A}$$

27. Find current through 20Ω



→ Soln.





loop1 :-

$$100 - 10I_1 - 10[I_1 - I_2] - 100 - 10[I_1 - I_3] - 100 = 0$$

$$30I_1 - 10I_2 - 10I_3 = -100 \rightarrow (1)$$

loop2 :-

$$100 - 10[I_2 - I_1] - 10I_2 - 10[I_2 - I_3] = 0$$

$$-10I_1 + 30I_2 - 10I_3 = 100 \longrightarrow (2)$$

loop3:-

$$100 - 10[I_3 - I_1] - 10[I_3 - I_2] - 20I_3 = 0.$$

$$-10I_1 - 10I_2 + 40I_3 = 100 \longrightarrow (3)$$

$\therefore I_1 = -0.833$

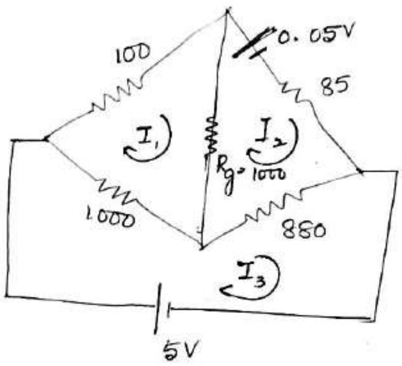
$I_2 = 4.16$

$I_3 = 3.33A$

$$V_{20\Omega} = 20I_3 = 66.6V$$

$$\therefore I_{20\Omega} = I_3 = 3.33A$$

28 Determine the current through galvanometer of the bridge n/w, assume galvanometer resistance $R_g = 1000\Omega$, use loop analysis



→ Soln:

loop1:

$$2100I_1 - 1000I_2 - 1000I_3 = 0 \rightarrow (1)$$

Loop2:

$$-1000I_1 + 1965I_2 - 880I_3 = -0.05 \rightarrow (2)$$

Loop3:

$$-1000I_1 - 880I_2 + 1880I_3 = +5 \rightarrow (3)$$

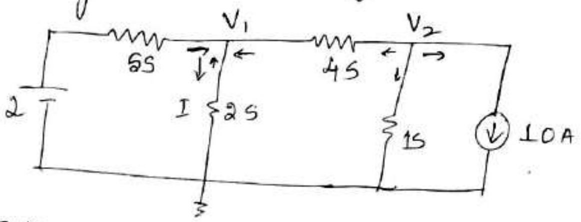
$\therefore$ $I_1 = 0.02675187A$

$I_2 = 0.026762505A$

$I_3 = 0.029416422A$

$\therefore I_g = I_1 - I_2 = -10.635\mu A$

29 Find the current in conductance of '2S' in n/w using node analysis.



Conductance, $G = \frac{1}{R}$. siemens/mho

→ Soln:

$$i = \frac{V_1 - V_2}{R} = (V_1 - V_2)G$$

Node1: $(2 - V_1)5 + (0 - V_1)2 + (V_2 - V_1)4 = 0$.

$$11V_1 - 4V_2 = 10 \rightarrow (1)$$

Node2: $(V_2 - V_1)4 + (V_2)1 + 10 = 0$

$$-4V_1 + 5V_2 = -10 \rightarrow (2)$$

$\therefore V_1 = 0.2564V$; $V_2 = -1.79V$.

$\therefore I_{2S} = 2V_1 = 2 \times 0.2564$

$\boxed{I_{2S} = 0.5128A}$

* **Supernode analysis :-**

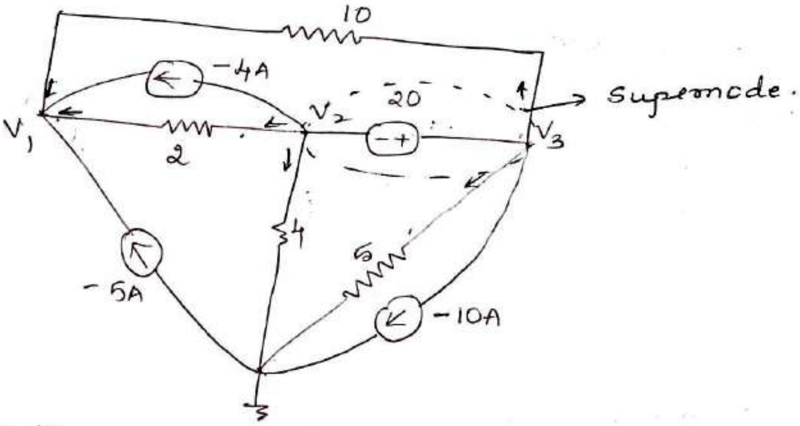
When a n/w. contains a V.S. without a element connected in series with it, KCL cannot be applied directly.

Remedies:

* By assuming an arbitrary current through V.S.
* Super node analysis

A super node analysis is superposition of two / more nodes to avoid V.S.

30. Find V_1, V_2, V_3



⇒ **Soln:-**

Node 1: $-4-5+\frac{V_2-V_1}{2}+\frac{V_3-V_1}{10}=0$

$-0.6V_1+0.5V_2+0.1V_3=9 \longrightarrow (1)$

$V_3-V_2=20 \longrightarrow (2)$

Node 2: $2\cancel{0}V, 4\Omega, 2\Omega, -4A$

Node 3: $10\Omega, 20\cancel{V}, 5\Omega, -10A$

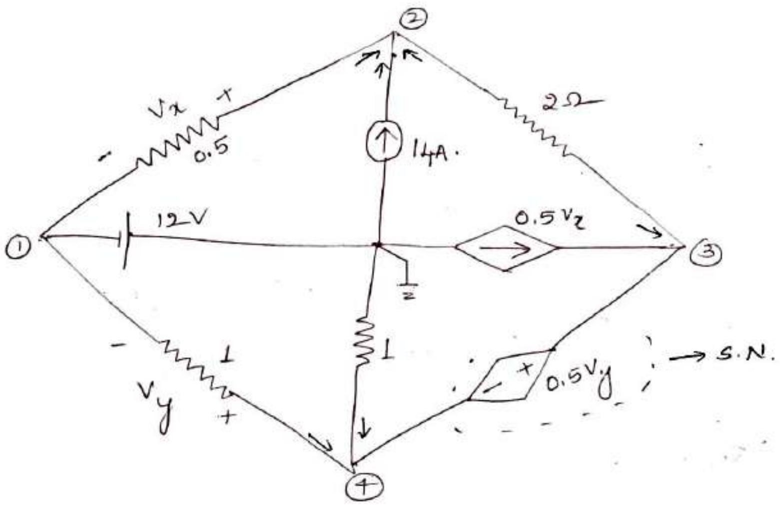
∴ Supernode eqn. is,

$$-4+\frac{V_2-V_1}{2}+\frac{V_2}{4}+\frac{V_3}{5}-10+\frac{V_3-V_1}{10}=0$$

$$-0.6V_1+0.75V_2+0.3V_3=14 \longrightarrow (3)$$

$$\therefore \boxed{V_1=-9.44V\ ;\ V_2=2.22V;\ V_3=22.22V}$$

#31



→ Sol^n:

Node 1: $V_1=-12V$.

Node 2: $14+\dfrac{V_1-V_2}{0.5}+\dfrac{V_3-V_2}{2}=0$

$$-2.5V_2+0.5V_3=10 \longrightarrow (1)$$

$$V_3-V_4=0.5V_y$$

But $V_y=V_4-V_1=V_4+12$.

$$\therefore V_3-V_4=0.5\left[V_4+12\right]$$

$$V_3-1.5V_4=6 \longrightarrow (2)$$

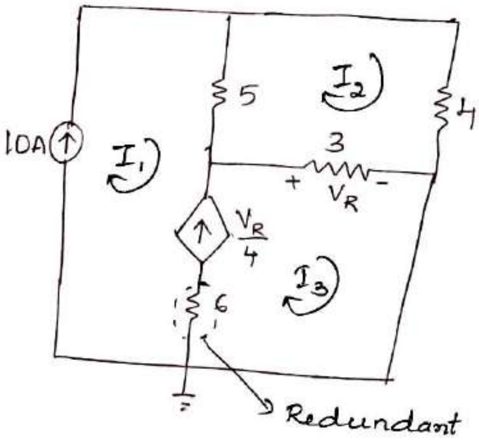
Supernode eq^n is,

$\dfrac{V_2-V_3}{2}+0.5V_x+\dfrac{0-V_4}{1}+\dfrac{V_1-V_4}{1}=0.$; put $V_x=V_2-V_1$ &
$V_1=-12V$

$$V_2-0.5V_3+2V_4=6 \longrightarrow (3)$$

$$\therefore \quad \boxed{\begin{aligned} V_1 &= -12\,V \\ V_2 &= -4.24\,V \\ V_3 &= -1.224\,V \\ V_4 &= -4.8\,V \end{aligned}}$$

32. Find I_1, I_2 & I_3



$\Longrightarrow$ <u>soln</u>:- $I_1 = 10\,A$

<u>loop2</u>:-

$$-5[I_2 - I_1] - 4I_2 - 3[I_2 - I_3] = 0$$

$$12I_2 - 3I_3 = 50 \rightarrow (1)$$

<u>loop3</u>:-

$$I_3 - I_1 = \frac{V_R}{4}$$

But $V_R = 3(I_3 - I_2)$

$$\therefore 4(I_3 - I_1) = 3(I_3 - I_2)$$

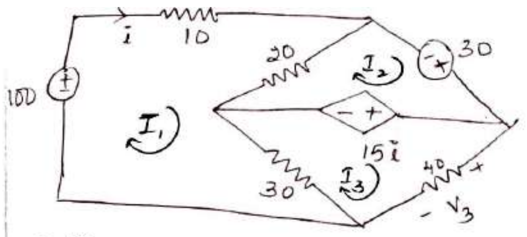
$$4I_3 - 3I_3 - 4I_1 + 3I_2 = 0$$

$$-4I_1 + 3I_2 + I_3 = 0$$

$$3I_2 + I_3 = 40 \rightarrow (2)$$

$$\therefore \quad \boxed{\begin{aligned} I_1 &= 10\,A \\ I_2 &= 8.095\,A \\ I_3 &= 15.714\,A \end{aligned}}$$

33. Use mesh analysis to find 'V_3'



→ Soln:-

Loop1:- $100 - 10I_1 - 20[I_1 - I_2] - 30[I_1 - I_3] = 0$

$$60I_1 - 20I_2 - 30I_3 = 100 \longrightarrow (1)$$

Loop2:- $-20[I_2 - I_1] + 30 - 15i = 0$

But $i = I_1$

$$-20I_2 + 20I_1 + 30 - 15I_1 = 0$$

$$5I_1 - 20I_2 = -30$$

$$-5I_1 + 20I_2 = 30 \longrightarrow (2)$$

Loop3:- $-30[I_3 - I_1] - 40I_3 + 15I_1 = 0$

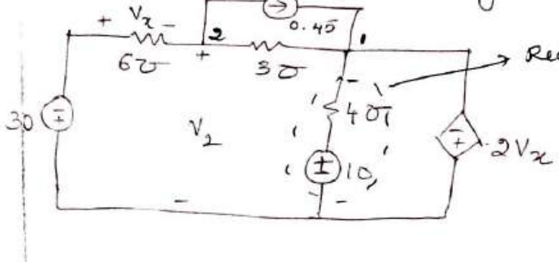
$$45I_1 - 70I_3 = 0 \longrightarrow (3)$$

$\therefore I_1 = 3.64A,\ I_2 = 2.41A,\ I_3 = 2.34V.$

$\therefore V_3 = 40I_3 = 40 \times 2.34$

$$\boxed{V_3 = 93.6V}$$

34. Find V_2 & V_x using node analysis



Redundant 4Ω is in series with 10V is redundant in terms of vtg. ∴ this element can be neglected.

Node 2 :- $0.45 + (V_2 - V_1)3 + (V_2 + 30)6 = 0$.

$$0.45 + 3V_2 - 3V_1 + 6V_2 + 180 = 0$$

$$-3V_1 + 9V_2 = -180.45$$

$$-3(-2V_x) + 9V_2 = -180.45$$

$$6V_x + 9V_2 = -180.45 \longrightarrow (1)$$

But $V_1 = -2V_x$

& $V_x = -30 - V_2$

∴ put in (1)

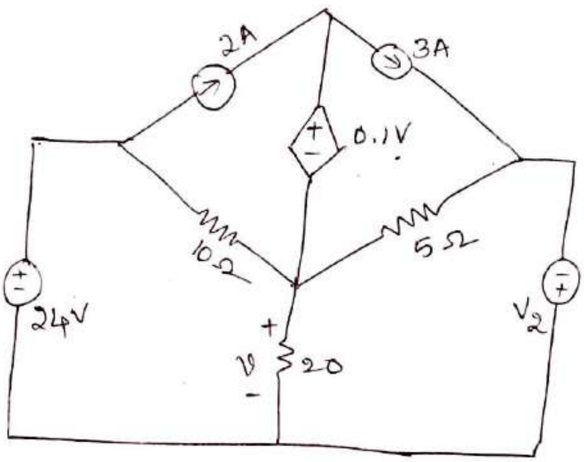
$$6[-30 - V_2] + 9V_2 = -180.45$$

$$\therefore \boxed{V_2 = -0.15\,V}$$

& $V_x = -30 - V_2 = -30 + 0.15$

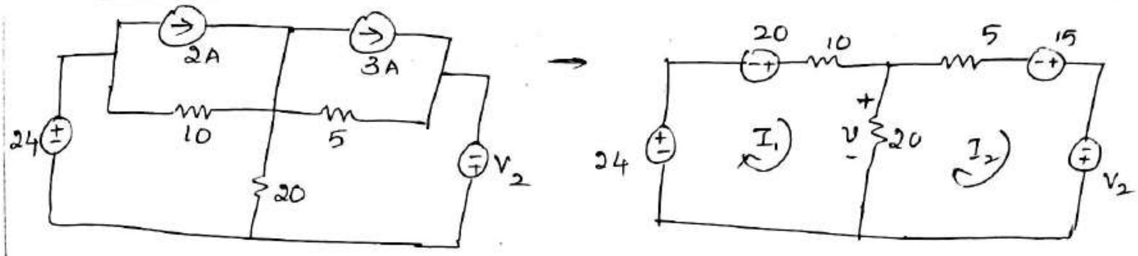
$$\boxed{V_x = -29.85\,V}$$

35. Use mesh analysis to find what value of 'V_2' in the n/w. causes $v = 0$, where 'v' is voltage across 20Ω.



Since it is necessary to find 'V_2' to make $v = 0$, dependent v.s. 0.1V becomes '0' [SC].

The two c.s. can be converted into v.s.



$$v = 0 = 20\left[I_1 - I_2\right]$$

$$I_1 - I_2 = 0 \; ; \; I_1 = I_2 .$$

loop1 :- $24 + 20 - 10I_1 - \cancel{20\left[I_1 - I_2\right]}^{\,0} = 0$

$$44 = 10 I_1$$

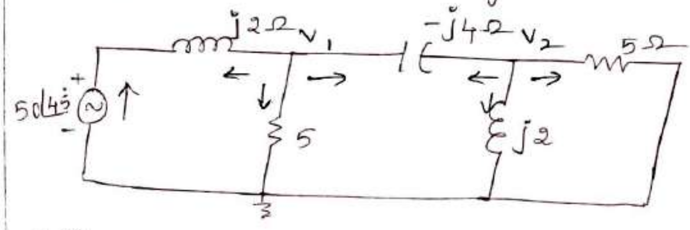
$$I_1 = 4.4A = I_2$$

loop2 :- $-5I_2 + 15 + v_2 - \cancel{v}^{\,0} = 0.$

$$-5(4.4) + 15 + v_2 = 0$$

$$\therefore \boxed{V_2 = 7V}$$

36 Find current through '5Ω' using node analysis



$j = \sqrt{-1}$; $j^2 = -1$

$j1 = 1\angle 90°$

$-j1 = 1\angle -90°$

$\frac{1}{j} = -j$; $\frac{1}{-j} = j$.

Soln :-

Node1 :- $\dfrac{V_1 - 50\angle 45°}{j2} + \dfrac{V_1}{5} + \dfrac{V_1 - V_2}{-j4} = 0$

$$(0.2 - j0.25)V_1 - j0.25V_2 = 25\angle -45° \longrightarrow (1)$$

Node 2 :- $\frac{V_2 - V_1}{-j4} + \frac{V_2}{j2} + \frac{V_2}{5} = 0$

$-j0.25V_1 + (0.2 - j0.25V_2) = 0$

$$\begin{bmatrix} (0.2-j0.25) & -j0.25 \\ -j0.25 & (0.2-j0.25) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 25\angle{-45} \\ 0 \end{bmatrix}$$

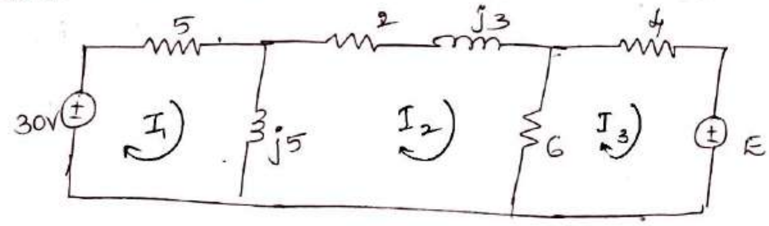
$$I_{5\Omega} = \frac{V_1}{5}$$

$$V_1 = \frac{\begin{vmatrix} 25\angle{-45^\circ} & -j0.25 \\ 0 & (0.2-j0.25) \end{vmatrix}}{\Delta}$$

$$V_1 = \frac{(25\angle{-45^\circ})(0.2-j0.25)}{(0.2-j0.25)^2 + (0.25)^2} = 74.31\angle{-28.1}$$
$$= (65.5 - j35.03)\,V$$

$$\therefore \boxed{I_{5\Omega} = (13.1 - 7j)A}$$

$\overset{*}{37}$ Use mesh analysis to determine current in $(2+j3)\,\Omega$ if $E = 20V$. Calculate 'E' to make the current in $(2+j3)\Omega = 0$



→ Sol^n:- i) $E = 20V$

$$\begin{bmatrix} (5+j5) & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -20 \end{bmatrix}$$

$$I_2 = \frac{(5+j5)(-120) - 30\ (-j50)}{(5+j5)\left[(80+j80)-36\right] + j5(-j50)} = \frac{-600+900j}{70+620j}$$

$$I_2 = 1.733 \underline{|40.13}\ A = 1.32 + j1.117$$

ii) $\underline{I_2 = 0}$

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -E_2 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 5+j5 & 30 & 0 \\ -j5 & 0 & -6 \\ 0 & -E_2 & 10 \end{vmatrix}}{\Delta} = 0.$$

$$(5+j5)(-6E) - 30\left[-10(5j)\right] = 0$$

$$\boxed{E = 35.35 \underline{|45^\circ}\ V}$$

38. The vtg of a node of a n/w is given by

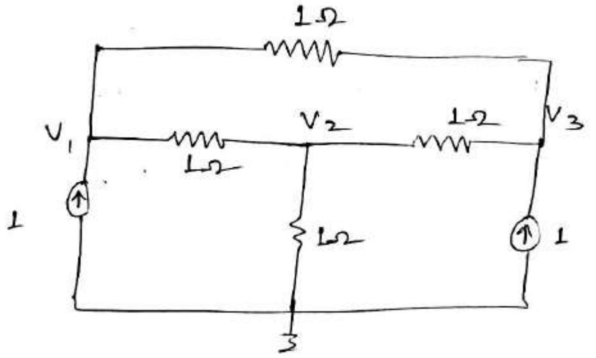
$$V_2 = \frac{\begin{vmatrix} 2 & 1 & -1 \\ -1 & 0 & -1 \\ -1 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{vmatrix}}$$

Draw the electrical n/w.

<u>Hint</u>:
If vtg is given :- Node analysis

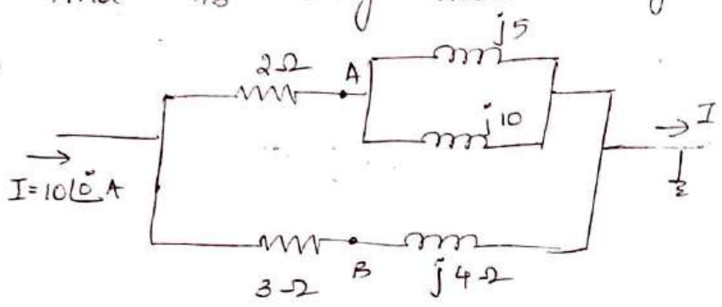
→ Soln:- $[Y][V] = [I]$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$



39. Find V_{AB} using node analysis.

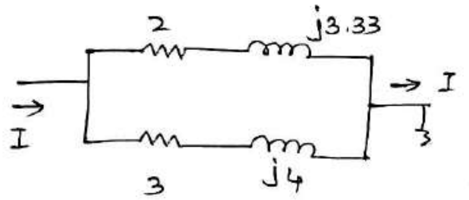
May-June-10



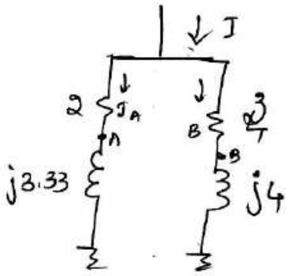
$V_{AB} = ?$

→ Soln:- $V_{AB} = V_A - V_B$.

$$j5 \| j10 = \frac{j5 \times j10}{j15} = \frac{-50}{j15} = j3.33\Omega$$



(or)

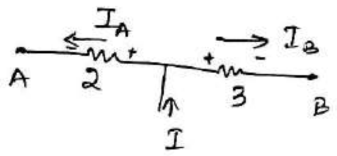


$$I_A = \frac{(3+4j) \times 10\angle 0°}{5+7.33j}$$

$I_A = 5.635 \angle -2.57 = (5.62 - 0.25j)$ A

$$I_B = \frac{(2+j3.33) \times 10\angle 0°}{5+7.33j}$$

$I_B = 4.377 \angle 13.309°$ A

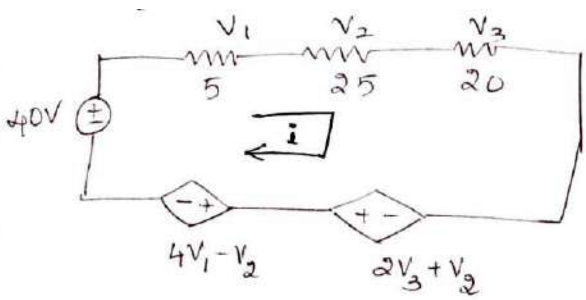


$V_{AB} = 3I_B - 2I_A$.

$\hookrightarrow = 2.2562 \angle 34.264°$ V

$\hookrightarrow = 1.8647 + j1.2703$ V

40 Find power across 5Ω, 20Ω, 25Ω using KVL



$$40 - 5i - 25i - 20i + 2V_3 + V_2 - 4V_1 + V_2 = 0$$

$$40 - 50i + 2V_3 + 2V_2 - 4V_1 = 0$$

But $V_1 = 5i$, $V_2 = 25i$, $V_3 = 20i$

$$40 - 50i + 2(5i) + 2(25i) - 4(20i) = 0$$

$$\therefore \quad i = \frac{-40}{26} = -2A$$

$$P_{5\Omega} = V_1 i = 5i \times i = 20W$$

$$P_{25\Omega} = V_2 i = 25i \times i = 100W$$

$$P_{20\Omega} = V_3 i = 20i \times i = 80W$$

$$P_{(2V_3+V_2)} = (2V_3 + V_2)\, i = [40i + 25i]\, i = 220W.$$

$$P[4V_1 - V_2) = (4V_1 - V_2)\, i = -20W.$$

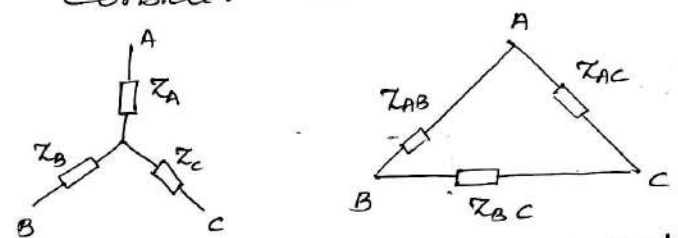
* STAR- DELTA TRANSFORMATION :-

Y T $\rightarrow$ star, Y; T n/w.

$\triangle$ π ∇ $\rightarrow$ delta, π n/w.

When a n/w. contains large no of loops (or) nodes, application of KVL & KCL becomes complicated. In such cases it is possible to simplify the n/w. by using star-Δ transform.

Consider the star & delta n/ws given below.



For two n/ws to be equivalent, they should have same i/p, o/p & transfer impedances. The impedance b/w 'A' & 'B' are:

$$Z_A + Z_B = Z_{AB} \parallel (Z_{BC} + Z_{AC})$$

$$\hookrightarrow = \frac{Z_{AB}(Z_{BC} + Z_{AC})}{Z_{AB} + Z_{BC} + Z_{AC}} = \frac{Z_{AB}Z_{BC} + Z_{AB}Z_{AC}}{\Sigma Z_{AB}} \rightarrow (1)$$

Impedance b/w 'B' & 'C'.

$$Z_B + Z_C = Z_{BC} \parallel (Z_{AC} + Z_{AB}) = \frac{Z_{BC}(Z_{AC} + Z_{AB})}{\Sigma Z_{AB}}$$

$$\hookrightarrow = \frac{Z_{AC}Z_{BC} + Z_{AB}Z_{BC}}{\Sigma Z_{AB}} \rightarrow (2)$$

$$Z_C + Z_A = Z_{AC} \parallel (Z_{BC} + Z_{AB}) = \frac{Z_{AC}Z_{BC} + Z_{AC}Z_{AB}}{\Sigma Z_{AB}} \rightarrow (3)$$

(i) Delta - star conversion :-

(1) - (2)

$$Z_A + \cancel{Z_B} - \cancel{Z_B} - Z_C = \frac{Z_{AB}Z_{AC} + \cancel{Z_{AB}Z_{BC}} - \cancel{Z_{AB}Z_{BC}} - Z_{BC}Z_{AC}}{\Sigma Z_{AB}}$$

$$Z_A - Z_C = \frac{Z_{AB}Z_{AC} - Z_{BC}Z_{AC}}{\Sigma Z_{AB}} \longrightarrow (4)$$

(3) + (4)

$$Z_A + \cancel{Z_C} + Z_A - \cancel{Z_C} = \frac{Z_{AC}Z_{AB} + \cancel{Z_{AC}Z_{BC}} + Z_{AB}Z_{AC} - \cancel{Z_{BC}Z_{AC}}}{\Sigma Z_{AB}}$$

$$2Z_A = \frac{2Z_{AB}Z_{AC}}{\Sigma Z_{AB}}$$

$$\therefore \boxed{Z_A = \frac{Z_{AB}Z_{AC}}{\Sigma Z_{AB}}} \longrightarrow (5)$$

$$Z_B = \frac{Z_{AB}Z_{BC}}{\Sigma Z_{AB}} \longrightarrow (6)$$

$$Z_C = \frac{Z_{AC}Z_{BC}}{\Sigma Z_{AB}} \longrightarrow (7)$$

Eqns. (5), (6) & (7) are used for Δ-Y conversion

ii) star - delta conversion :-

(5) × (6)

$$Z_A Z_B = \frac{Z_{AB}^2 Z_{BC} Z_{AC}}{\Sigma^2 Z_{AB}} \longrightarrow (8)$$

(6) × (7)

$$Z_B Z_C = \frac{Z_{BC}^2 Z_{AB} Z_{AC}}{\Sigma^2 Z_{AB}} \longrightarrow (9)$$

(5) × (7)

$$Z_A \cdot Z_C = \frac{Z_{AB}\ Z_{BC}\ Z_{AC}^2}{\Sigma Z_{AB}^2} \rightarrow (10)$$

(8) + (9) + (10)

$$Z_A Z_B + Z_B Z_C + Z_A Z_C = \frac{Z_{AB}\ Z_{BC}\ Z_{AC}(Z_{AB} + Z_{BC} + Z_{AC})}{\Sigma Z_{AB}^2}$$

$$\hookrightarrow = \frac{Z_{AB} Z_{BC} Z_{AC}}{\Sigma Z_{AB}}$$

substitute eqn. (7)

$$Z_A Z_B + Z_B Z_C + Z_A Z_C = Z_{AB} Z_C$$

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C}$$

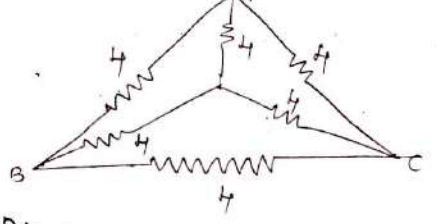
$$\therefore\ Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_C} \rightarrow (11)$$

$$Z_{BC} = Z_B + Z_C + \frac{Z_B Z_C}{Z_A} \rightarrow (12)$$

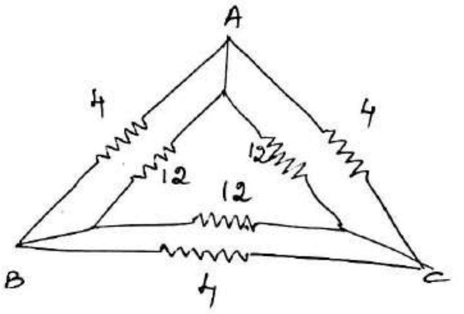
$$Z_{AC} = Z_A + Z_C + \frac{Z_A Z_C}{Z_B} \rightarrow (13)$$

(11), (12) & (13) are used for Y to Δ

42. Six equal resistors of 4 Ω are connected as shown in fig. Find equivalent R b/w any two nodes.



* NOTE: Start simplification from innermost part of the n/w.



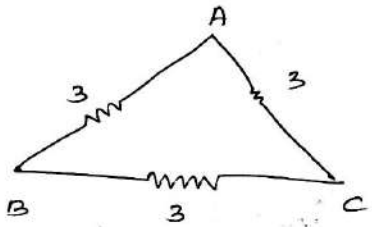
Soln:- Y - Δ.

$$R_{AB} = R_{BC} = R_{AC} = R_A + R_B + \frac{R_A R_B}{R_C}$$

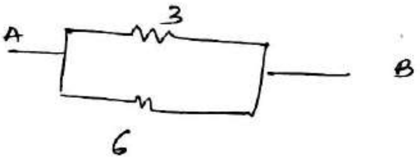
$$= 4 + 4 + \frac{16}{4}$$

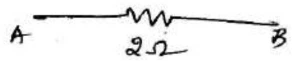
$$\hookrightarrow = 12\,\Omega$$

$$12 \| 4 = \frac{12 \times 4}{16} = 3\,\Omega$$

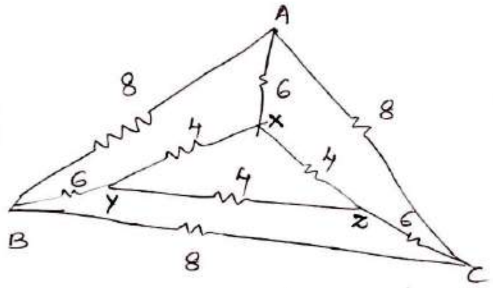


$$R_{AB} = 3 \| 6 = \frac{3 \times 6}{9} = 2\,\Omega$$





43.

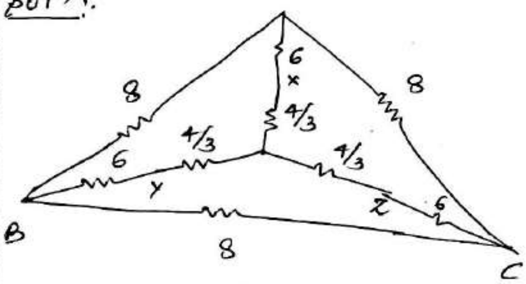


$\underline{\Delta \text{ to } Y}$:

$R_x = R_y = R_z$.

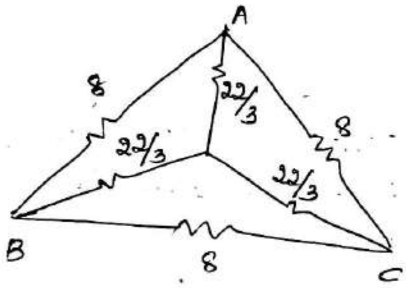
$$\therefore R_x = \frac{R_{xy} R_{xz}}{\Sigma R_{xz}}$$

→ $\underline{\text{Soln}}$:



$$\hookrightarrow = \frac{4 \times 4}{12} = 4/3$$

$$\frac{4}{3} + 6 = \frac{22}{3}$$



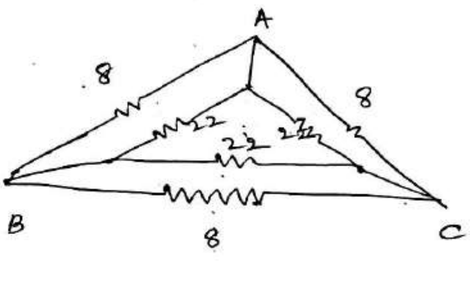
$\underline{Y \text{ to } \Delta}$:-

$R_{AB} = R_{BC} = R_{CA}$

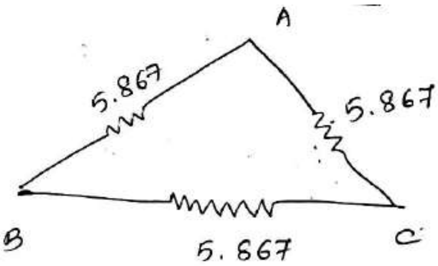
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

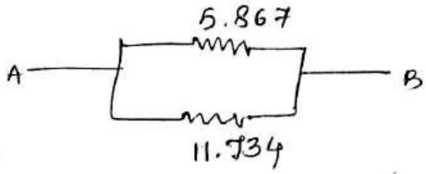
$$= \frac{22}{3} + \frac{22}{3} + \frac{\left(22/3\right)^2}{22/3}$$

$R_{AB} = 22\,\Omega$



$$8 \| 22 = \frac{8 \times 22}{30} = 5.867\,\Omega$$

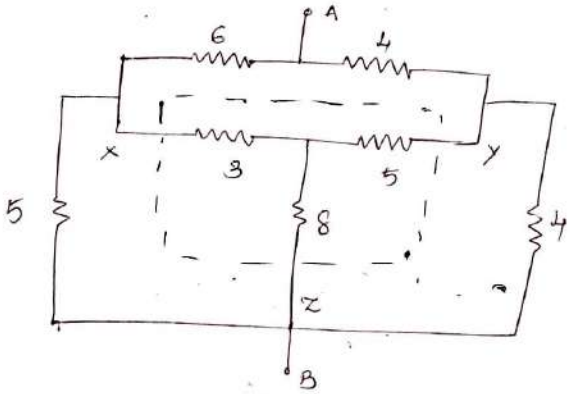




$R_{eq} = R_{AB} = 5.867 \,||\, 11.734$

$\therefore \boxed{R_{AB} = 3.9113\,\Omega}$

44 Determine (R) b/w A & B



$R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$

$\hookrightarrow = 3 + 5 + \frac{15}{8} = 9.875\,\Omega$

$R_{yz} = R_y + R_z + \frac{R_y R_z}{R_x}$

$= 5 + 8 + \frac{40}{3}$

$\hookrightarrow = 26.33\,\Omega$

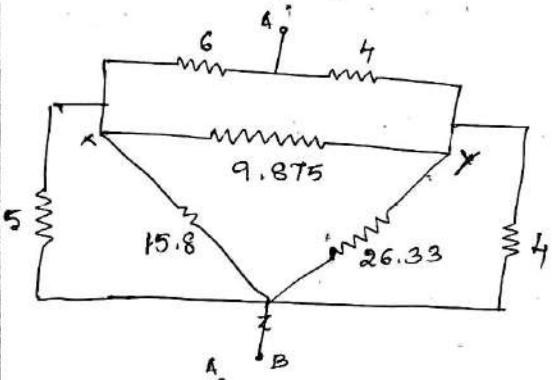
$R_{xz} = R_x + R_z + \frac{R_x R_z}{R_y}$

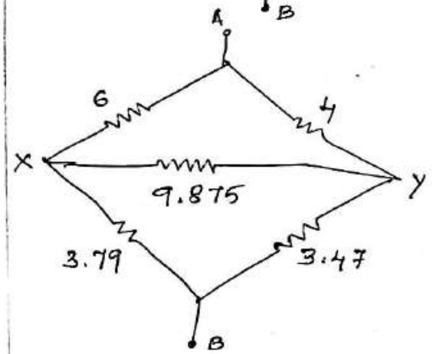
$= 3 + 8 + \frac{24}{5}$

$\hookrightarrow = 15.8\,\Omega$

$5 \,||\, 15.8 = 3.79\,\Omega$

$4 \,||\, 26.33 = 3.472\,\Omega$



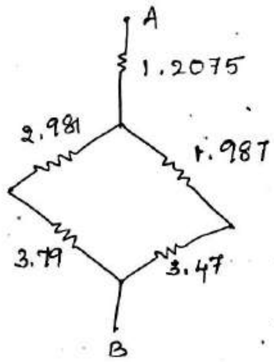


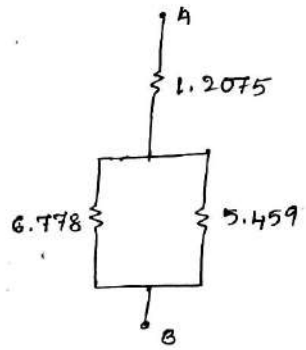
Δ to Y

$R_A = \frac{R_{Ay} R_{Ax}}{\Sigma_{RAx}} = \frac{6 \cdot 4}{19.875} = 1.2075\,\Omega$

$R_x = \frac{R_{xy} \cdot R_{Ax}}{\Sigma_{RAx}} = \frac{9.875 \cdot 6}{19.875} = 2.981\,\Omega$

$R_y = \frac{R_{xy} \cdot R_{Ay}}{\Sigma_{RAx}} = \frac{9.875 \cdot 4}{19.875} = 1.9874\,\Omega$

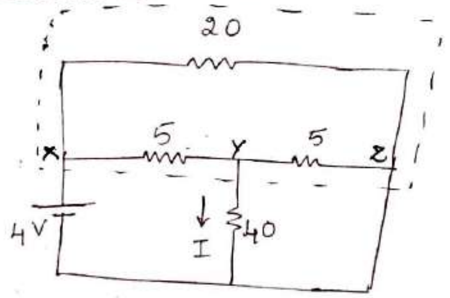


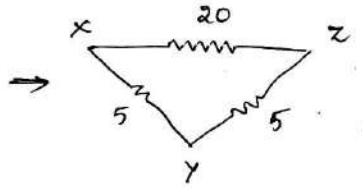


$$R_{AB} = 1.2075 + [6.778 \,||\, 5.459]$$

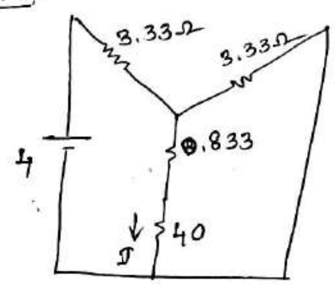
$$\boxed{R_{AB} = 4.231\Omega}$$

45 Calculate current in 40Ω resistor.





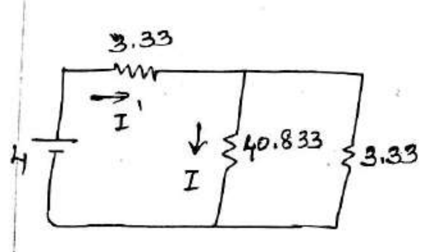
Sol^n:-



$$R_x = \frac{R_{xy} R_{xz}}{\Sigma R_{xy}} = \frac{5 \cdot 20}{30} = 3.33\Omega$$

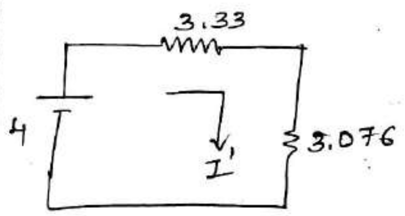
$$R_y = \frac{R_{xy} R_{yz}}{\Sigma R_{xy}} = \frac{5 \times 5}{30} = 0.833\Omega$$

$$R_z = \frac{R_{xz} R_{yz}}{\Sigma R_{xy}} = \frac{20 \times 5}{30} = 3.33\Omega$$



$$I = \frac{3.33 \times I'}{40.833 + 3.33} = 0.0754 I'$$

$$40.633 \,||\, 3.33 = 3.076\Omega$$

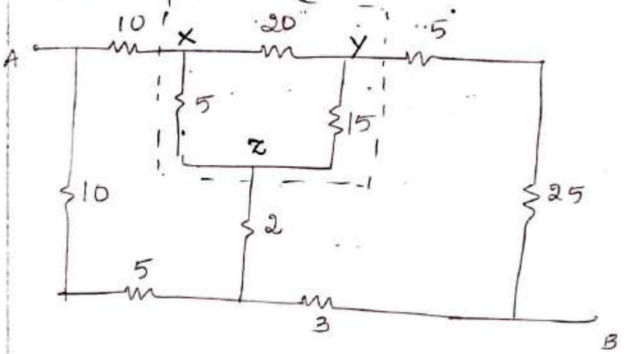


$$I' = \frac{4}{3.33 + 3.076} = 0.624A$$

$$\therefore I = 0.0754 \times 0.624$$

$$\boxed{I = 0.047A}$$

46. Find resistance b/w 'A' & B

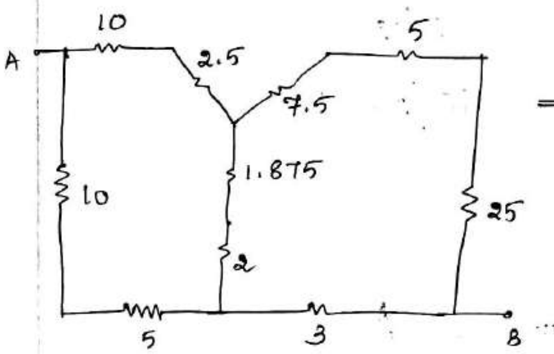


Δ to Y:

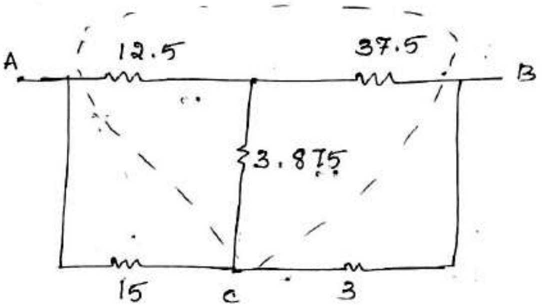
$$R_x = \frac{R_{xy} R_{xz}}{\Sigma R_{xy}} = \frac{5 \cdot 20}{40} = 2.5$$

$$R_y = \frac{R_{xy} R_{yz}}{\Sigma R_{xy}} = \frac{20 \cdot 15}{40} = 7.5$$

$$R_z = \frac{R_{yz} R_{xz}}{\Sigma R_{xy}} = \frac{15 \times 5}{40} = 1.875$$



$\Longrightarrow$



Y to Δ

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$= 12.5 + 37.5 + \frac{12.5 \times 37.5}{3.875}$$

$$\hookrightarrow = 170.96\Omega$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

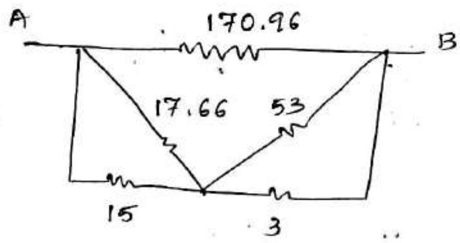
$$= 37.5 + 3.875 + \frac{37.5 \times 3.875}{12.5}$$

$$\hookrightarrow = 53\Omega$$

$$R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$$

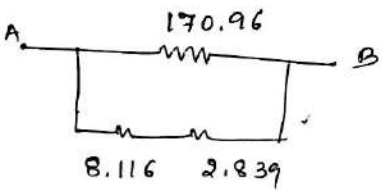
$$= 12.5 + 3.875 + \frac{12.5 \times 3.875}{37.5}$$

$$\hookrightarrow = 17.66\Omega$$



$$15 \| 17.66 = 8.116\Omega$$

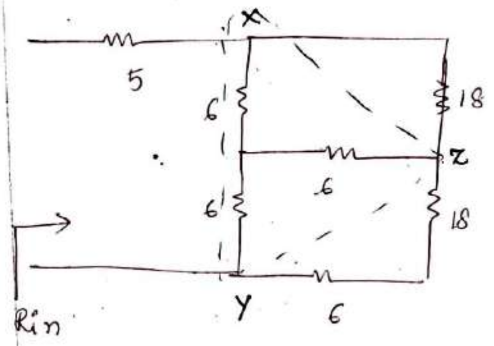
$$53 \| 3 = 2.839\Omega$$



$$\therefore R_{AB} = (8.116 + 2.839) \| 170.96$$

$$\therefore R_{AB} = 10.29\Omega$$

47. Find R_{in}



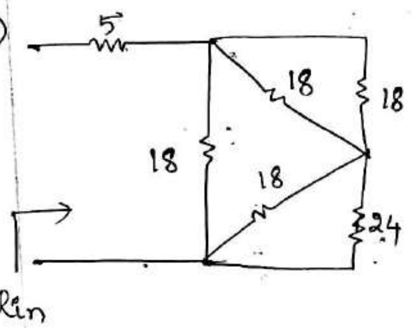
$R_{xy} = R_{yz} = R_{zx}$

$$R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$$

$$= 6 + 6 + \frac{6 \times \cancel{6}}{\cancel{6}}$$

$$\hookrightarrow = 18$$

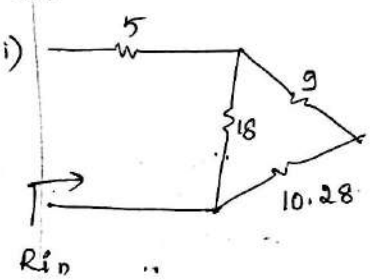
i)



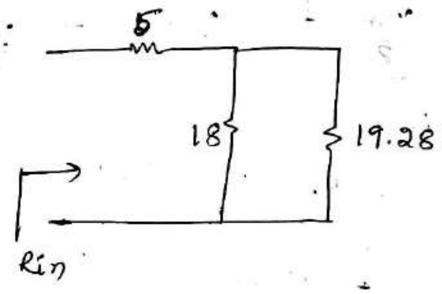
$18 \parallel 18 = 9\,\Omega$

$18 \parallel 24 = 10.28$

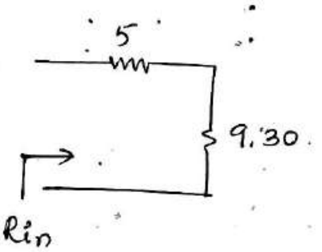
ii)



iii)

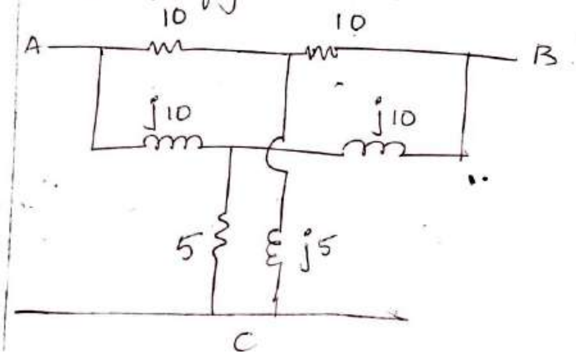


iv)

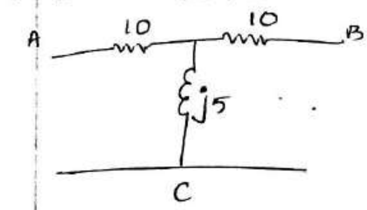


$\therefore$ $\boxed{R_{in} = 14.30\,\Omega}$

※ 48 Simplify the given twin-T n/w.



Soln:- Y − Δ



$$Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_C}$$

$$= 10 + 10 + \frac{100}{j5}$$

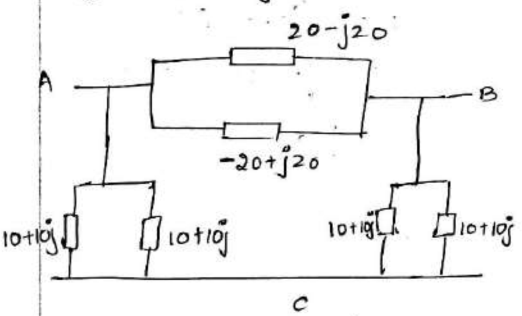
$$\hookrightarrow = 20 - j20$$

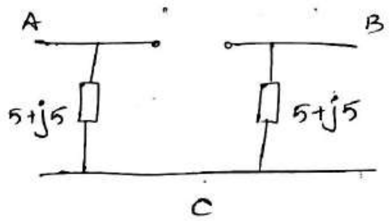
$$Z_{BC} = 10 + j5 + \frac{50j}{10}$$

$$\hookrightarrow = 10 + 10j$$

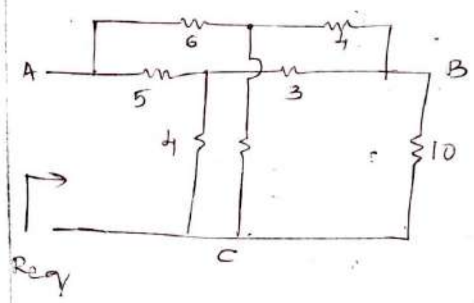
$$Z_{AC} = 10 + j5 + \frac{50j}{10}$$

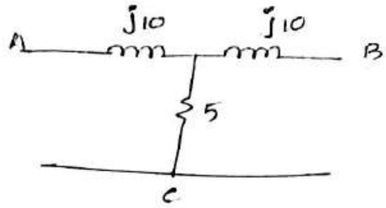
$$\hookrightarrow = 10 + 10j$$





49.





$$Z_{AB} = j10 + j10 + \frac{100j^2}{5}$$

$$\hookrightarrow = 20j - 20$$

$$Z_{BC} = j10 + 5 + \frac{50j}{10j}$$

$$\hookrightarrow = 10 + j10$$

$$Z_{AC} = j10 + 5 + 5$$

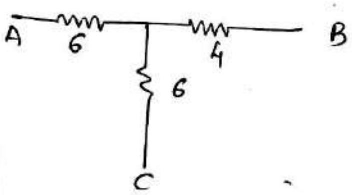
$$\hookrightarrow = 10 + 10j$$

$$(20 - j20) \,||\, (-20 + 20j)$$

$$\frac{(20 - j20) \cdot (-20 + 20j)}{20 - j20 - 20 + j20} = \infty \,[O.C]$$

$$(10 + j10) \,||\, (10 + j10) = (5 + j5)\,\Omega$$

Y - Δ



$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$= 6 + 4 + \frac{24}{6}$$

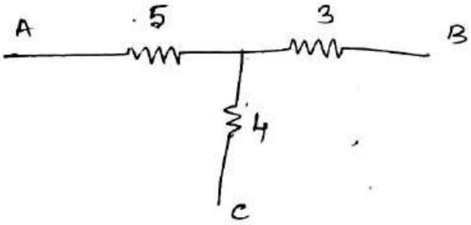
$$\hookrightarrow = 14\,\Omega$$

$$R_{BC} = 4 + 6 + \frac{24}{6}$$

$$\hookrightarrow = 14\,\Omega$$

$$R_{AC} = 6 + 6 + \frac{36}{4}$$

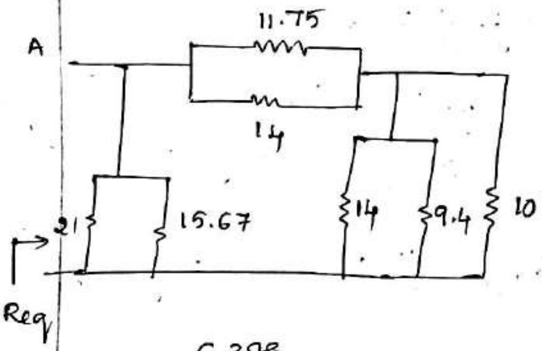
$$\hookrightarrow = 21\,\Omega$$



$$R_{AB} = 5 + 3 + \frac{15}{4} = 11.75$$

$$R_{BC} = 3 + 4 + \frac{12}{5} = 9.4\,\Omega$$

$$R_{CA} = 5 + 4 + \frac{20}{3} = 15.67\,\Omega$$

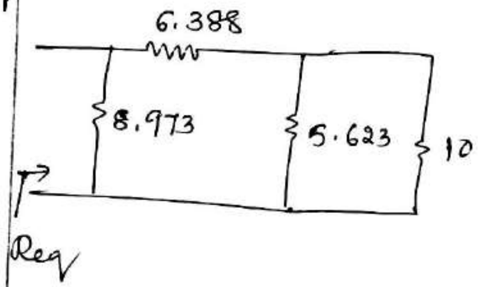


$$11.75 \parallel 14 = 6.388\,\Omega$$

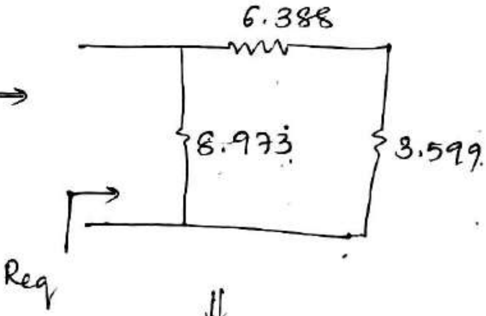
$$21 \parallel 15.67 = 8.973\,\Omega$$

$$14 \parallel 9.4 = 5.623\,\Omega$$

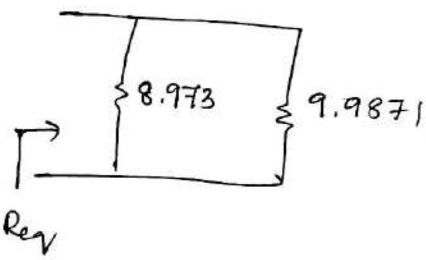
8.9



⟹



⇓



$$\therefore R_{eq} = \frac{89.6133}{18.96}$$

$$\boxed{R_{eq} = 4.72\,\Omega}$$

MODULE 2

NETWORK THEOREMS

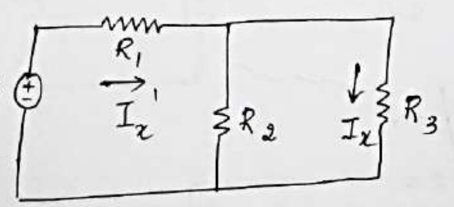
Mesh & nodal analysis are general analysis which can be used for detailed analysis of a given n/w. In many application it may be necessary to find current & voltage in particular branch, in such cases n/w. theorems are used.

1. RECIPROCITY THEOREM :-

This theorem is applicable only to single src n/w.

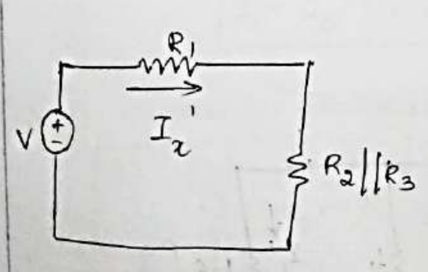
Statement :- In a single src linear bilateral n/w the ratio of response to excitation remains the same when the positions of src. & response are interchanged.

Proof :- consider the linear n/w. shown below. Let the response be I_x



$$I_x = \frac{R_2 \times I_x'}{R_2 + R_3}$$

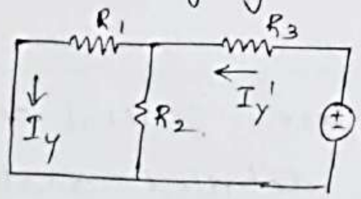
But, $$I_x' = \frac{V}{R_1 + (R_2 || R_3)}$$



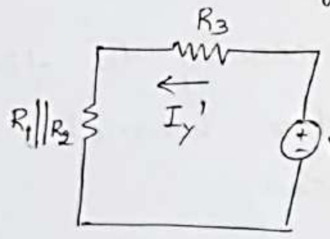
$$\therefore I_x = \frac{R_2}{R_2+R_3} \cdot \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$\hookrightarrow = \frac{R_2 V}{R_1 R_2 + R_1 R_3 + R_2 R_3} \rightarrow (1)$$

Interchanging the positions of src & response



* NOTE:-
Independent V.S. should be replaced by short ckt. & C.S. by open ckt.



$$I_y = \frac{R_2}{R_1+R_2} \cdot I_y'$$

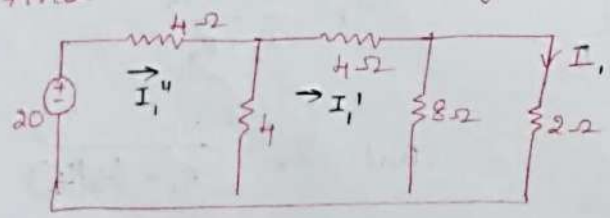
But $$I_y' = \frac{V}{R_3 + (R_1||R_2)}$$

$$\therefore I_y = \frac{R_2}{R_1+R_2} \cdot \frac{V}{R_3 + \frac{R_1R_2}{R_1+R_2}}$$

$$I_y = \frac{R_2 V}{R_1R_2 + R_1R_3 + R_2R_3} \longrightarrow (2)$$

$\therefore$ From (1) & (2) $\boxed{I_x = I_y}$

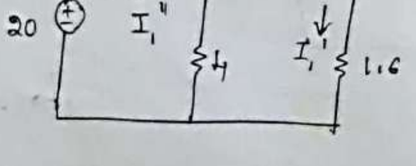
1. Find current through 2Ω



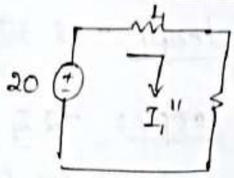
→ Case 1:

$$I_1 = \frac{8}{10} \times I_1' \longrightarrow (1)$$

$8||2 = 1.6$



$$\therefore \quad I_1' = \frac{4}{9.6} \times I_1''$$



$4 \| 5.6 = 2.33$

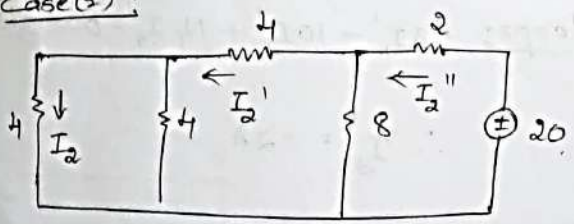
$$\therefore \quad I_1'' = \frac{20}{4+2.33}$$

$$\hookrightarrow = 3.159\,A$$

$$\therefore \quad I_1' = \frac{4 \times 3.159}{9.6} = 1.3162\,A$$

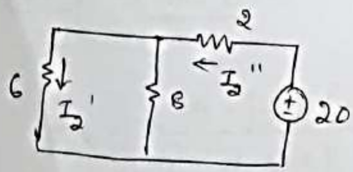
$$I_1 = \frac{8 \times 1.3162}{10} = 1.053\,A$$

Case(2):



$$I_2 = \frac{4}{8} \times I_2'$$

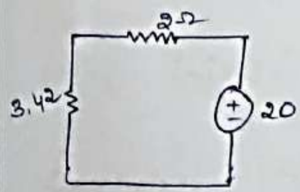
$$4 \| 4 = 2\Omega$$



$$I_2' = \frac{8 \times I_2''}{8+6}$$

$$6 \| 8 = 3.42$$

$$\therefore \quad I_2'' = \frac{20}{2+3.42} = 3.69\,A$$

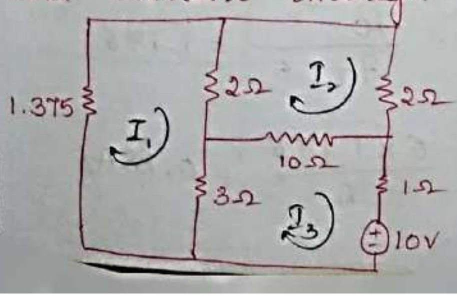


$$\therefore \quad I_2' = \frac{8 \times 3.69}{8+6} = 2.10\,A$$

$$I_2 = \frac{4 \times 2.10}{8} = 1.05\,A$$

$$\therefore \boxed{I_1 = I_2 = 1.05\,A}$$

(2) Find current through 1.375Ω.



→ Soln:-

Case(1):-

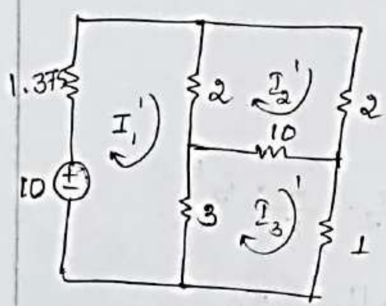
loop1 : $6.375I_1 - 2I_2 - 3I_3 = 0 \rightarrow (1)$

loop2 : $-2I_1 + 14I_2 - 10I_3 = 0 \rightarrow (2)$

loop3 : $-3I_1 - 10I_2 + 14I_3 = -10 \rightarrow (3)$

$\therefore \quad I_1 = -2A$

Case(2):-



loop1: $6.375I_1' - 2I_2' - 3I_3' = 10 \rightarrow (1)$

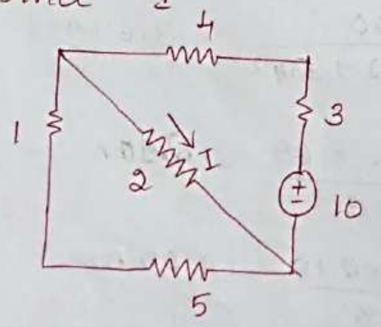
loop2: $-2I_1' + 14I_2' - 10I_3' = 0 \rightarrow (2)$

loop3: $-3I_1' - 10I_2' + 14I_3' = 0 \rightarrow (3)$

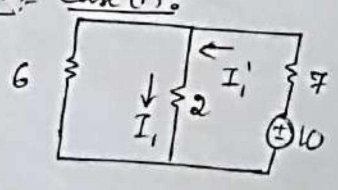
$\therefore \quad I_3' = -2A$

$$\therefore \boxed{I_1 = I_3' = -2A}$$

3. Find 'I'



→ Soln:- Case(1):

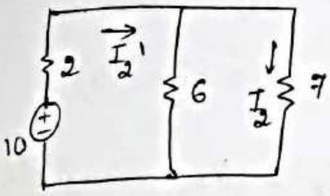


$I_1 = \frac{6 \times I_1'}{8}$ $\quad 6 \| 2 = 1.5$

But $I_1' = \frac{10}{7+1.5} = 1.176$

$\therefore \quad I_1 = \frac{6 \times 1.176}{8} = 0.88A$

Case (2):



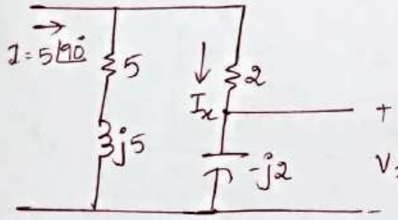
$$I_2 = \frac{6 \times I_2'}{7+6} \qquad 6 \| 7 = 3.23$$

$$I_2' = \frac{10}{2+3.23} = 1.911$$

$$I_2 = \frac{6 \times 1.911}{13} = 0.882\,A.$$

$$\therefore \boxed{I_1 = I_2 = 0.882A} = I$$

+ Find 'V_x'

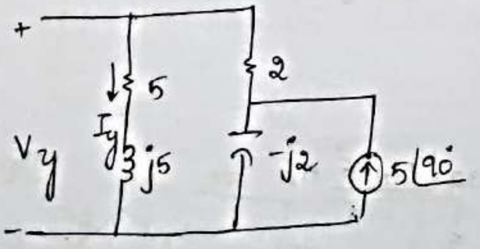


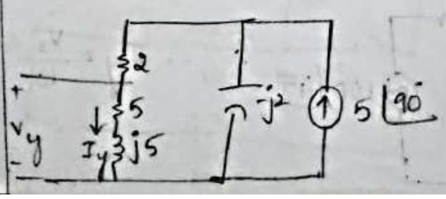
⇒ Sol^n:- Case (1):- $V_x = (-j2) I_x$

$$I_x = \frac{(5+j5)(5\angle 90°)}{7+3j} = 4.6424 \angle 111.801\,A$$

$$V_x = (-j2)(4.6424 \angle 111.801) = 9.2848 \angle 21.8°\ V$$

Case(2):-





$$V_y = (5+j5) I_y.$$

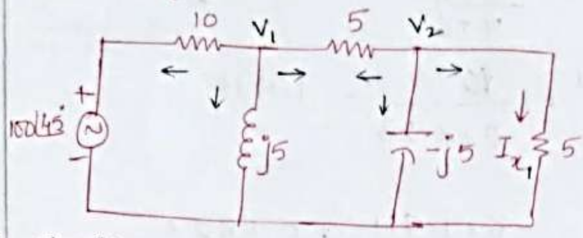
$$I_y = \frac{0-j2}{7+j3} \times 5\angle 90°$$

$$I_y = 1.313 \angle -23.198\ A$$

$$\therefore V_y = 9.2848 \angle 21.8°\ V$$

$$\therefore \boxed{V_x = V_y}$$

5. Find I_x



$$I_{x_1} = \frac{V_2}{5}$$

Case(1):

Node(1):

$$\frac{V_1 - 100\angle 45^\circ}{10} + \frac{V_1}{j5} + \frac{V_1 - V_2}{5} = 0$$

$$(0.3 - j0.2)V_1 - 0.2V_2 = 10\angle 45^\circ \longrightarrow (1)$$

Node(2):

$$\frac{V_2 - V_1}{5} + \frac{V_2}{-j5} + \frac{V_2}{5} = 0$$

$$-0.2V_1 + (0.4 + j0.2)V_2 = 0 \longrightarrow (2)$$

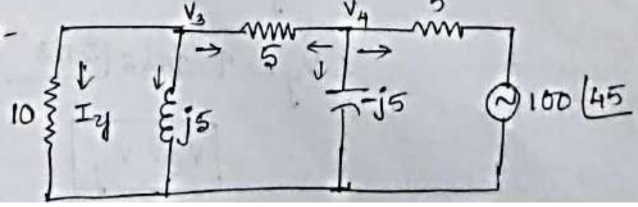
$$\begin{bmatrix} (0.3-j0.2) & -0.2 \\ -0.2 & (0.4+j0.2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \end{bmatrix}$$

$$V_2 = \frac{\begin{vmatrix} 0.3-j0.2 & 10\angle 45^\circ \\ -0.2 & 0 \end{vmatrix}}{\Delta} = \frac{2\angle 45^\circ}{(0.3-j0.2)(0.4+j0.2) - (0.2)^2}$$

$$V_2 = 16.66\angle 54.46^\circ \text{ V}$$

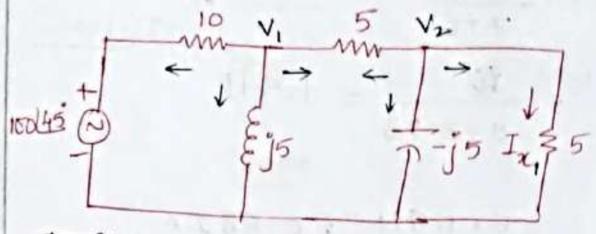
$$\therefore I_{x_1} = \frac{16.66\angle 54.46}{5} = 3.332\angle 54.46 \text{ A}$$

Case(2):-



$$\therefore I_y = \frac{V_3}{10}$$

5. Find I_x



$$I_{x_1} = \frac{V_2}{5}$$

Case(1):

Node(1):

$$\frac{V_1 - 100\angle 45^\circ}{10} + \frac{V_1}{j5} + \frac{V_1 - V_2}{5} = 0$$

$$(0.3 - j0.2)V_1 - 0.2V_2 = 10\angle 45^\circ \longrightarrow (1)$$

Node(2):

$$\frac{V_2 - V_1}{5} + \frac{V_2}{-j5} + \frac{V_2}{5} = 0$$

$$-0.2V_1 + (0.4 + j0.2)V_2 = 0 \longrightarrow (2)$$

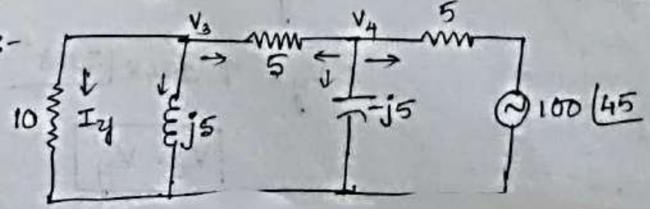
$$\begin{bmatrix} (0.3-j0.2) & -0.2 \\ -0.2 & (0.4+j0.2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 45^\circ \\ 0 \end{bmatrix}$$

$$V_2 = \frac{\begin{vmatrix} 0.3-j0.2 & 10\angle 45^\circ \\ -0.2 & 0 \end{vmatrix}}{\Delta} = \frac{2\angle 45^\circ}{(0.3-j0.2)(0.4+j0.2) - (0.2)^2}$$

$$V_2 = 16.66\angle 54.46^\circ \text{ V}$$

$$\therefore I_{x_1} = \frac{16.66\angle 54.46}{5} = 3.332\angle 54.46 \text{ A}$$

Case(2):-



$$\therefore I_y = \frac{V_3}{10}$$

Node 3:

$$\frac{V_3}{10} + \frac{V_3}{j5} + \frac{V_3 - V_4}{5} = 0$$

$$(0.3 - j0.2)V_3 - 0.2V_4 = 0 \longrightarrow (3)$$

Node 4:

$$\frac{V_4 - V_3}{5} + \frac{V_4}{-j5} + \frac{V_4 - 100\angle 45^\circ}{5} = 0$$

$$-0.2V_3 + (0.4 + j0.2)V_4 = 20\angle 45^\circ \longrightarrow (4)$$

$$\begin{bmatrix} (0.3 - j0.2) & -0.2 \\ -0.2 & (0.4 + j0.2) \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 20\angle 45^\circ \end{bmatrix}$$

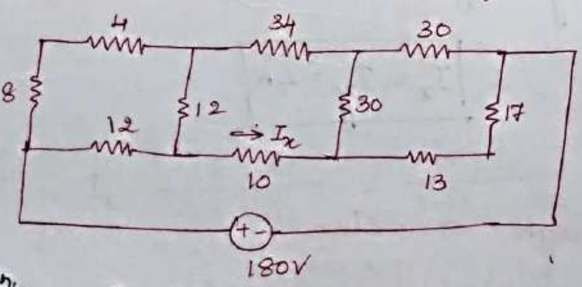
$$V_3 = \frac{\begin{vmatrix} 0 & -0.2 \\ 20\angle 45^\circ & 0.4 + j0.2 \end{vmatrix}}{\Delta} = \frac{0.2 \times 20\angle 45^\circ}{(0.3 - j0.2)(0.4 + j0.2) - (0.2)^2}$$

$$V_3 = 33.33\angle 54.46^\circ \text{ V}$$

$$\therefore I_y = \frac{V_3}{10} = 3.333\angle 54.46^\circ \text{ V}$$

$$\therefore \boxed{I_x = I_y}$$

6. Determine current through '10Ω'

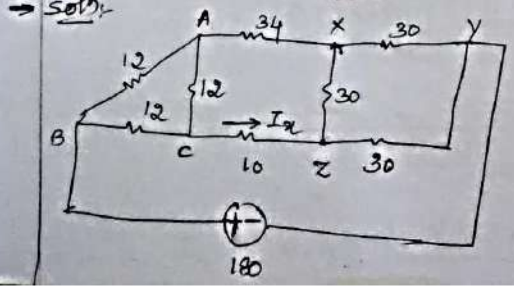


Δ to Y

$R_A = R_B = R_C$

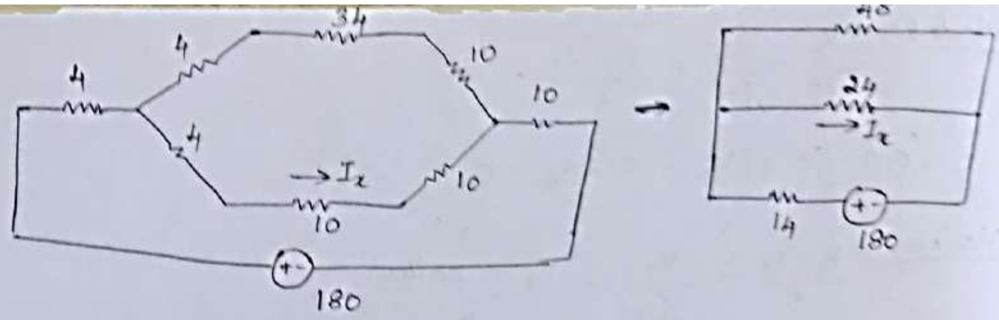
$$R_A = \frac{R_{AB} R_{AC}}{\Sigma R_{AB}} = \frac{12 \times 12}{36} = 4\Omega$$

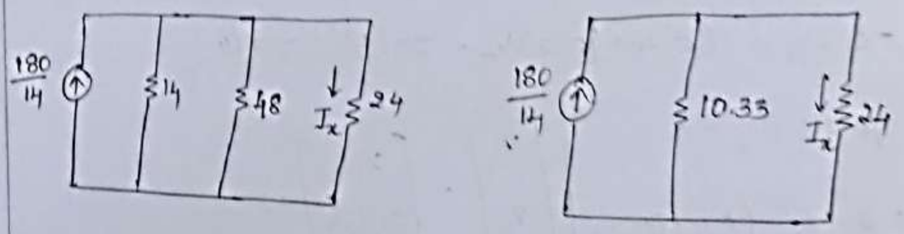
→ soln:



$R_x = R_y = R_z$

$$R_x = \frac{R_{xy} R_{xz}}{\Sigma R_{xz}} = \frac{30 \times 30}{90} = 10\Omega$$

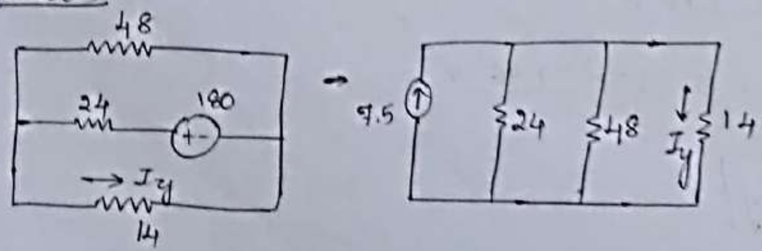




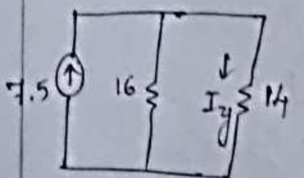
$14 \| 48 = 10.33$

$$I_x = \frac{10.33 \times (180/14)}{34.33} \quad ; \quad \boxed{I_x = 4A}$$

Case(2):-



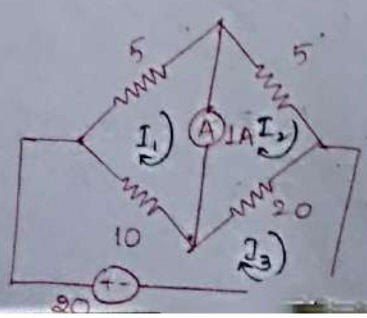
$24 \| 48 = 16\Omega$



$$I_y = \frac{16 \times 7.5}{30} = 4A$$

$$\therefore \boxed{I_x = I_y = 4A}$$

7. Find current through ammeter.



Soln:- Case1:

loop1:- $16I_1 - I_2 - 10I_3 = 0 \longrightarrow (1)$

loop2:- $-I_1 + 26I_2 - 20I_3 = 0 \longrightarrow (2)$

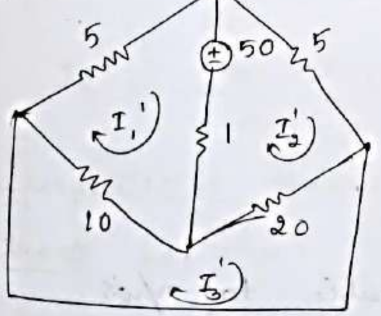
loop3:-

$-10I_1 - 20I_2 + 30I_3 = 50 \longrightarrow (3)$

$I_1 = 4.59A$; $I_2 = 5.41A$

$\therefore \; I_A = I_2 - I_1 = 0.82A$

Case(2):



loop1:

$16I_1' - I_2' - 10I_3' = -50 \rightarrow (1)$

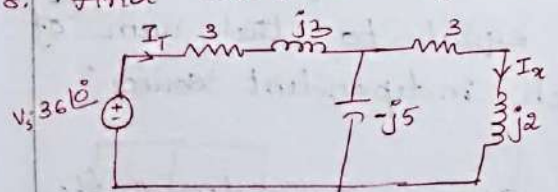
loop2: $-I_1' + 26I_2' - 20I_3' = 50 \rightarrow (2)$

loop3: $-10I_1' - 20I_2' + 30I_3' = 0 \rightarrow (3)$

$I_3' = 0.82A.$

$$\therefore \boxed{I_A = I_3' = 0.82A}$$

8. Find current I_x in $j2\Omega$ impedance.

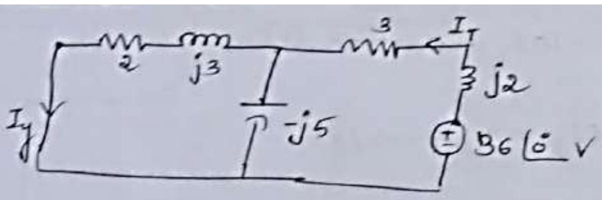


→ Soln:-
$$I_x = \frac{-j5 \cdot I_T}{-j5 + 3 + j2} = \frac{-j5 \cdot I_T}{3 - j3}$$

$$Z_T = (2 + j3) + \left[(-j5) \,||\, (3 + j2)\right] = 6.537 \angle 19.36°$$

$$\therefore \; I_T = \frac{36\angle 0°}{6.537\angle 19.36°} = 5.507\angle -19.36° \; A$$

$$\therefore \; I_x = 6.49 \angle -64.36° \; A$$



$$I_y = \frac{-j5}{2-j2} \cdot I_T'$$

$$\therefore\ I_T' = \frac{36\angle 0°}{Z_T'} = 3.672\angle -19.36°\ A$$

$$\therefore\ Z_T' = \left[(2+j3)\,||\,(-j5)\right] + (3+j2)$$

$$\hookrightarrow\ = 9.804\angle 19.36°\ \Omega$$

$$\therefore\ I_y = 6.49\angle -64.36°\ A$$

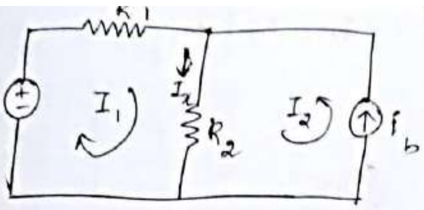
2 SUPERPOSITION THEOREM:

This theorem is applicable to n/w's containing multiple src^s.

Statement:- Superposition theorem states that," in a linear bilateral n/w. containing several independent src^s., the overall response at any time in the n/w. is equal to the sum of responses due to each independent source."

other than the source considered the remaining src^s. are set to zero. i.e. independent Voltage srcs. are replaced by short ckt. & independent current src^s. are replaced by open ckt. Dependent srcs. are left as they are

Proof:-



$\therefore I_x = I_1 + I_2$

$I_2 = i_b$

$$\therefore V_a = R_1 I_1 + R_2(I_1 + I_2) = (R_1 + R_2) I_1 + R_2 i_b$$

$$\therefore I_1 = \frac{V_a - R_2 i_b}{R_1 + R_2}$$

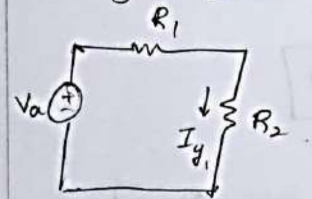
W.K.T.

$$I_x = I_1 + I_2 = \frac{V_a - R_2 i_b}{R_1 + R_2} + i_b$$

$$I_x = \frac{V_a + R_1 i_b}{R_1 + R_2} \longrightarrow (1)$$

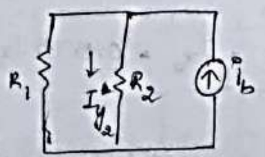
Superposition theorem method:

Case(1): Let V.S. 'V_a' be present & C.S. 'i_b' be set to zero [open ckt.]



$$\therefore I_{y_1} = \frac{V_a}{R_1 + R_2}$$

Case(2):- Let C.S. be present & V.S. be set to zero [short ckt]



$$\therefore I_{y_2} = \frac{R_1 i_b}{R_1 + R_2}$$

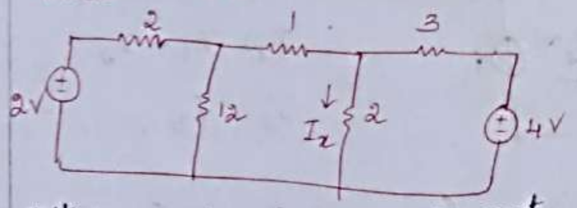
$\therefore$ By SP. theorem, $I_y = I_{y_1} + I_{y_2}$

$$\therefore I_y = \frac{V_a}{R_1 + R_2} + \frac{R_1 i_b}{R_1 + R_2}$$

$$I_y = \frac{V_a + R_1 i_b}{R_1 + R_2} \rightarrow (2)$$

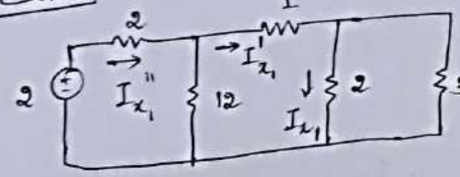
$\therefore (1) = (2)$; $\boxed{I_x = I_y}$ Hence superposition theorem verified

1 Find the current in $R = 2\Omega$



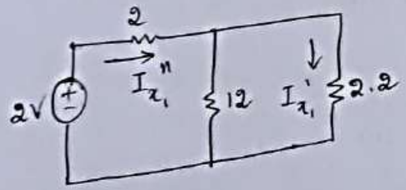
⇒ Soln :-

Case(1) :- Let 2V be present & 4V = 0



$I_{x_1} = \frac{3}{5} \times I_{x_1}' = 0.6 I_{x_1}'$

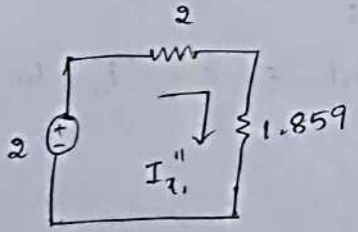
$[2||3] + 1 = 2.2\Omega$



$\therefore I_{x_1}' = \frac{12}{14.2} \times I_{x_1}'' = 0.845 I_{x_1}''$

$12||2.2 = \frac{12 \times 2.2}{14.2}$

$= 1.859\Omega$

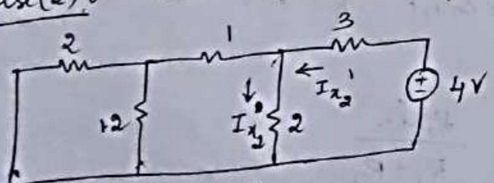


$\therefore I_{x_1}'' = \frac{2}{3.859} = 0.5182A$

$\therefore I_{x_1}' = 0.845 \times 0.5182 =$

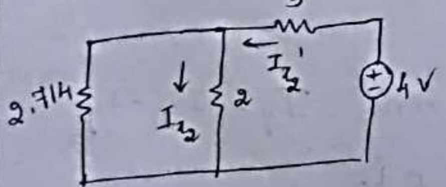
$\boxed{I_{x_1} = 0.2627A}$

Case(2) :- Let 4V be present



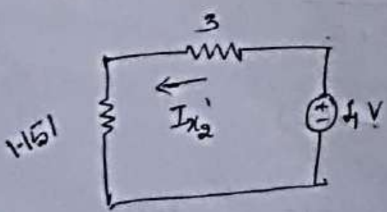
$[2||12] + 1 = \frac{24}{14} + 1$

$= 2.714$



$\therefore I_{x_2} = \frac{2.714}{4.714} \times I_{x_2}' = 0.575 I_{x_2}'$

$[2||2.714] = 1.151\Omega$



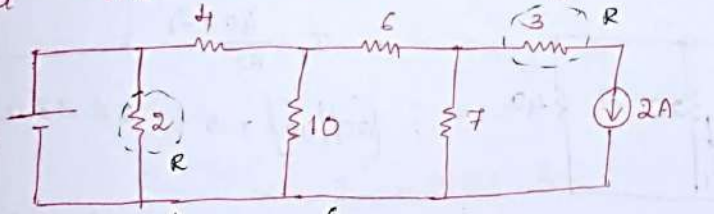
$I_{x_2}' = \frac{4}{4.151} = 0.9635A$

$\therefore I_{x_2} = 0.5540A$

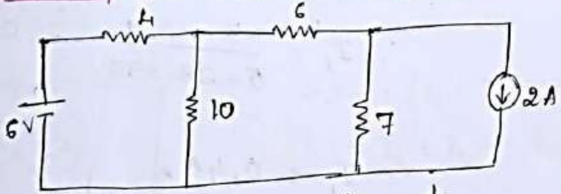
$I_x = I_{x_1} + I_{x_2} = 0.2627 + 0.5540$

$\boxed{I_x = 0.8167A}$

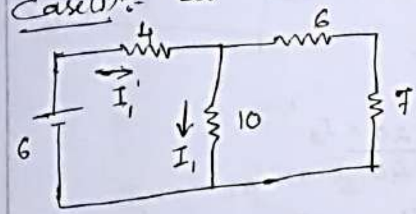
2. Find the current through 10Ω



Sol^n :-



Case(1):- Let 6V be present



$6 + 7 = 13$

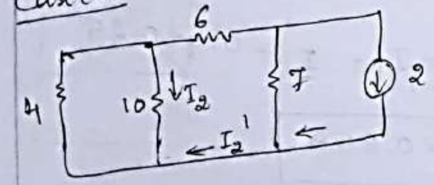
$I_1 = \frac{13}{23} \times I_1' = 0.5652 I_1'$

$13 \| 10 = 5.652$

$\therefore I_1' = \frac{6}{4+5.652} = 0.6216$

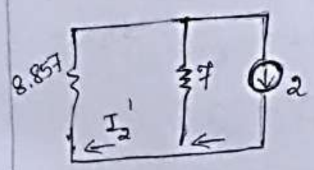
$\therefore I_1 = 0.3513A$

Case(2):- Let 2A be present



$\therefore I_2 = \frac{4}{14} \times (-I_2)'$

$[4 \| 10] + 6 = 8.857$



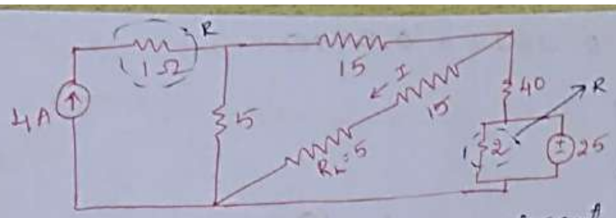
$\therefore I_2' = \frac{7 \times 2}{15.857} = 0.8828$

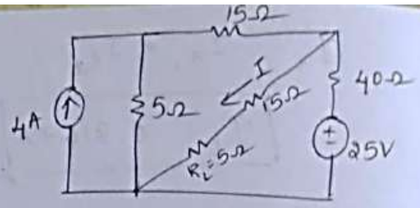
$\therefore I_2 = -0.2522A$

$\therefore I = I_1 + I_2 = 0.3513 - 0.2522$

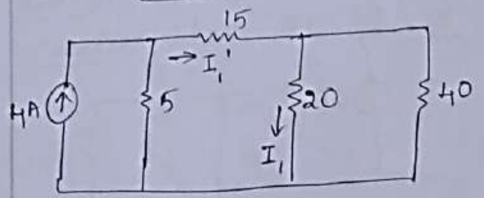
$\boxed{I = 0.099A}$

3. Find current through R_L'



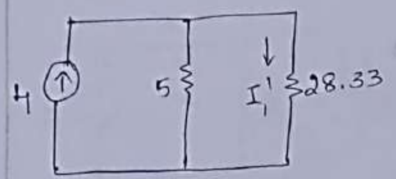


⇒ Sol^n:- Case(1): Let 4A be present



$\therefore I_1 = \frac{40 \times I_1'}{60}$

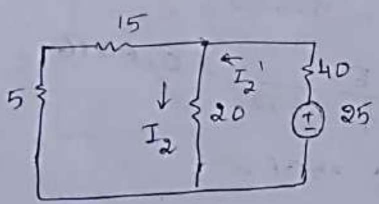
$\therefore [20 \| 40] + 15 = 28.33\Omega$.



$\therefore I_1' = \frac{5 \times 4}{5 + 28.333} = 0.6$

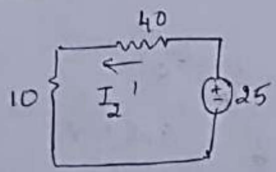
$\therefore I_1 = 0.4A$

Case(2): Let 25V be present



$I_2 = \frac{20 \times I_2'}{40}$

$20 \| 20 = 10$



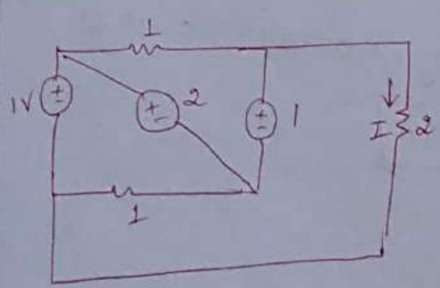
$I_2' = \frac{25}{50} = 0.5$

$\therefore I_2 = 0.25$

$\therefore I = I_1 + I_2 = 0.4 + 0.25$

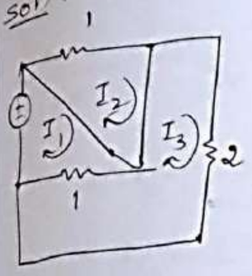
$\boxed{I = 0.65A}$

4- Find the current through 2Ω



soln:

Case (1) : 1V



$1 - (I_1 - I_3) = 0$

$-I_1 + I_3 = -1 \rightarrow (1)$

$I_2 = 0$

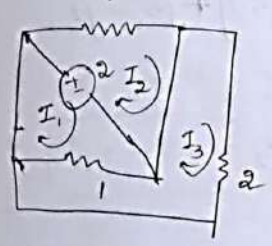
$-2I_3 - 1(I_3 - I_1) = 0$

$-3I_3 + I_1 = 0 \rightarrow (2)$

$\therefore I_1 = 1.5A ; I_2 = 0 ; I_3 = 0.5A$

$I'_{2\Omega} = 0.5A = I_3$

Case (2) :- 2V



$-2 - 1(I_1 - I_3) = 0$

$-I_1 + I_3 = 2 \rightarrow (1)$

$2 - I_2 = 0 ; I_2 = 2$

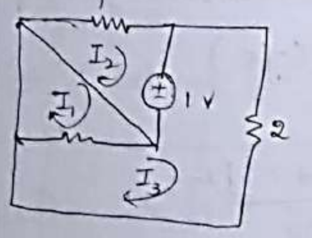
$-2I_3 - 1(I_3 - I_1) = 0$

$I_1 - 3I_3 = 0 \rightarrow (2)$

$I_1 = -3A ; I_2 = 2A ; I_3 = -1A$

$I''_{2\Omega} = I_3 = -1$

Case (3) :- 1V



$-I_1 + I_3 = 0 \rightarrow (1)$

$-I_2 - 1 = 0$

$I_2 = -1$

$1 - 2I_3 - (I_3 - I_1) = 0$

$I_1 - 3I_3 = -1 \rightarrow (2)$

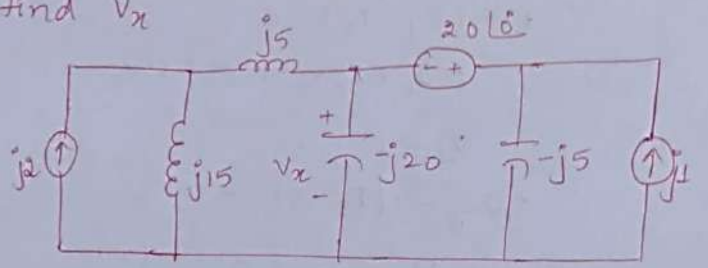
$I_1 = 0.5 ; I_2 = -1A ; I_3 = 0.5$

$\therefore I'''_{2\Omega} = I_3 = 0.5$

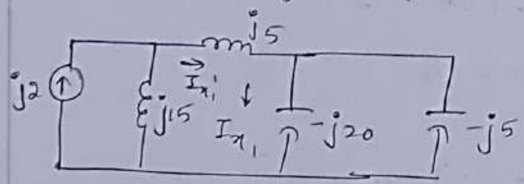
$\therefore I_{2\Omega} = I'_{2\Omega} + I''_{2\Omega} + I'''_{2\Omega} = 0.5 - 1 + 0.5$

$\boxed{I_{2\Omega} = 0A}$

5. Find 'V$_x$'

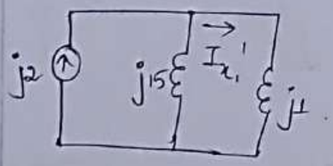


$\Rightarrow$ Soln:- Case(1): $j2A$ = present.



$\therefore I_{x_1} = \frac{-j5}{-j25} \times I_{x_1}' = 0.2 I_{x_1}'$

$[(-j20)||(-j5)] + j5 = j1$

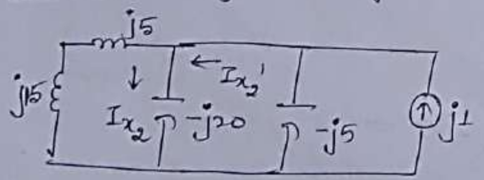


$\therefore I_{x_1}' = \frac{j15 \times j2}{j16}$

$I_{x_1}' = j1.875A$

$\therefore I_{x_1} = j0.375A$

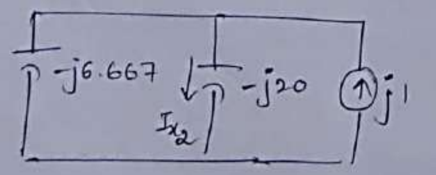
Case(2):- Let $j1A$ be present



$I_{x_2} = \frac{j20}{j20 - j20} \times I_{x_2}' = \infty$

so take $||^{le}$ combination of $(j20)||(-j5)$

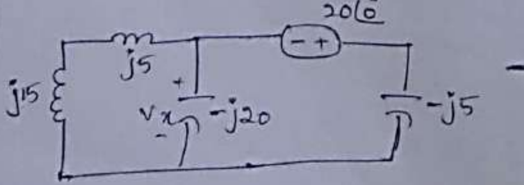
$\therefore j20||(-j5) = -j6.667$



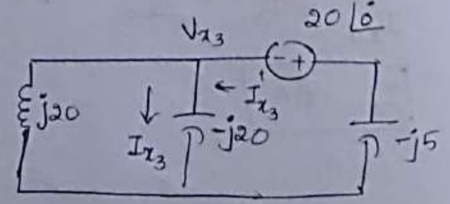
$I_{x_2} = \frac{-j6.667 \times j1}{-j26.67}$

$I_{x_2} = 0.247j$

Case(3):- Let $20\angle 0°$ be present



$\rightarrow$



Node analysis,

$$\frac{V_{x_3}-0}{j20} + \frac{V_{x_3}}{-j20} + \frac{V_{x_3}+20\angle 0^\circ}{-j5} = 0$$

$$V_{x_3} = -20\angle 0^\circ$$

$$\therefore I_{x_3} = \frac{-20\angle 0^\circ}{-j20} = -j1$$

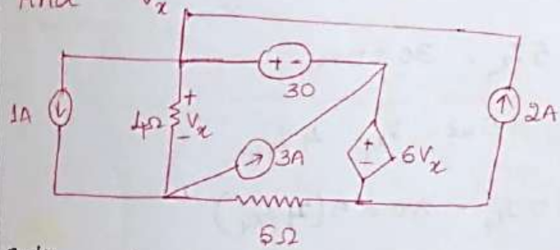
By superposition thm,

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} = -j0.375A$$

$$\therefore V_x = I_x R = -j0.375 \times (-j20)$$

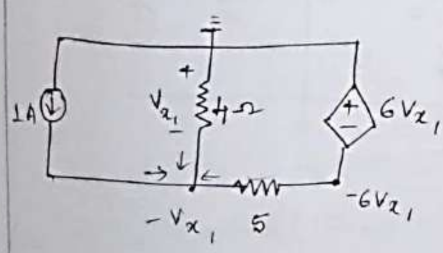
$$\boxed{V_x = -7.5V}$$

6. Find 'V_x'



⇒ Soln: While applying SPT, dependent srcs. are not set to zero.

Case(1): Let 1A c.s. be present



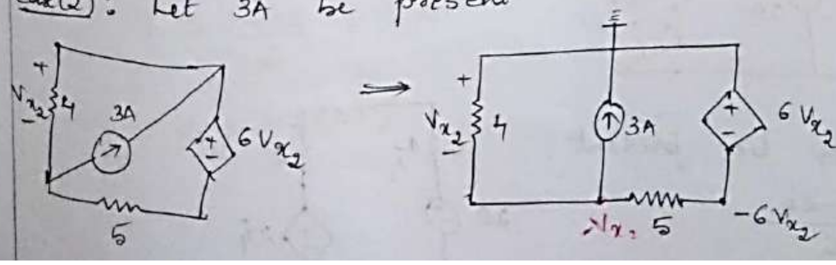
$$1 + \frac{0-(-V_{x_1})}{4} + \frac{(-6V_{x_1})+V_{x_1}}{5} = 0$$

$$1 + 0.25V_{x_1} - V_{x_1} = 0$$

$$1 - 0.75V_{x_1} = 0$$

$$V_{x_1} = 1.33V$$

Case(2): Let 3A be present

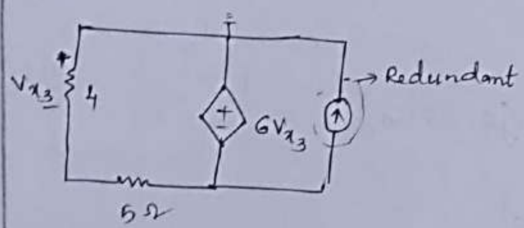


$$3 + \frac{-V_{x_2}}{4} + \frac{[-V_{x_2} - (-6V_{x_2})]}{5} = 0$$

$$3 - 0.25V_{x_2} + V_{x_2} = 0$$

$$3 + 0.75V_{x_2} = 0 \; ; \quad V_{x_2} = -4V$$

Case(3):- Let 2A C.S. be present

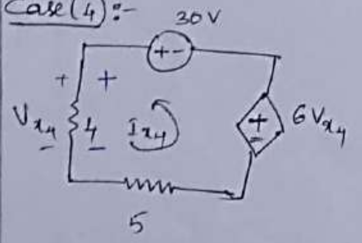


There is no independent src

$\therefore V_{x_3} = 0$

[∵ Anything in $||^{le}$ with V.S. is redundant]

Case(4):-



$$I_{x_4} = \frac{30 - V_{x_4} + 6V_{x_4}}{5} = \frac{30 + 5V_{x_4}}{5}$$

$$5I_{x_4} = 30 + 5V_{x_4}$$

But $V_{x_4} = -4I_{x_4}$

$$\therefore 5I_{x_4} = 30 + 5(-4I_{x_4})$$

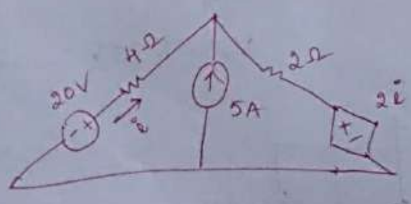
$$-15I_{x_4} = 30; \quad I_{x_4} = -2A$$

$$\therefore V_{x_4} = 4I_{x_4} = -8V$$

∴ By S.P.T.

$$V_x = V_{x_1} + V_{x_2} + V_{x_3} + V_{x_4} = -10.667V$$

7. Find current 'i'

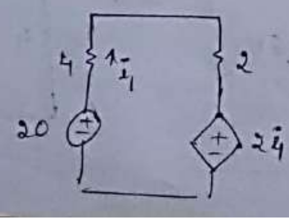


Soln:-

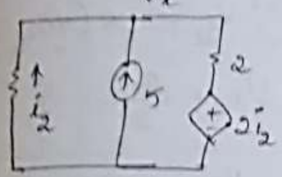
Case(1): Let 20V be present

$$i_1 = \frac{20 - 2i_1}{6}$$

$$\therefore i_1 = 2.5A$$



Case (2): Let 5A C.S. be present



$$i_2 = \frac{0 - V_x}{4}$$

Node analysis.

$$5 + i_2 + \frac{2i_2 - V_x}{2} = 0$$

$$10 + 2i_2 + 2i_2 - V_x = 0$$

$$10 + 4i_2 - V_x = 0.$$

$$10 + 4i_2 = V_x = -4i_2$$

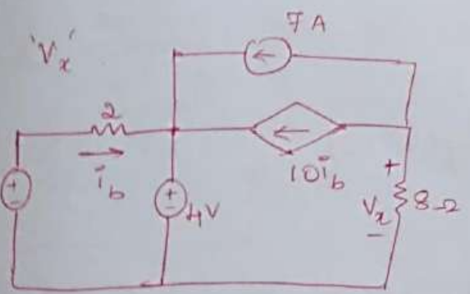
$$\therefore 8i_2 = -10; \quad i_2 = -1.25A$$

By SPT,

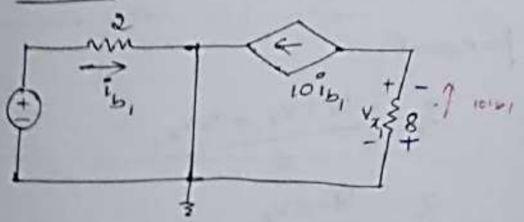
$$i = i_1 + i_2 = 2.5 - 1.25$$

$$i = 1.25A$$

8. Find 'V_x'



Soln: Case (1): Let 12V be present.



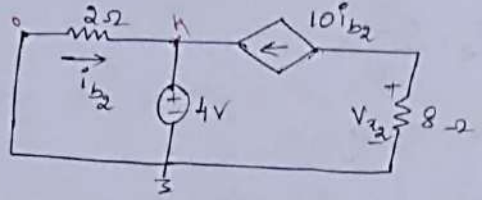
* NOTE:- 8Ω connected across '$10i_{b_1}$' is redundant interms of current but not in terms of voltage

$$\therefore V_{x_1} = -8(10i_{b_1}) = -80i_{b_1}$$

$$i_{b_1} = \frac{12 - 0}{2} = 6A$$

$$\therefore V_{x_1} = -480V$$

Case(2):- Let 4V V.S. be present

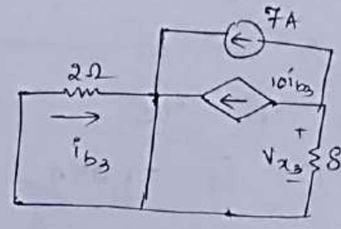


$\therefore V_{x_2} = -8(10 i_{b_2})$

$i_{b_2} = \frac{0-4}{2} = -2$

$V_{x_2} = +160V.$

Case(3):- Let 7A C.S. be present



$\therefore V_{x_3} = -8[10 i_{b_3} + 7)$

$i_{b_3} = 0$

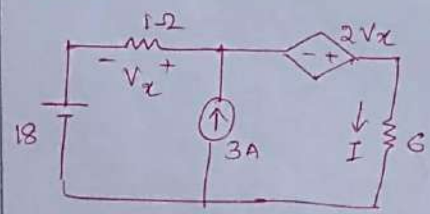
$V_{x_3} = -56V$

By SPT.

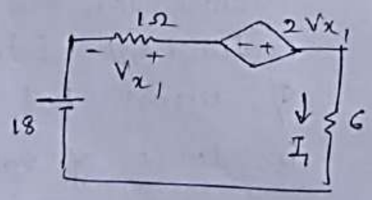
$V_x = V_{x_1} + V_{x_2} + V_{x_3}$

$$\boxed{V_x = -376V}$$

9\. Find current through 6Ω



→ Soln. Case(1):- 18V be presem



$I_1 = \frac{18 + V_{x_1} + 2V}{}$

I_1

$6I_1 = 18 + 3V_{x_1}$

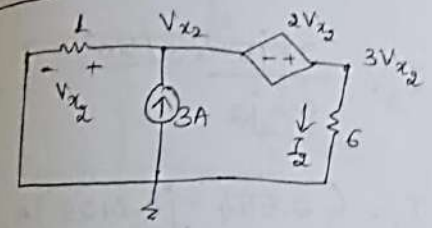
But $V_{x_1} = -I_1$

$\therefore 6I_1 = 18 + 3(-I_1) = 18 - 3I_1$

$9I_1 = 18$

$I_1 = 2A$

Case(2): Let 3A be present



Hint:- If c.s. is present apply node analysis

$$3 + \frac{0 - V_{x_2}}{1} + \frac{0 - 3V_{x_2}}{6} = 0$$

$$3 - V_{x_2} - 0.5V_{x_2} = 0$$

$$3 - 1.5V_{x_2} = 0$$

$$V_{x_2} = 2V$$

$$\therefore I_2 = \frac{3V_{x_2} - 0}{6}$$

$$\hookrightarrow = \frac{6-0}{6}$$

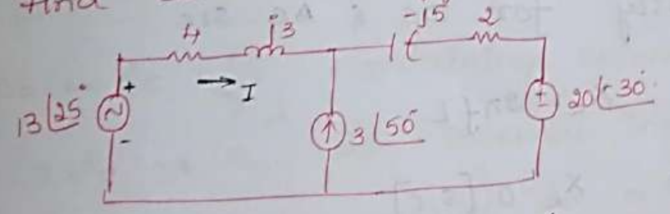
$$I_2 = 1A$$

By SPT,

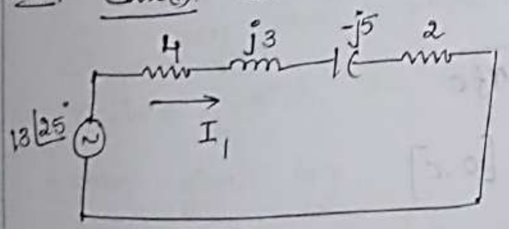
$$I = I_1 + I_2$$

$$\hookrightarrow = 3A$$

10. Find current through $(4+j3)\Omega$



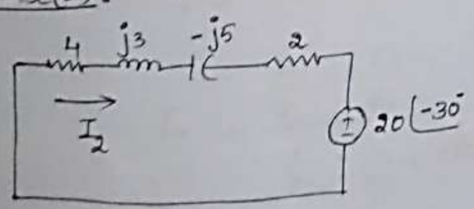
→ Sol^n: Case(1): Let $13\angle 25^\circ$ be present



$$\therefore I_1 = \frac{13\angle 25^\circ}{6 + j3 - j5}$$

$$I_1 = (1.4925 + j1.4132)A$$

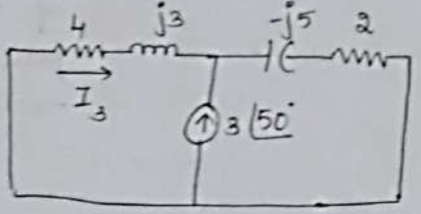
Case(2): Let $20\angle -30^\circ$ be present



$$I_2 = \frac{-20\angle -30^\circ}{6 - j2}$$

$$I_2 = (-3.098 + j0.6339)A$$

Case(3): Let $3\angle 50^\circ$ A be present



$$I_3 = \frac{2-j5}{6-j2} \cdot 3\angle 50^\circ$$

$$I_3 = (2.554 - j0.0105)A$$

∴ By SPT,

$$I = I_1 + I_2 + I_3$$

$$I = (-4.1595 + j2.0366)A$$

* Practical application of superposition thm.:-

* NOTE: When a n/w contain both AC & DC sources to find the response, SPT must be used because ckt element behave differently for DC & AC src.

L ; $X_L = 2\pi f L$

L: DC → SC, AC → O.C

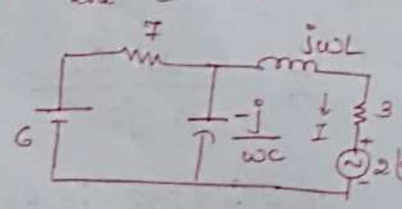
For $f_{dc} = 0$: $X_L = 0$ [S.C]

C ; $X_C = \frac{1}{2\pi f C}$

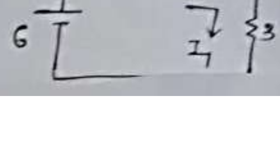
C: DC → OC, AC → SC

For $f_{dc} = 0$; $X_C = \infty$ [O.C]

11. Find the current 'I' through 3Ω [Assume ωL = 1 & ωC = 1]

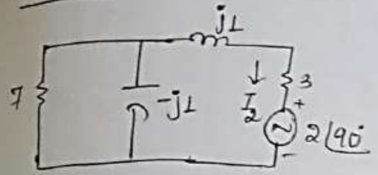


→ Soln:- Case(1): Let 6V be present

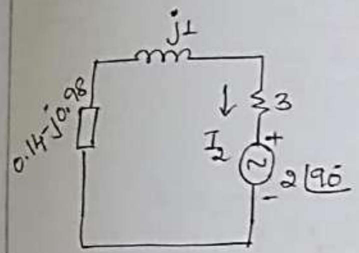


$$I_1 = \frac{6}{10} = 0.6A$$

Case(2) :- Let $2\angle 90^\circ$ be present



$7 \parallel (-j1) = 0.14 - j0.98$



$$I_2 = \frac{-2\angle 90^\circ}{0.14 - j0.98 + j1 + 3}$$

$$I_2 = -0.00405 - j0.6369$$

$\therefore$ By SPT, $I = I_1 + I_2 = 0.6 - 0.00405 - j0.6369$

$$\boxed{I = 0.59595 - j0.636\ A}$$

3 ## MILLMAN'S THEOREM :-

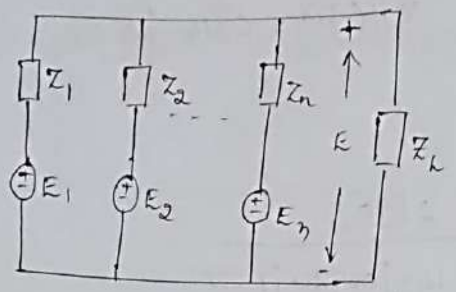
A n/w. containing several independent v.s $E_1, E_2 \ldots E_n$ with internal impedance $Z_1, Z_2 \ldots Z_n$ connected in parallel to a load impedance 'Z_L' may be replaced by a single v.s. 'E' in series with an impedance 'Z_L'. where 'E' & 'Z' are given by.

$$E = IZ = \frac{I}{Y} = \frac{I_1 + I_2 + \cdots + I_n}{Y_1 + Y_2 + \cdots + Y_n}$$

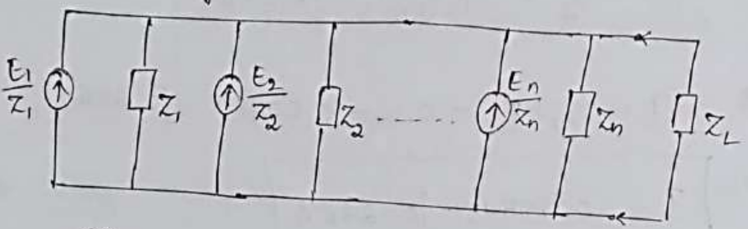
$$\hookrightarrow = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \cdots + \frac{E_n}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

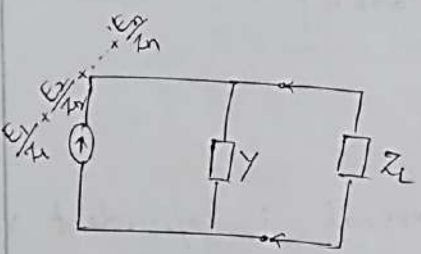
$$\therefore Z = \frac{1}{Y} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

Proof: Consider the n/w. given below



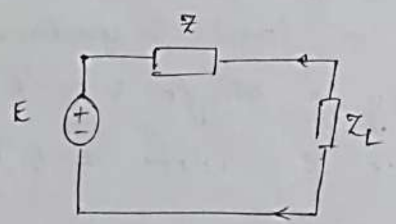
Converting v.s to c.s.





$$Y = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

$$\therefore Z = \frac{1}{Y} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$



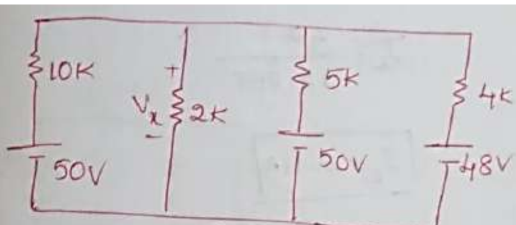
$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \cdots + \frac{E_n}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}}$$

* **Note:**

With 3 srcs.

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

1. Find vtg 'V_x' across 2k resistor by using Millman's theorem.



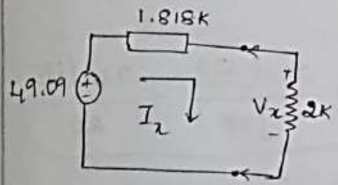
Soln:

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\frac{50}{10,000} + \frac{50}{5000} + \frac{48}{4000}}{\frac{1}{10,000} + \frac{1}{5000} + \frac{1}{4000}}$$

$$E = \frac{0.027}{5.5 \times 10^{-4}}$$

$$E = 49.09\,V$$

$$Z = \frac{1}{Y} = \frac{1}{5.5 \times 10^{-4}} = 1.818\,k\Omega$$

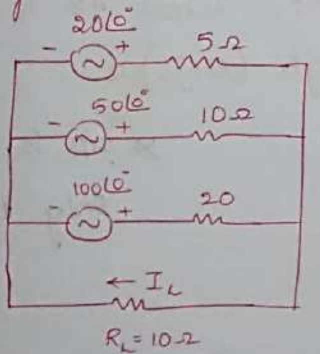


$$I_x = \frac{49.09}{3.818 \times 10^3} = 0.0129\,A$$

$$\therefore V_x = 2000\, I_x = 2000 \times 0.0154$$

$$\boxed{V_x = 25.8\,V}$$

2. Using Millman's theorem, find current through $R_L = 10\Omega$



Soln:-

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{\frac{20\angle 0^\circ}{5} + \frac{50\angle 0^\circ}{10} + \frac{100\angle 0^\circ}{20}}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} = \frac{14}{0.35}$$

$$E = 40\,V$$

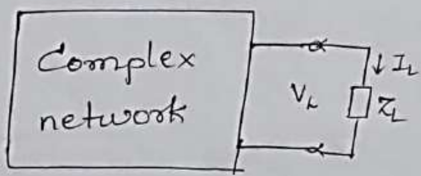
$$Z = \frac{1}{Y} = \frac{1}{0.35} = 2.857\,\Omega$$

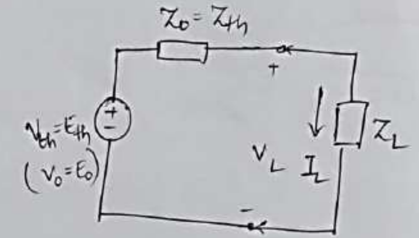
4 THEVENIN'S THEOREM:-

A linear bilateral n/w. however complicated the n/w. may be connected to a load impedance 'Z_L' may be replaced by a single equivalent ckt. containing a voltage source in series with an impedance.

The voltage of voltage source is equal to open circuited voltage across load terminals.

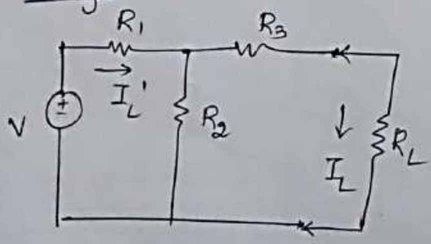
The value of impedance is equal to equivalent impedance of the n/w. as viewed from the load terminals into the n/w replacing all independent voltage source by short ckt & current source by open ckt.





$$\therefore \boxed{I_L = \frac{V_0}{Z_0 + Z_L}} \quad ; \quad \boxed{V_L = \frac{V_0 \times Z_L}{Z_0 + Z_L}}$$

Proof:- Consider a linear n/w. given below,



$$I_L = \frac{R_2 \times I_L'}{R_2 + R_3 + R_L}$$

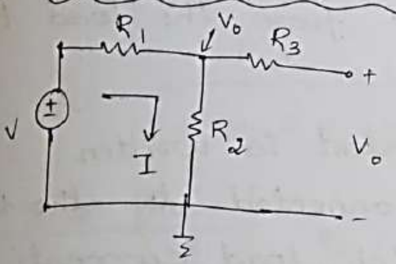
$$\therefore I_L' = \frac{V}{R_1 + \left[R_2 || (R_3 + R_L)\right]}$$

$$I_L = \frac{R_2}{R_2 + R_3 + R_L}\left[\frac{V}{R_1 + \frac{R_2(R_3 + R_L)}{R_2 + R_3 + R_L}}\right]$$

$$I_L = \frac{R_2 V}{R_1R_2 + R_1R_3 + R_2R_3 + R_1R_L + R_2R_L} \longrightarrow (1)$$

To find thevenin's equivalent:-

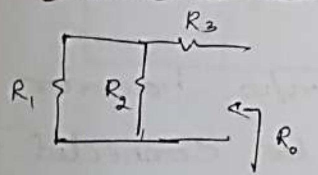
i) To find thevenin's vtg $[V_0]$:



$$V_0 = I R_2$$

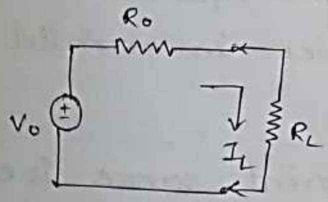
$$\hookrightarrow = \frac{VR_2}{R_1+R_2}$$

ii) To find thevenin's resistance 'R_0':



$$R_0 = [R_1 \| R_2] + R_3 = \frac{R_1R_2}{R_1+R_2} + R_3$$

$$R_0 = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1+R_2}$$



$$I_L = \frac{V_0}{R_0 + R_L}$$

$$I_L = \frac{\frac{VR_2}{R_1+R_2}}{\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1+R_2} + R_L}$$

$$I_L = \frac{VR_2}{R_1R_2 + R_2R_3 + R_1R_3 + R_1R_L + R_2R_L} \longrightarrow (2)$$

(1) = (2)

Hence proved.

* Procedure for solving a n/w using thevenin's theorem

Step1:- The load impedance 'Z_L' through which voltage (or) current is to be found is removed & an open ckt. is created across load terminal.

step2: The open ckted. vtg. 'V_0' across load terminal is measured.

step3: All the independent v.s. are replaced by short ckt. & independent c.s. by open ckt.

step: The equivalent impedance 'Z_0' looking into the n/w. as viewed from the load terminal is found.

step4: The thevenin's equivalent is written.

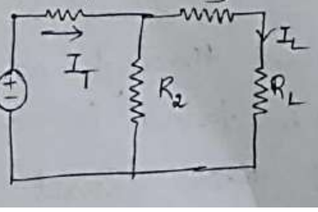
step5: The load 'Z_L' is connected to the thevenin equivalent from which load current/ load vtg. can be found.

5. NORTON'S THEOREM:-

In a linear bilateral n/w. however complicated the n/w. may be connected to 'Z_L' it maybe replaced by simple equivalent ckt. containing a current source in parallel with an impedance.

The current of the current source is equal to short ckt current flowing through load terminal & the value of 'Z' is equal to the equivalent impedance of the n/w. as viewed from the load terminal into the n/w. replacing all independent v.s. by short ckt. & current src by open ckt.

Proof:-



By current division,

$$I_L = \frac{R_2 \cdot I_T}{R_2 + R_3 + R_L} \longrightarrow (1)$$

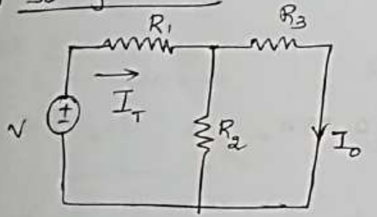
$$I_T = \frac{V}{R_T} \longrightarrow (2)$$

$$R_T = \frac{R_1R_2 + R_2R_3 + R_3R_1 + R_1R_L + R_2R_L}{R_2 + R_3 + R_L} \rightarrow (3)$$

Subs. (3) in (2) & (2) in (1)

$$I_L = \frac{R_2V}{R_1R_2 + R_2R_3 + R_3R_1 + R_1R_L + R_2R_L} \longrightarrow (4)$$

i) To find I_o:-



$$I_o = \frac{R_2 \cdot I_T}{R_2 + R_3} \longrightarrow (5)$$

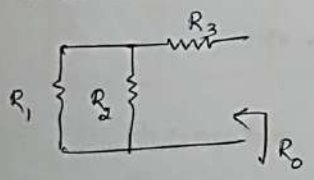
$$I_T = \frac{V}{R_T} \longrightarrow (6)$$

$$R_T = R_1 + \frac{R_2R_3}{R_2 + R_3} \longrightarrow (7)$$

Sub eqn. (7) in (6) & (6) in (5)

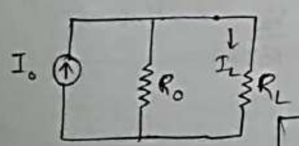
$$\therefore I_o = \frac{VR_2}{R_1R_2 + R_2R_3 + R_1R_3} \longrightarrow (8)$$

ii) To find R_o:-



$$R_o = \frac{R_1R_3 + R_2R_3 + R_1R_2}{R_1 + R_2} \rightarrow (9)$$

$\therefore$ Norton's eqn. ckt is,



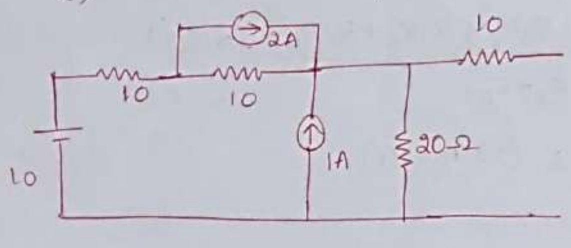
$$\therefore I_L = \frac{R_o \cdot I_o}{R_o + R_L} \longrightarrow (10)$$

subs. (8) & (9) in (10),

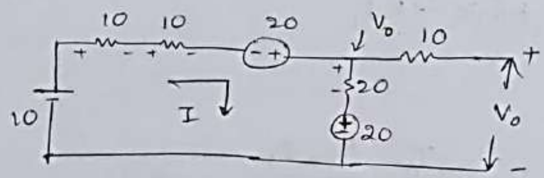
$$\boxed{I_L = \frac{VR_2}{R_1R_2 + R_2R_3 + R_3R_1 + R_1R_L + R_2R_L}} \rightarrow (11)$$

$\therefore$ Norton's thm. is proved.

1) For the n/w shown find.
(i) Thevenin's Equivalent (ii) Norton's equivalent



⇒ Soln:- i) Thevenin's equivalent :- (a) thevenin's vtg:



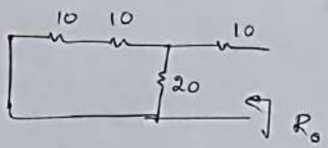
$\therefore V_0 = 20 + 20I$

$$\therefore I = \frac{10+20-20}{10+10+20} = 0.25A$$

$\therefore V_0 = 20 + 20(0.25)$

$\boxed{V_0 = 25V}$

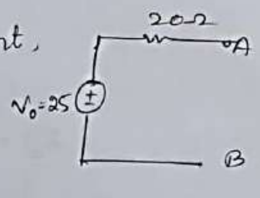
(b) thevenin's resistance:-



$R_0 = [20 \| 20] + 10 = 10 + 10$

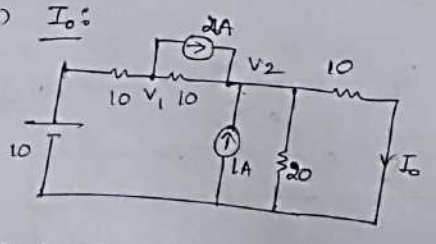
$R_0 = 20\Omega$

$\therefore$ Thevenin's equivalent,



ii) Norton's equivalent :-

(a) I_0:



$I_0 = \frac{V_2}{10}$

Node1 :- $\frac{10-V_1}{10} + \frac{V_2-V_1}{10} - 2 = 0$

$0.2V_1 - 0.1V_2 = -1 \longrightarrow (1)$

Node(2):- $1 + 2 + \frac{V_1 - V_2}{10} + \frac{0 - V_2}{20} + \frac{0 - V_2}{10} = 0$

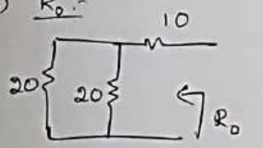
$$-0.1V_1 + 0.25V_2 = 3 \longrightarrow (2)$$

$$\begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.25 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$\therefore V_1 = 1.25V \;;\; V_2 = 12.5V$

$\therefore I_o = \frac{V_2}{10} = \frac{12.5}{10} \;;\; \boxed{I_o = 1.25A}$

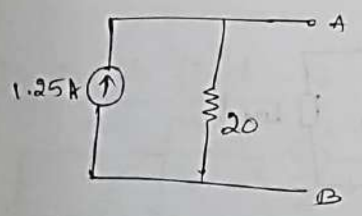
(b) R_o:-



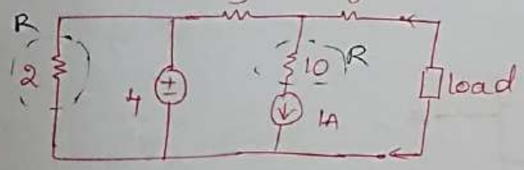
$R_o = [20 \| 20] + 10$

$\hookrightarrow = 20\Omega$

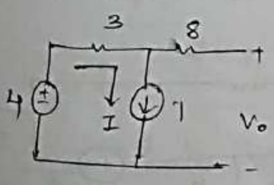
Norton's equivalent,



3. Find thevenin's & norton's equivalent ckt



⇒ soln:- i) thevenin's equivalent:-



$V_o = 4 - 3 \cdot I$

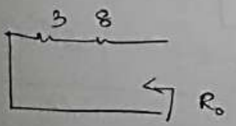
But $I = 1A$

$\therefore V_o = 4 - 3 \;;\; V_o = 1V$

(or) $\frac{V_o - 4}{3} + 1 + 0 = 0$

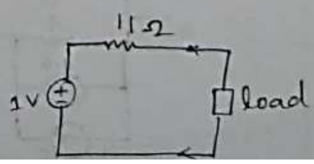
$3 + V_o - 4 = 0$

$\boxed{V_o = 1V}$

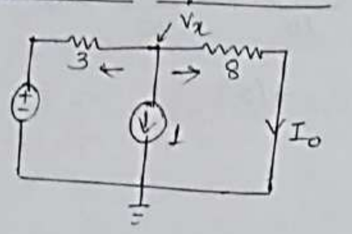


$R_o = 3 + 8$

$\hookrightarrow = 11\Omega$



ii) Norton's equivalent :-



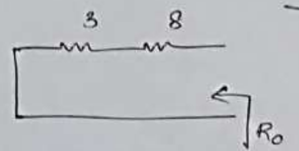
$I_o = \frac{V_x - 0}{8}$

Node analysis, $\frac{V_x - 4}{3} + 1 + \frac{V_x}{8} = 0$

$V_x = 8/11$

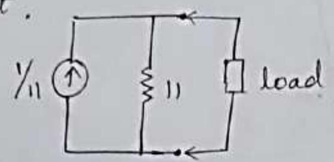
$\therefore I_o = \frac{8/11}{8}$

$\boxed{I_o = \frac{1}{11} A}$

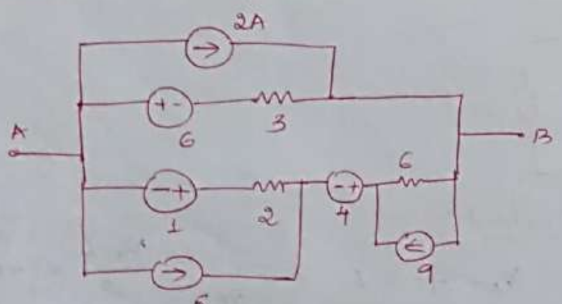


$R_o = 11\Omega$

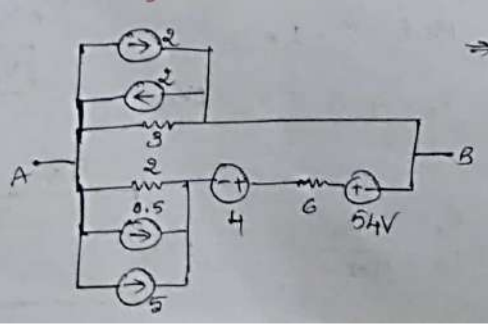
∴ Norton's equivalent.



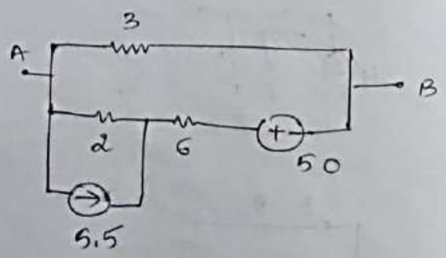
3. Obtain T.E & N.E

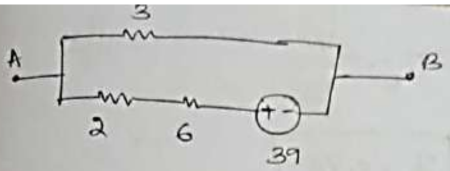


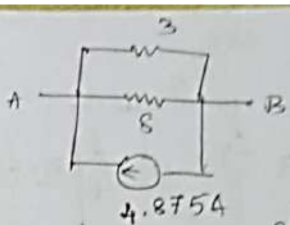
sol^n:-



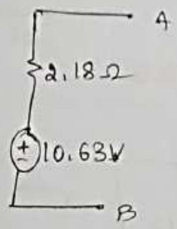
⇒

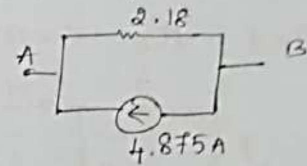






$3 \| 8 = 2.18\ \Omega$

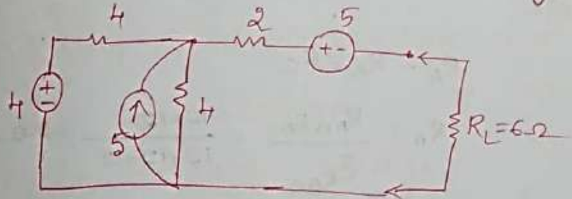




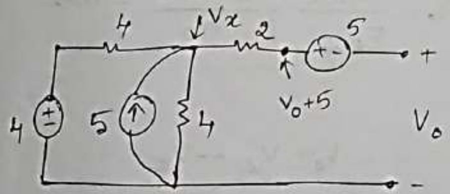
TE

N.E

4. Find the current through $R_L = 6\Omega$



Soln:- i) To find 'V_o':-



$\therefore V_x = 5 + V_o$

$$5 + \frac{4 - V_x}{4} + \frac{0 - V_x}{4} + 0 = 0$$

$$24 - 2V_x = 0$$

$$2V_x = 24$$

$$\boxed{V_x = 12}$$

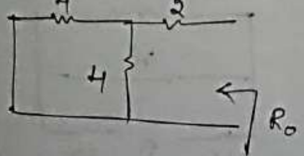
$\therefore V_x = 5 + V_o$

$$V_o = V_x - 5 = 12 - 5$$

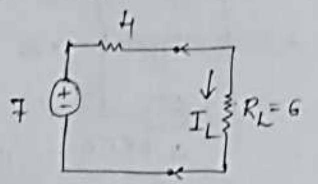
$$\boxed{V_o = 7V}$$

ii) To find 'R_o':-

$$R_o = [4 \| 4] + 2 = 4\Omega$$



Th.E. ckt,

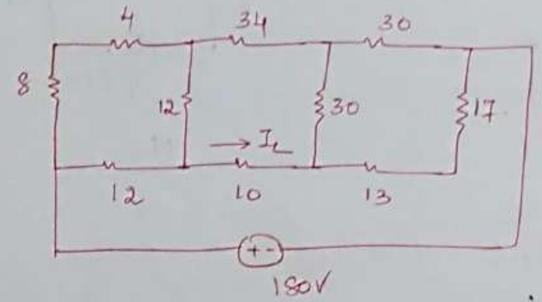


$$I_L = \frac{7}{10}$$

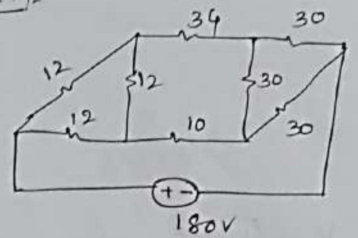
$$\boxed{I_L = 0.7A}$$

M-J-10

5) Find the current in 10Ω



→ soln.-

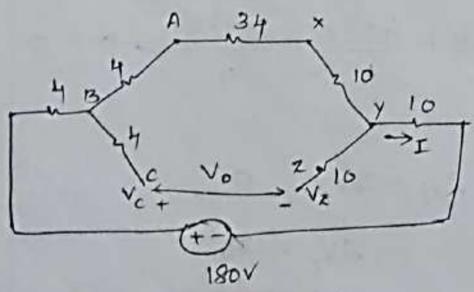


$R_A = R_B = R_C$

$$R_A = \frac{R_{AB}R_{AC}}{\Sigma R_{AB}} = \frac{12 \times 12}{12+12+12} = 4\Omega$$

$R_x = R_y = R_z$

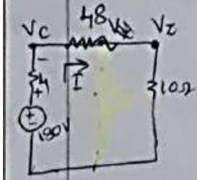
$$R_x = \frac{R_{xy}R_{xz}}{\Sigma R_{xy}} = \frac{30 \times 30}{90} = 10\Omega$$

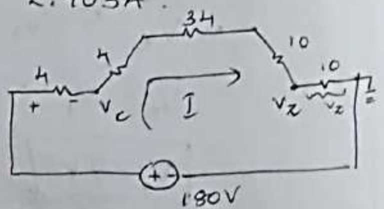


$$\therefore V_0 = V_C - V_z$$

$$I = \frac{180}{4+4+34+10+10}$$

$$I = 2.903A.$$





$$V_C = 180 - 4I = 168.39V$$

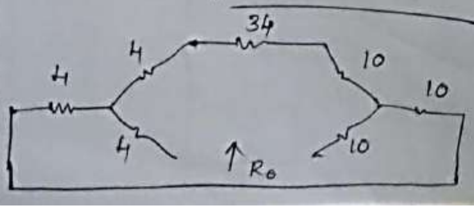
$$V_Z = 10I = 29.03V$$

$$\therefore V_0 = 168.39 - 29.03$$

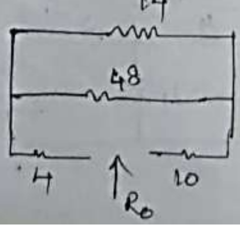
$$\boxed{V_0 = 139.36V}$$

$V_C = 180 - 4I$

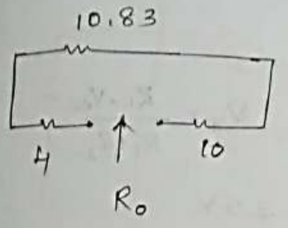
$V_Z = 10I$



⟹

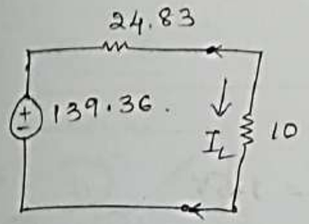


$$14\|48 = \frac{14\times 48}{14+48} = 10.83\Omega$$



$R_o = 10+4+10.83$

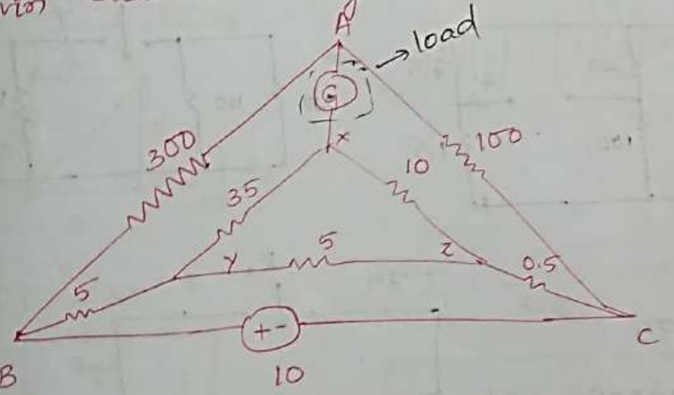
$R_o = 24.83\Omega$



$$I_L = \frac{139.36}{34.83}$$

$$\boxed{I_L = 4A}$$

6. Using T.T. determine the current through the galvanometer of 16Ω resistance in Kevin double bridge.

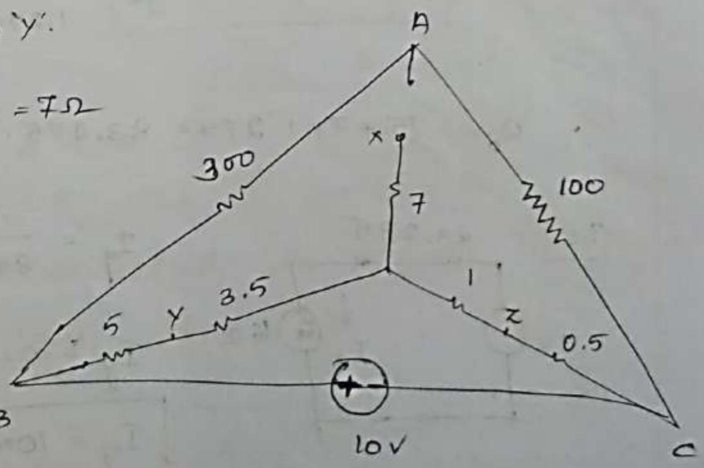


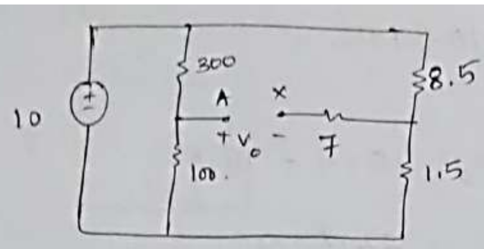
→ soln. i) V_o: Convert 'Δ' to 'Y'.

$$R_x = \frac{R_{xy}R_{xz}}{\Sigma R_{xz}} = \frac{35\times 10}{50} = 7\Omega$$

$$R_y = \frac{5\times 35}{10} = 3.5\Omega$$

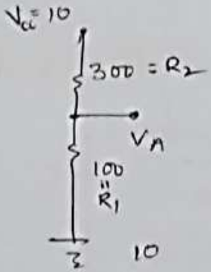
$$R_z = \frac{10\times 5}{50} = 1\Omega$$



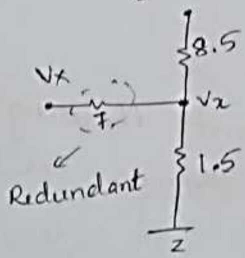


$V_o = V_A - V_x$

$V_A = \dfrac{R_1 \times V_{cc}}{R_1 + R_2}$



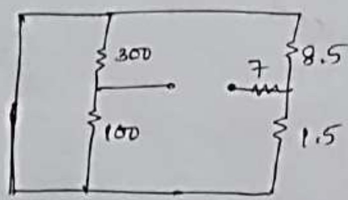
$V_A = \dfrac{100 \times 10}{100+300} = 2.5V$



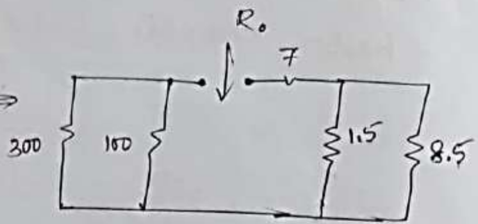
$V_x = \dfrac{1.5 \times 10}{1.5 + 8.5} = 1.5V$

$\therefore V_o = 2.5 - 1.5 = 1V$

ii) R_o:

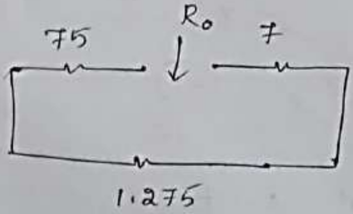


$\Rightarrow$



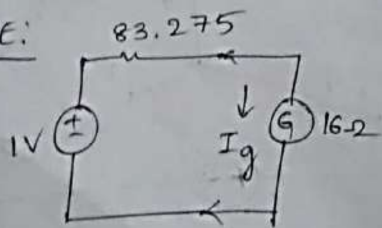
$300 \parallel 100 = 75$

$1.5 \parallel 8.5 = 1.275$



$\therefore R_o = 75 + 7 + 1.275 = 83.275$.

T.E:

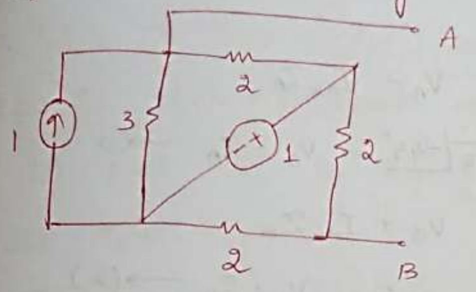


$I_g = \dfrac{1}{83.275 + 16}$

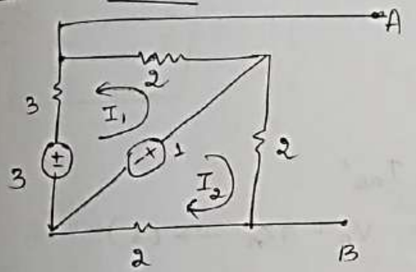
$I_g = 0.0100$

$\boxed{I_g = 10mA}$

Determine current through 1Ω resistor connected b/w A & B using T.T.



Soln: i) V_0:



$V_0 = 2I_2 - 2I_1$

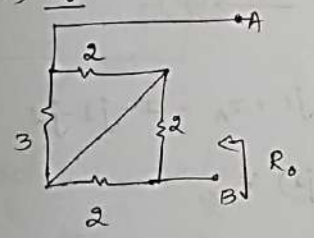
$I_1 = \frac{-3+1}{5} = -\frac{2}{5} = -0.4A$

$I_2 = \frac{1}{4} = 0.25A$

$V_0 = (2 \times 0.25) - (2 \times (-0.4))$

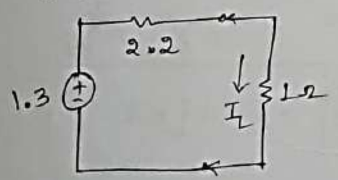
$V_0 = 1.3V$

ii) R_0:



$[2||3] + [2||2] = 1.2 + 1 = 2.2\Omega$

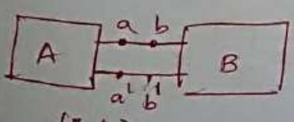
T.E ckt.



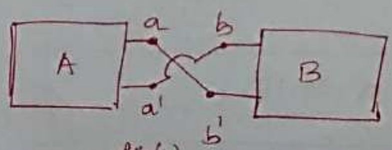
$I_L = \frac{1.3}{3.2} = 0.40625$

$\boxed{I_L = 406.25mA}$

When 2 n/w's A & B are connected as shown in fig(a) $I_{ab} = 1A$, $V_{aa'} = \sqrt{2}\angle{-45^\circ}V$. When the same n/w is connected as shown in fig(b) $I_{a'b} = 3A$, $V_{aa'} = \sqrt{2}\angle{45^\circ}V$. Find T.E.

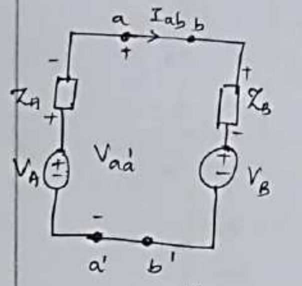


fig(a)



fig(b)

→ Sol? Let the T.E of n/w A & B be V_A in series with Z_A & V_B in series with Z_B



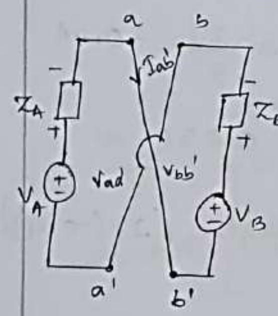
$V_{aa'} = V_A - I_{ab} Z_A$

$1 - j1 = \sqrt{2}\angle{-45^\circ} = V_A - Z_A \rightarrow (1)$

$V_{bb'} = V_B + I_{ab} Z_B$

$\sqrt{2}\angle{45^\circ} = 1 + j1 = V_B + Z_B \rightarrow (2)$

$V_{aa'} = V_{bb'}$ [Same vtg.]



$V_{aa'} = -V_{bb'}$

$\therefore V_{aa'} = V_A - Z_A I_{ab'}$

$\sqrt{2}\angle{45^\circ} = 1 + j1 = V_A - 3Z_A \rightarrow (3)$

$V_{bb'} = V_B - 2Z_B = -\sqrt{2}\angle{45^\circ}$

$-(1 + 1j) = V_B - 2Z_B \longrightarrow (4)$

(1) − (3)

$$\begin{array}{l} V_A - Z_A = 1 - j1 \\ V_A - 3Z_A = 1 + j1 \\ (-) \quad (+) \quad (-)(-) \\ \hline 2Z_A = -j2 \\ \hline Z_A = -j1\Omega. \end{array}$$

$\therefore V_A = 1 - j1 + Z_A = 1 - j1 - j1$

$V_A = (1 - j2)\ V$

(2) − (4)

$$\begin{array}{l} V_B + Z_B = 1 - j1 \\ V_B - 3Z_B = -1 - j1 \\ (-) \quad (+) \quad (+)(+) \\ \hline 4Z_B = 2 \\ \hline Z_B = 0.5\Omega \end{array}$$

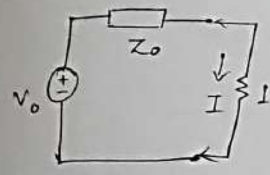
$\therefore V_B = 1 - j1 - Z_B$

$= 1 - j1 - 0.5$

$V_B = (0.5 - j1)\ V$

9. A LTI n/w. shown when terminated with (i) $R = 1\Omega$, $I = 5\angle -45^\circ$ A ; (ii) $X_C = 1\Omega$, $I_C = 10\angle -45^\circ$ A. Find the T.E. of the n/w. what will be the current if it is terminated with $X_L = 1\Omega$

soln:- (i) Let 'V_0' is in series with 'Z_0', the TE of the n/w,

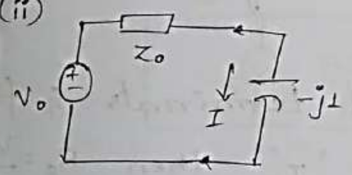


$$I = \frac{V_0}{Z_0 + 1} = 5\angle -45^\circ$$

$$Z_0 + 1 = \frac{V_0}{5\angle -45^\circ}$$

$$Z_0 + 1 = 0.2\angle 45^\circ \, V_0 \longrightarrow (1)$$

(ii)



$$I = \frac{V_0}{Z_0 - j1} = 10\angle -45^\circ$$

$$Z_0 - j1 = 0.1\angle -45^\circ \, V_0 \longrightarrow (2)$$

Solve (1) & (2),

$$\cancel{Z_0} + 1 = 0.2\angle 45^\circ \, V_0$$

$$\underset{(-)}{\cancel{Z_0}} \underset{(+)}{-j1} = \underset{(-)}{0.1\angle -45^\circ} \, V_0$$

$$1 + j1 = \left[0.2\angle 45^\circ - 0.1\angle -45^\circ\right] V_0.$$

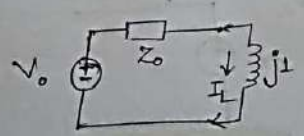
$$V_0 = \frac{1 + j1}{0.2\angle 45^\circ - 0.1\angle -45^\circ} = 14.14\text{V}$$

From eqn.(1),

$$Z_0 = 0.2\angle 45^\circ V_0 - 1 = \left[0.2\angle 45^\circ \times 14.14 - 1\right]$$

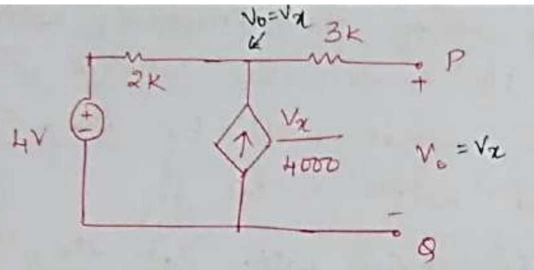
$$Z_0 = (1 + j2)\ \Omega$$

$$I_L = \frac{14.14}{1 + j2 + j1}$$



$$\boxed{I_L = 4.46\angle -71.56^\circ \text{ A}}$$

10



⟹ Solⁿ:- i) V_o :

$$\frac{V_x}{4000} + \frac{4 - V_x}{2000} + \cancel{\frac{V_x - V_x}{3k}} = 0$$

$$\boxed{V_x = 8V = V_o}$$

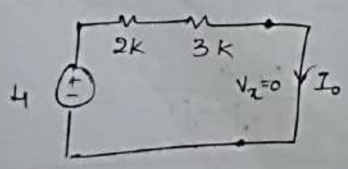
ii) R_o :- When a n/w. contains dependent src. to find $R_o [Z_o]$ the following techniques can be used,

1) Short ckt. the load terminals & find short ckted current 'I_o', then $R_o = \frac{V_o}{I_o}$.

2) Replace all independent v.s. by short ckt. & c.s. by open ckt. Connect a V.S. across the load terminals of value 'V' volt 1V, 10V... etc, find the current 'I' drawn by the n/w. then $R_o = \frac{V}{I}, \frac{1}{I}, \frac{10}{I}$

I_o:
when o/p terminals are short ckted. to find I_o, V_x becomes '0'. This makes dependent c.s.

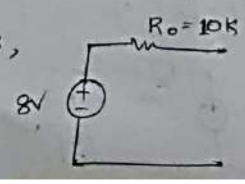
$\frac{V_x}{4000} = 0$ [o.c]



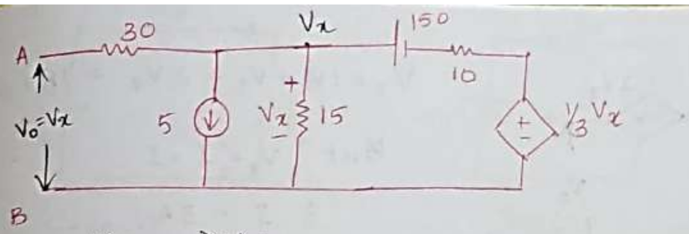
$\therefore I_o = \frac{4}{5k} = 0.8mA$

$R_o = \frac{8}{0.8} = 10k\Omega$

∴ TE is,



#11.



Soln :- i) V_o :

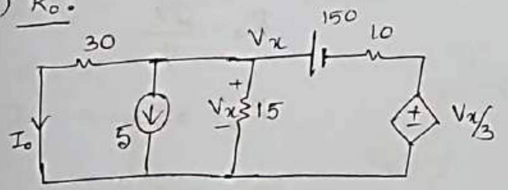
$$0 + 5 + \frac{V_x - 0}{15} + \frac{V_x - 150 - \frac{1}{3}V_x}{10} = 0$$

$$150 + 2V_x + 3\left[V_x - 150 - \frac{V_x}{3}\right] = 0$$

$$4V_x = 300$$

$$\boxed{V_x = 75V = V_o}$$

ii) R_o :



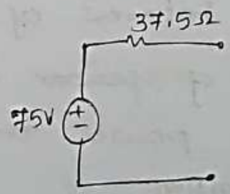
$$\frac{V_x - 0}{30} + 5 + \frac{V_x - 0}{15} + \frac{(V_x - 150) - V_x/3}{10} = 0$$

$$5V_x = 300$$

$$\boxed{V_x = 60V} \quad ; \quad I_o = \frac{V_x}{30} = \frac{60}{30} = 2A.$$

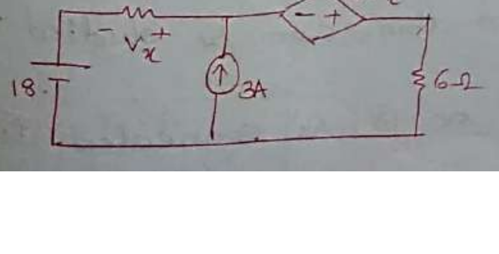
$$R_o = \frac{V_o}{I_o} = \frac{75}{2}$$

$$\boxed{R_o = 37.5\,\Omega}$$

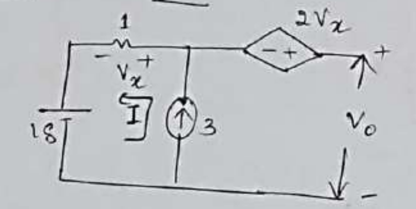


m-J07

12. Find thevenin's vtg., short ckt current & determine the actual current flowing through 6Ω.



⟹ soln :- i) V_o :



$V_o = 18 + V_x + 2V_x = 18 + 3V_x$

But $V_x = 1 \times I$

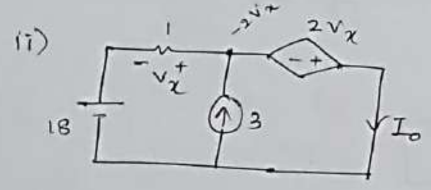
& $I = 3A$

$\therefore V_x = 3V$

$\therefore V_o = 18 + 9$

$V_o = 27V$

(ii)



$3 = I_o + \frac{-2V_x - 18}{1}$

But, $V_x = 1(3 - I_o)$

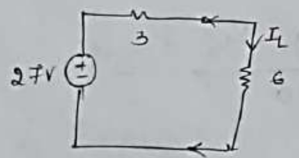
$3I_o = 27$

$I_o = 9A$

$R_o = \frac{27}{9}$

$R_o = 3\Omega$

$\therefore$ T.E is,



$I_L = \frac{27}{9}$

$\boxed{I_L = 3A}$

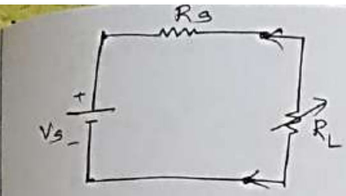
* **Maximum power transfer theorem:-**

General interest of any electrical ckt. is the transfer of power from src. to load. At all conditions the power transferred to the load will not be maximum.

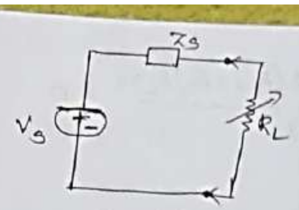
The condition for maximum power depends upon the type of src. i.e. DC/AC & the type of load. i.e. (R)/(Z).

Max. power transfer thm. can be studied under following cases.

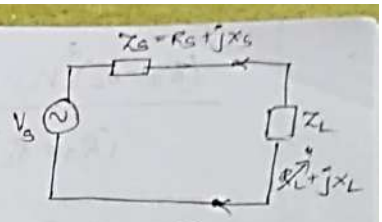
Case (1): DC src with src. (R) 'Rs' connected to load resistance 'RL' [RL is variable]



Case (1)



Case (2)

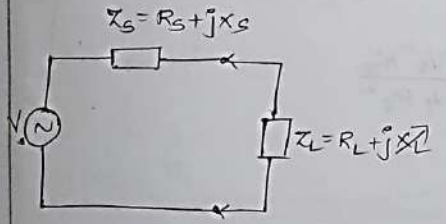


Case (3)

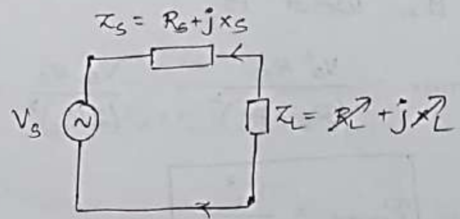
Case(2): AC src. with src. impedance 'Z_s' is equal to $R_s + jX_s$ connected to a load resistance 'R_L' [R_L = Variable]

Case(3): AC src with src impedance $Z_s = R_s + jX_s$ connected to a load impedance, $Z_L = R_L + jX_L$ [R_L = Variable, X_L = Constant]

Case(4): AC src with src impedance $Z_s = R_s + jX_s$ connected to load impedance. $Z_L = R_L + jX_L$ $\begin{cases} R_L = \text{Constant} \\ X_L = \text{Variable} \end{cases}$



Case (4)

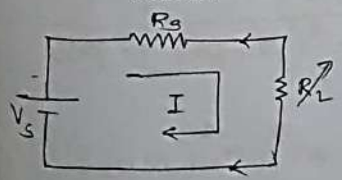


Case (5)

Case(5): AC src. with src impedance $Z_s = R_s + jX_s$ connected to load impedance $Z_L = R_L + jX_L$ [R_L = Variable, X_L = Variable]

Proof:-

Case(1): DC src. with src. resistance 'R_s' connected to load resistance 'R_L'



$$I = \frac{V_s}{R_s + R_L}$$

$$P = I^2 R_L = \frac{V_s^2 \cdot R_L}{(R_s + R_L)^2}$$

For max. power transfer, $\frac{dP}{dR_L} = 0$.

$$\frac{d}{dR_L}\left[\frac{V_s^2 \cdot R_L}{(R_s + R_L)^2}\right] = 0$$

$$\frac{(R_S+R_L)^2 V_S - 2(R_S+R_L)R_L V_S}{(R_S+R_L)^4} = 0$$

$$(R_S+R_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + \cancel{2R_LR_S} - \cancel{2R_LR_S} - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 = 0$$

$$R_S^2 = R_L^2$$

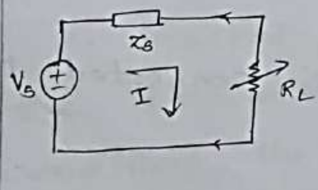
$\boxed{R_S = R_L}$ This is the condition for max. power transfer.

∴ Under this condition the max. power transferred to the load is,

$$P_{max} = \frac{V_S^2 R_S}{(R_S+R_S)^2} = \frac{V_S^2 R_S}{(2R_S)^2} = \frac{V_S^2 R_S}{4R_S^2}$$

$$\therefore \boxed{P_{max} = \frac{V_S^2}{4R_S}}$$

<u>Case(2)</u> AC src with $Z_S = R_S + jX_S$ connected to load resistance 'R_L' [R_L = variable]



$$I = \frac{V_S}{R_S + R_L + jX_S}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + X_S^2}}$$

$$P = |I|^2 R_L = \frac{V_S^2 \cdot R_L}{(R_S+R_L)^2 + X_S^2} \longrightarrow (*)$$

For max. power transfer, $\frac{dP}{dR_L} = 0$:

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2 + X_S^2}\right] = 0 \qquad \frac{[(R_S+R_L)^2 + X_S^2](1) - 2R_L(R_S+R_L)}{[(R_S+R_L)^2 + X_S^2]^2} = 0$$

$$(R_S+R_L)^2+X_S^2 - R_L[2(R_S+R_L)] = 0$$

$$R_S^2 + R_L^2 + 2\cancel{R_S R_L} + X_S^2 - \cancel{2R_L R_S} - 2R_L^2 = 0$$

$$R_S^2 + X_S^2 - R_L^2 = 0$$

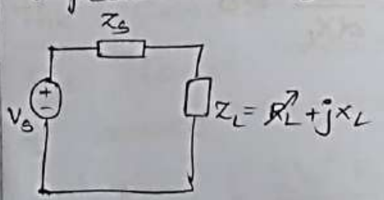
$$R_L^2 = R_S^2 + X_S^2 \; ; \quad \boxed{R_L = \sqrt{R_S^2 + X_S^2} = |Z_S|}$$

This is the Condition for max power transfer.

Replace $R_L \longrightarrow |Z_S|$ in (*)

$$\therefore \; P_{max} = \frac{V_S^2 \, |Z_S|}{(R_S + |Z_S|)^2 + X_S^2}$$

Case(3):- AC src with $Z_S = R_S + jX_S$ connected to load impedance $Z_L = R_L + jX_L$



$$I = \frac{V_S}{R_S + jX_S + R_L + jX_L}$$

$$\Rightarrow = \frac{V_S}{(R_S+R_L) + j(X_S+X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

$$\therefore \; P = |I|^2 R_L = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$$

For max. power transfer, $\frac{dP}{dR_L} = 0$

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}\right] = 0$$

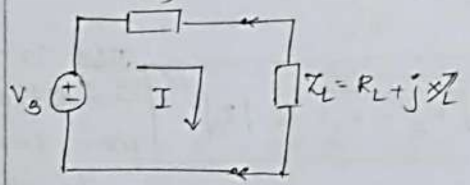
$$(R_S+R_L)^2 + (X_S+X_L)^2 - 2R_L(R_S+R_L) = 0$$

$$R_S^2 + R_L^2 + 2\cancel{R_S}R_L + X_S^2 + X_L^2 + 2X_SX_L - 2\cancel{R_S}R_L - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 + X_S^2 + X_L^2 + 2X_SX_L = 0$$

$$\therefore \; \boxed{R_L = \sqrt{R_S^2 + (X_S+X_L)^2}} \quad \text{(or)} \quad \boxed{R_L = |Z_S + jX_L|}$$

Case(4):- AC src. with $Z_S = R_S + jX_S$ connected to $Z_L = R_L + j\cancel{X_L}$



$$I = \frac{V}{Z_S + Z_L} = \frac{V_S}{(R_S+R_L) + j(X_S+X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

$$\therefore \quad P = |I|^2 R_L = \frac{V_S^2 R_L}{\left[\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}\right]^2} = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$$

For max. power ~~transfer~~, $\frac{dP}{dX_L} = 0$

$$0 - R_L\left[2(X_S+X_L)\right] = 0$$

$$X_S + X_L = 0$$

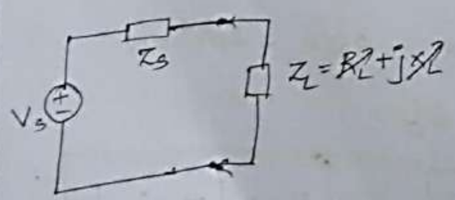
$$X_L = -X_S$$

The condition for max. power transfer is load reactance should be conjugate of source reactance. Under this condition,

$$P_{max} = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (\cancel{-X_L} + \cancel{X_L})^2}$$

$$P_{max} = \frac{V_S^2 R_L}{(R_S+R_L)^2}.$$

** Case(5):- AC source with $Z_S = R_S + jX_S$ connected to a $Z_L = \cancel{R_L} + j\cancel{X_L}$



$$I = \frac{V_S}{(R_S+R_L) + j(X_S+X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

$$P = |I|^2 R_L = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}$$

For max. power transfer, $\frac{dP}{dX_L} = 0$

$$\frac{d}{dX_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}\right] = 0.$$

$$0 - R_L\left[0 + 2(X_S+X_L)\right] = 0$$

$$X_S + X_L = 0$$

$$X_L = -X_S.$$

The condition for max. power transfer is the load reactance should be conjugate of source reactance. Under this condition,

$$P_{max} = \frac{V_S^2 R_L}{(R_S+R_L)^2}$$

$\therefore \frac{dP}{dR_L} = 0,$

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2}\right] = 0$$

$$(R_S+R_L)^2(1) - R_L\left[2(R_S+R_L)\right] = 0.$$

$$R_S^2 + R_L^2 + \cancel{2R_S R_L} - \cancel{2R_S R_L} - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 = 0$$

$$R_S^2 = R_L^2$$

WKT,

$Z_L = R_L + jX_L$

put, $R_L = R_S$ & $X_L = -X_S$

$\therefore Z_L = R_S - jX_S$

$\boxed{Z_L = Z_S^*}$

$\therefore$ The condition for max. power transfer is the load impedance should be complex conjugate of source impedance.

* Summary :-

i) $R_L = R_S = R_{th}$

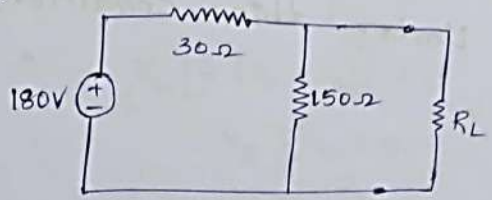
ii) $R_L = |Z_S|$

iii) $R_L = |Z_S + jX_L|$

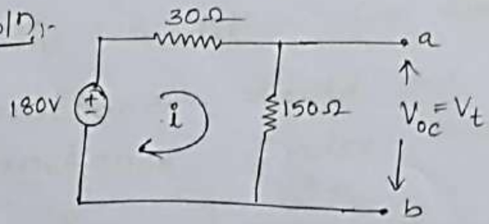
iv) $X_L = -X_S$

v) $Z_L = Z_S^* = Z_{th}^*$

1. Find the load 'R_L' that will result in maximum power delivered to the load for the ckt. of fig. Also determine the max power P_{max}.

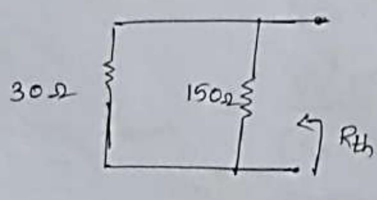


⟹ soln:-



$$i = \frac{180}{30+150} = 1A$$

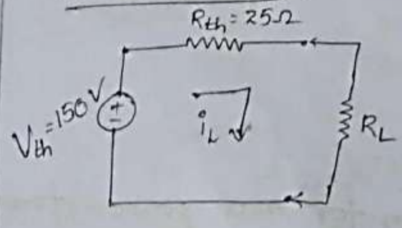
$$V_{oc} = V_{th} = 150 \cdot i = 150V$$



$$R_{th} = 30 || 150 = \frac{30 \times 150}{180}$$

$$R_{th} = 25\Omega$$

Thevenin's ckt,



∴ For max power transfer,

$$R_L = R_{th} = 25\Omega$$

$$\therefore P_{max} = \frac{V_{th}^2}{4R_L} = \frac{150^2}{4 \cdot 25}$$

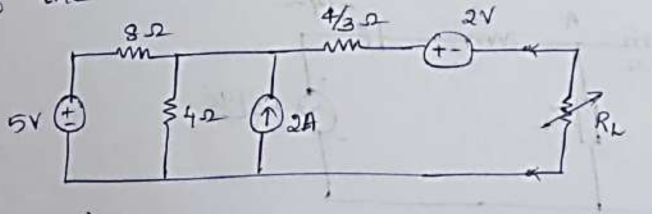
$$\boxed{P_{max} = 225W}$$

$\therefore$ Total power, $P_t = 150 \cdot i_L$

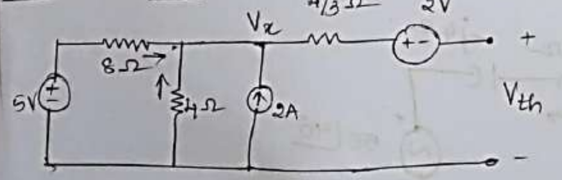
$$P_t = 150 \cdot \left[\frac{150}{25+25}\right]$$

$$\boxed{P_t = 450 \text{ Watts}}$$

2. Find the value of 'R_L' for max. power to be transferred to the load. Also find max. power transferred.



→ Soln:- i) V_{th}:



$V_{th} = V_x - 2$

$$2 + \frac{0 - V_x}{4} + \frac{5 - V_x}{8} = 0$$

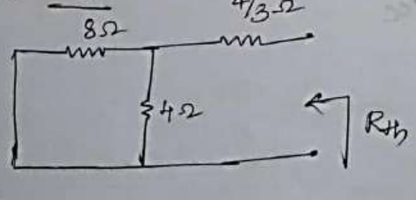
$$16 - 2V_x + 5 - V_x = 0$$

$$-3V_x = -21$$

$$V_x = 7\text{ V}$$

$$\therefore V_{th} = 7 - 2 = 5\text{V}$$

ii) R_{th}:

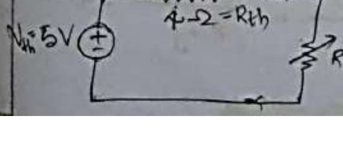


$$R_{th} = \frac{4}{3} + [8 \| 4]$$

$$= \frac{4}{3} + \frac{4 \times 8}{12}$$

$$= 4\Omega$$

Thevenin's ckt,



$\therefore$ For max. power transfer, $R_L = R_{th}$

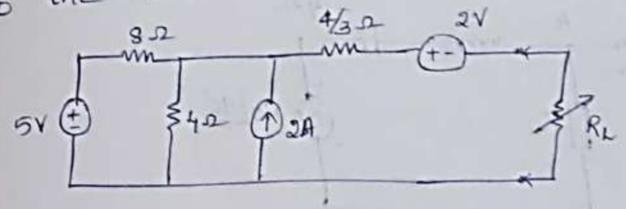
$R_L = 4\Omega$

$\therefore$ Total power, $P_t = 150 \cdot i_L$

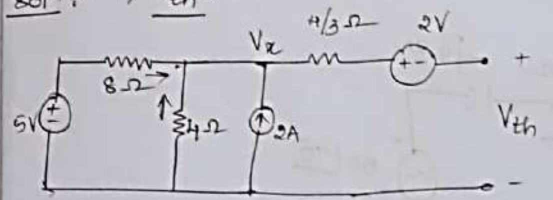
$$P_t = 150 \cdot \left[\frac{150}{25+25}\right]$$

$$\boxed{P_t = 450 \text{ Watts}}$$

2. Find the value of 'R_L' for max. power to be transferred to the load. Also find max. power transferred.



$\rightarrow$ Soln:- i) V_{th}:



$$V_{th} = V_x - 2$$

$$2 + \frac{0 - V_x}{4} + \frac{5 - V_x}{8} = 0$$

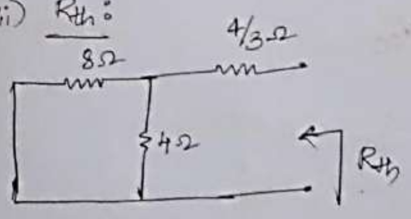
$$16 - 2V_x + 5 - V_x = 0$$

$$\cancel{-}3V_x = \cancel{-}21$$

$$V_x = 7\text{ V}$$

$$\therefore V_{th} = 7 - 2 = 5\text{V}$$

ii) R_{th}:

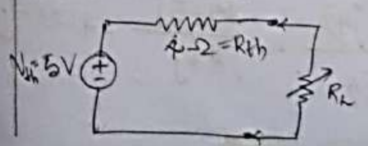


$$R_{th} = \frac{4}{3} + [8 \| 4]$$

$$= \frac{4}{3} + \frac{4 \times 8}{12}$$

$$= 4\Omega$$

Thevenin's ckt,



$\therefore$ For max. power transfer, $R_L = R_{th}$

$R_L = 4\Omega$

Maximum power transfer, $P_{max} = I^2 R_L$

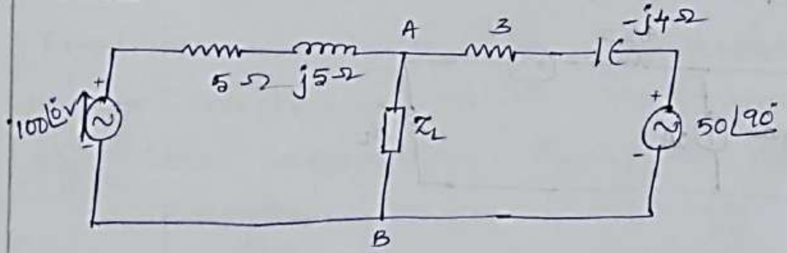
$$= 4 \times \left[\frac{5}{4+4}\right]$$

$$= \frac{20}{8}$$

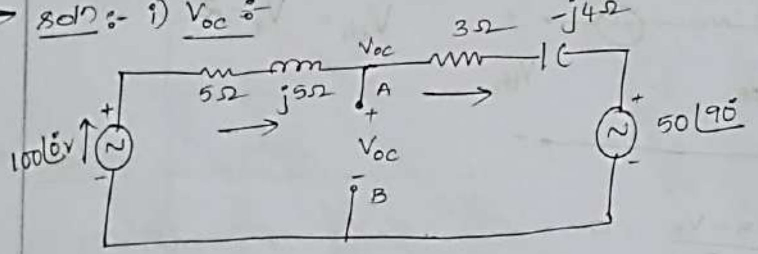
$$\boxed{P_{max} = 1.5625 \text{ Watts}}$$

May-June 10

3. Find the max. P.T. to load 'Z_L' of the n/w:



→ sol^n :- i) V_{oc} :-



$$\frac{100\angle 0° - V_{oc}}{5+j5} = \frac{V_{oc} - 50\angle 90°}{3-j4}$$

$$(3-j4)(100) - (3-j4)V_{oc} = (5+j5)V_{oc} - 50j(5+j5)$$

$$(300-j400) - (3-j4)V_{oc} - (5+j5)V_{oc} + 250j - 250 = 0$$

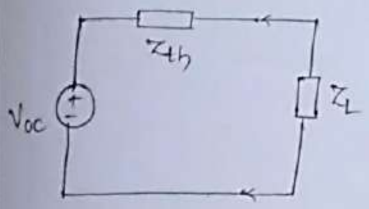
$$(50-j150) = [3+5-j4+j5]V_{oc}$$

$$V_{oc} = \frac{50-j150}{8+j}$$

$$V_{oc} = (3.84 - 19.2j)V$$

ii) Z_{th}: $(5+j5) \| (3-j4) = \dfrac{(5+j5)(3-j4)}{5+j5+3-j4} = \dfrac{35-5j}{8+j} = 4.23 - 1.15j$

Thevenin's ckt,



From fig., it belongs to case(5),

$$Z_L = Z_{th}^* = 4.23 + 1.15j$$

$$\therefore P_{max} = R_L \cdot I^2$$

$$I = \frac{3.84 - 19.2j}{4.23 \cancel{-1.15j} + 4.23 \cancel{+1.15j}} = 0.45 - 2.26j$$

$\therefore$