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KLE COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

KLE COLLEGE OF ENGINEERING AND TECHNOLOGY CHIKODI

CONTROL SYSTEMS NOTES (18EC43)

(As per Choice based Credit System (CBCS) Scheme)IVTH SEMESTER

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

KLE COLLEGE OF ENGINEERING AND TECHNOLOGY CHIKODI

"Don't see others doing better than you, beat your own records every day, because success is a fight between you and yourself"

CONTENTS

MODULE-1

Syllabus:

- Introduction to Control Systems:
- Types of Control Systems
- Effect of Feedback Systems
- Differential equation of Physical Systems
  - Mechanical Systems,
  - Electrical Systems, and
  - Analogous Systems.

Introduction to Control System :-

Control System :- It is an Arrangement of different physical Components such that it gives the desire Output for the given input by means of regulate or Control either direct or indirect method.

Plant :- It is defined as the portion of a system which is to be controlled or regulated. It is also called as Process.

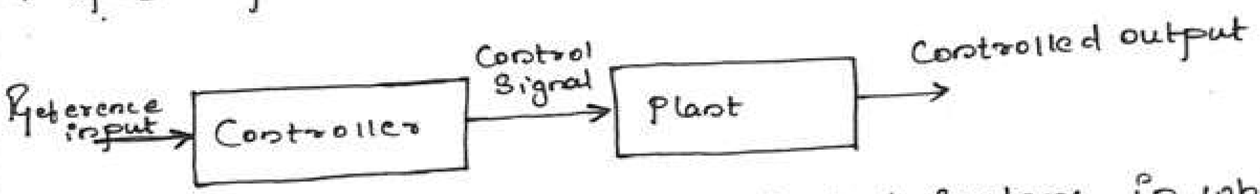
Controller :- It is an Element of the system itself or may be external to the System. It Controls the plant or the process.

Input :- The applied Signal or Excitation Signal that is applied to a Control system to get a specified Output is Called input.

Output :- The actual response that is Obtained from a control system due to the application of the input is termed as Output

Types Of Control Systems :-

* Open - loop Control System :-



* Open - Loop Control system are Control systems in which the output has no Effect upon the Control action.

* In such systems, there is no measurement of the Output and no-subsequent use of that Output to generate any Control action.

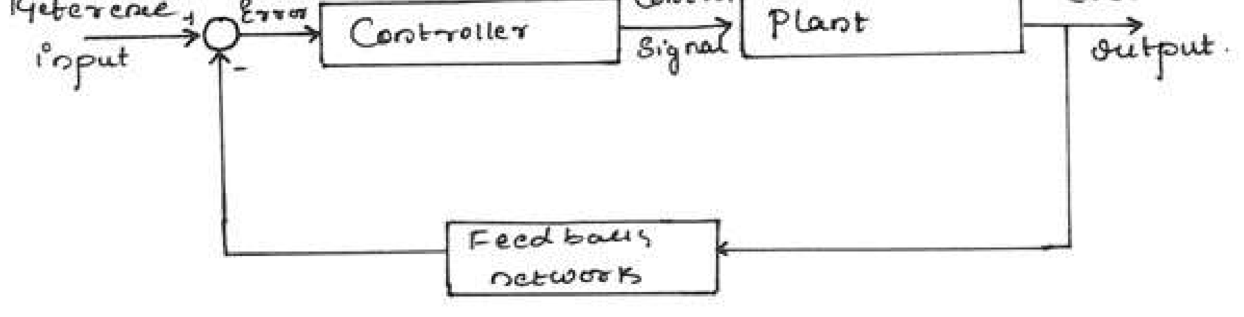
Advantages:-

* It is simple to Construct and Easy to maintain.
* It is less Expensive because of the use of minimum control devices.
* The problem of instability does not Exist.
* It is able to perform accurately once the calibration of the input is done.

Dis-advantages:-

* Disturbances, internal or External, Causes drifts in the desired Output.
* Changes in Calibration Cause Errors in the system.
* Re-Calibration of the system may be necessary from time-to-time in order to maintain the required quality of the Output.

Closed-Loop Control System:-



* A closed - loop Control System is one that measures its Output and adjusts its input accordingly by using a feedback signal.

* Feed back networks Consists of passive elements like R, L, C Which is been used to feed back the Obtained Output to the input.

* The Controller subsequently produces the necessary Control Signal, Which is then Applied to the plant or process to reduce the Error and bring the Output of the system to the desired value.

* In closed loop system the functionality of the system depends on the difference between input and the feedback.

* A closed loop Control System is also Called as feedback Control System.

Advantages:-

* Relatively more accurate and Expensive Components may be used to Obtain an accurate Control of a given process.

* The influence of internal and External disturbances On the Output can be made almost ineffective.

* Transient response of the system Can be improved.

* Steady - state Error can be reduced.

Dis advantages:-

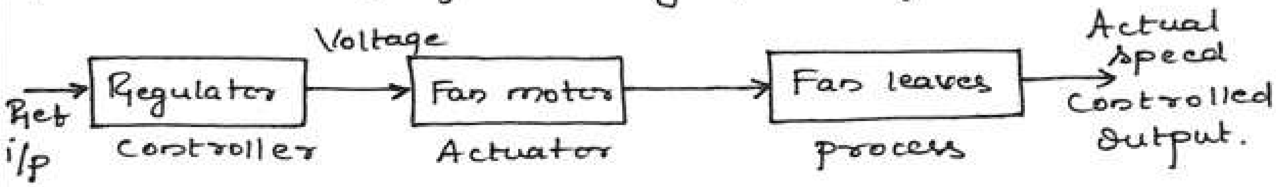
* It requires more Equipment and Components and it is costlier
* There is a tendency to over correct Errors, which may create oscillations in the system output. This may cause the system to drift to instability.

Comparision between Open-loop and Closed-Loop Control System.

SL.NO	Open-Loop Control System	Closed Loop Control System
1.	The Open-loop System are simple to construct and cheap.	The Closed Loop System are complicated to construct and costly.
2.	It consumes less power	It consumes more power.
3.	Any change in the output has no effect on the input i.e, feed back does not Exists	Changes in the output, affects the input which is possible by use of feed back.
4.	Highly sensitive to the disturbances	Less sensitive to the disturbances.
5.	It is inaccurate and un-reliable	Highly accurate and reliable.
6.	The open-loop systems are generally stable.	More care is required to design a stable System
7.	Highly sensitive to the Environmental changes	Less sensitive to the Environmental Changes.

Application of an Open-loop system :-

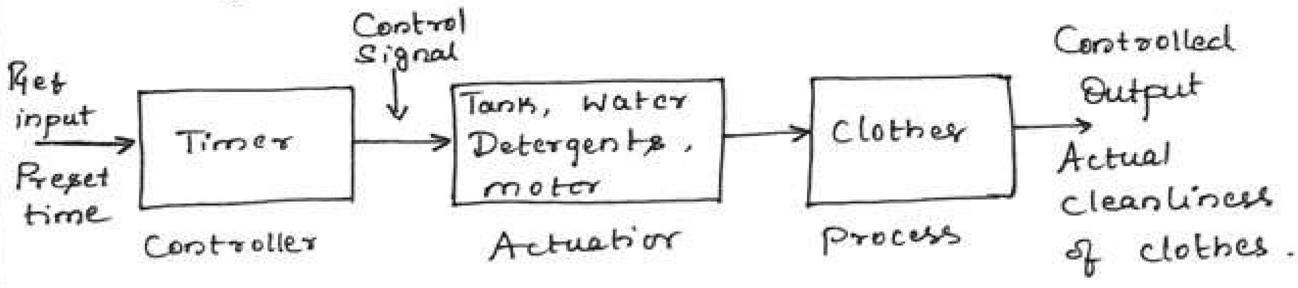
1) Control action of a Celing - fan regulator :-



Block diagram of Fan Regulator

A fan regulator is a Combination of a Series Switch and a speed regulator. For a given speed setting on the fan regulator, the fan runs at a specific speed. To obtain a different speed the setting on the regulator is to be changed This changes the voltage applied to the motor The specific setting on the regulator is the reference input and the Variable voltage applied to the fan motor is the Control Signal. The speed of the fan is the Controlled Output. the Output speed is not measured so the Control scheme is Open-loop The block diagram is shown in the figure above.

2) Working of an automatic Washing machine :-

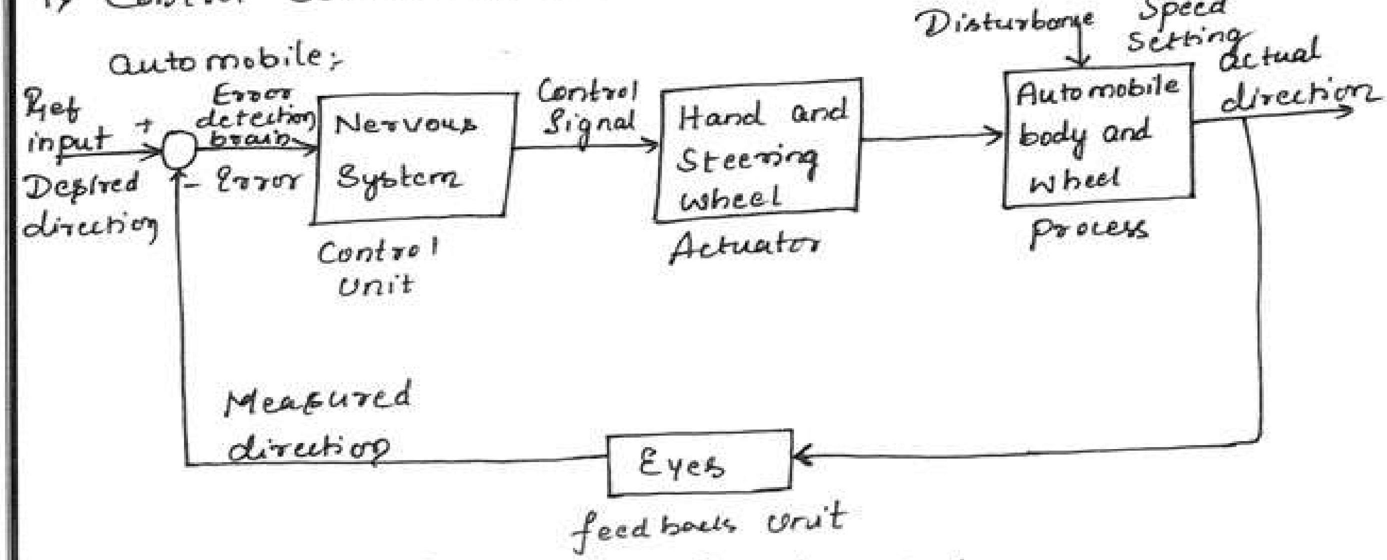


Block diagram of Automatic Washing Machine.

* An automatic washing machine is one with present wash times. The different operations like soaking, washing, rinsing, wringing and drying are all performed on a time basis.
* After the clothes to be washed are kept inside the tank, water and detergent are added in proper amounts
* The timer and the relays act as the controller. The tank, water, detergent and the motor constitute the actuator.
* The clothes that are to be washed from the process. There is no measurement of quality of wash. If cleanliness of clothes is the output parameter, there is no mechanism to judge it. The system is thus open-loop. The block diagram is shown above.

Applications of an closed loop Control System:-

1) Control Scheme in the manual steering system of an automobile;



(Automobile Steering control)

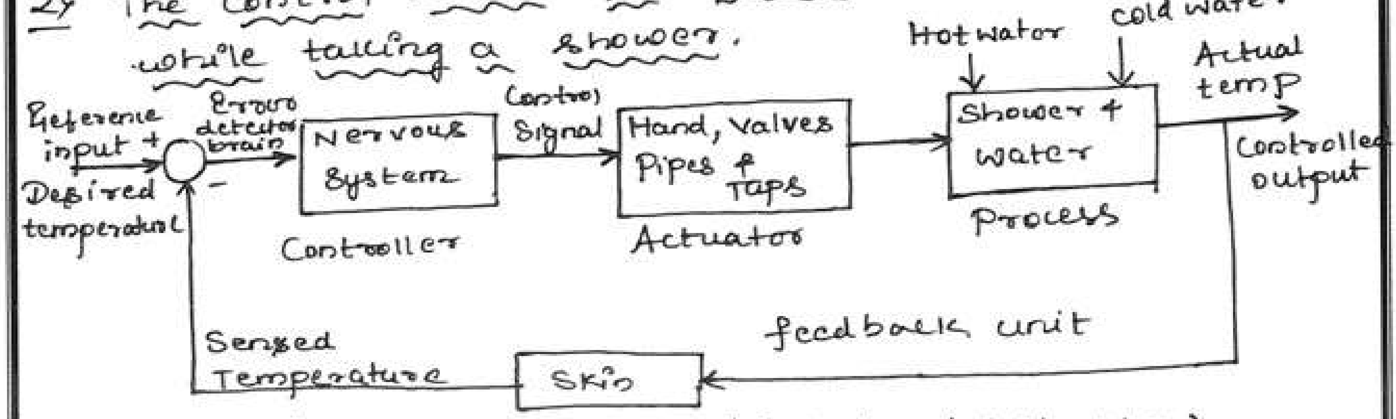
* The driver of an automobile watches the direction or heading of the vehicle with respect to a specified direction of the road. The eye senses the deviation, if any, and the information is fed to the brain through the nervous system.

* The brain processes the signal and generates a corrective signal and it is transmitted to the hand and then to the steering wheel. The direction of the road is the reference input and heading of the automobile is the controlled input.

* The human eye acts as the error detector. The brain and the nervous system acts as the comparator and the controller. The corrective signal generated in the brain is the control signal which is transmitted to the hands.

* The human hands and the steering wheel from the actuator unit, The automobile body and the wheels from the process. The scheme represents a manual feedback system and it is shown in the figure above.

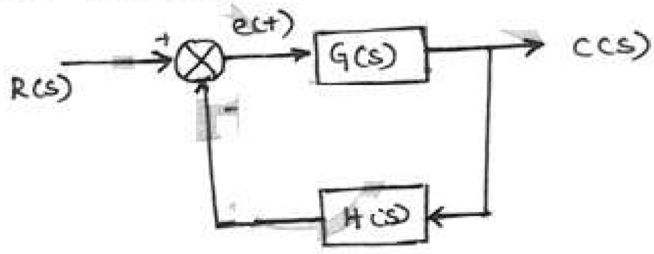
2) The control scheme for mixing cold and hot water while taking a shower.



(Block Diagram of mixing hot and cold water)

* One must have some idea of the water temperature he/ she wants while taking a shower. The skin acts as a temperature sensor, which measures the temperature not quantitatively but qualitatively, and conveys the information to the brain. These it is compared with the water temperature the person desires. the brain computes the differences in terms of 'too cold' or 'too hot' and activates the hand muscles to manipulate the hot and cold water valves to reduce the temperature if it is too hot or increase the temperature if it is too cold. This corrective action is reciprocal until the required water temperature is achieved. The block diagram is shown above.

Feedback System:-



* If Error signal e(t) is zero, output is controlled.
* If Error signal e(t) is not zero, output is not controlled.

For positive feedback, Error signal $= r(t) + c(t)$

For Negative feedback, Error signal $= r(t) - c(t)$

* The purpose of feedback is to reduce the Error between the reference input and the system output.

⇒ +ve feed back

Unity feed back ($H(S)=1$)

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1-G(S)}$$

Non-unity feed back ($H(S) \neq 1$)

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1-G(S)\,H(S)}$$

⇒ −ve feed back

Unity feed back ($H(S)=1$)

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$$

Non-Unity feed back ($H(S) \neq 1$)

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)\,H(S)}$$

Where $G(S)$ = T.F of the forward path.

$H(S)$ = T.F of the feed back path.

* The feed back has Effects on such System performance characteristics as stability, bandwidth, Overall gain, impedance and sensitivity.

* Effect of feed back On stability:

* Stability is a notion that describes whether the System will be able to follow the input Command.
* A System is said to be unstable, if its Output is Out of Control.
* The closed loop System stability depends On loop gain. If loop gain $GH = 1$, the Output of the System becomes infinity for any finite input. and the System is said to be unstable.

* If the loop gain > 0 then system stability is improved, The feedbacks can improve stability or be harmful to stability if it is not properly applied

⟹ Effect of feedback on Overall gain :-

* feedback affects the gain G of a non-feedback System by a factor of $1 \pm GH$. The general effect of feedbacks is that it may increase or decrease the gain.
* In a practical Control System G and H are functions of frequency, so the magnitude of $1+GH$ may be >1 in on frequency range, but <1 in another. Therefore, feedback could increase ~~the~~ gain of the system in one frequency range but decrease it in another.

⟹ Effect of feedbacks on Sensitivity

* In general a good control system should insensitive to parameter variations but sensitive to input command.
* Consider G as a parameter that may vary. The sensitivity of the gain of the overall system M to the variation in G is defined as

$$S_G^M = \frac{\partial M/M}{\partial G/G}$$

* Where ∂M denotes the incremental change in M due to incremental change in G

$\partial M/M$ and $\partial G/G$ denote the percentage change in M and G respectively.

$$S_G^M = \frac{\partial M}{\partial G} \times \frac{G}{M} = \frac{1}{1+GH}$$

This relation shows that the sensitivity function can be made arbitrarily small by increasing GH, provided that the system remains stable. In an open-loop system, the gain of the system will respond in a one-to-one fashion to the variation in 'G'. In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located.

⇒ Effects of feedback (In-Brief)

* Gain is reduced by a factor
* There is reduced of parameter variation by a factor $1 + G(s)H(s)$
* There is improvement in sensitivity.
* There may be reduction of stability.

The disadvantages of reduction of gain and reduction of stability can be overcome by gain amplification and good design respectively.

* Feedback reduces the Effect of noise and disturbance On system performance.
* Band width increases by the factor of $1+G(s)H(s)$
* The System becomes more accurate.

Mechanical Systems:-

Mechanical System are broadly classified into two groups.

1) Translational System.

2) Rotational system

Translational System: In translational System, the motion of the body is along a straight line

Rotational System:- In Rotational System, the motion of the body is about it's own axis. (Circular Path).

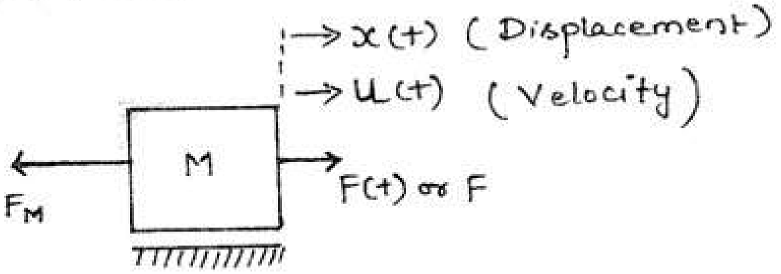
* The Basic Elements of translational System are.

① Mass

② Spring

③ Dashpot or friction.

① Mass:- Mass is the Energy (Kinetic) storage elements, where Energy Can be stored and retrieved without loss.



When a force $F(t)$ is applied on the mass, it Produce an oppossing force F_M and it is given by

$$F_M \propto a \qquad F_M = Ma$$

$$F_M = M\frac{du(t)}{dt} = M\frac{d}{dt}\left(\frac{d}{dt}x(t)\right)$$

$$\boxed{F_M = M\frac{d^2x(t)}{dt^2}}$$

Where
$x(t)$ and $u(t)$ are the displacement and the velocity respectively.

M is the mass, the force due to acceleration is given by

$$\boxed{F_M = M\frac{d^2x(t)}{dt^2}}$$

At Equilibrium according to network laws $F(t) = F_m$

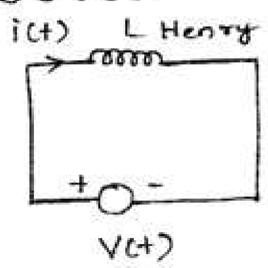
$$\therefore \boxed{F(t) = M\frac{d^2x(t)}{dt^2}}$$

W.K.T

M → Mass is an inertial element it stores Energy in the form of Kinetic Energy is given by.

$$\boxed{K.E = W = \frac{1}{2}mu^2}\ J \quad \text{--- (1)}$$

inductance:



$$v(t) = L\frac{d\,i(t)}{dt} \quad \text{--- (2)}$$

$$v(t) = L\frac{d^2q(t)}{dt^2} \quad \text{--- (3)}$$

$$\boxed{W = \frac{1}{2}LI^2}\ J \quad \text{--- (4)}$$

Two Systems are said to be Analogous if the Mathematical Equations of the two Systems are identical

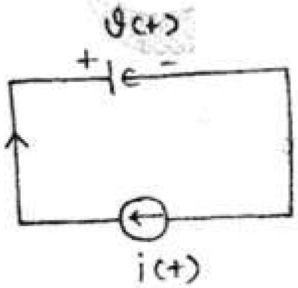
if $F(t) = V(t)$

and $M = L$ then $x(t) = q(t)$ or $u(t) = i(t)$

When force is compared with Voltage the Corresponding Electric Circuit is said to be force - Voltage Analogous Circuit.

Mechanical System	Electrical System F-V Analogy
Force	Voltage
Velocity	Current
Displacement	Charge
Mass	inductance.

Capacitance :-



$$i(t) = C \frac{d\,v(t)}{dt}$$

but $v(t) = \frac{d}{dt}\phi(t)$,

$$i(t) = C \frac{d^2 \phi(t)}{dt^2} \quad \text{— (5)}$$

$$\boxed{W = \frac{1}{2} C V^2(t)} \, J \quad \text{— (6)}$$

Comparing Equations (1) and (5) they are Analogous if

$F(t) = i(t)$

$M = C$

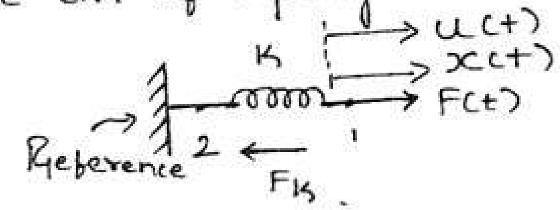
Then $x(t) = \phi(t)$

$u(t) = v(t)$

* Mass has only one displacement.
* Counter force produced by the mass is propotional to second derivative of displacement.
* When force is compared with voltage inductance is the electrical analog for mass and when force is compared with current capacitance is electrical analog for mass.

2) Spring :- Spring is the Energy (Potential) Storage Element where Energy can be stored and retrieved without loss.

(i) When one End of spring is connected to the reference.



For a linear spring counter force produced by the spring is propotional to net displacement of the spring.

$$F_K \propto (x(t) - 0)$$

$$F_K = K\,x(t)$$

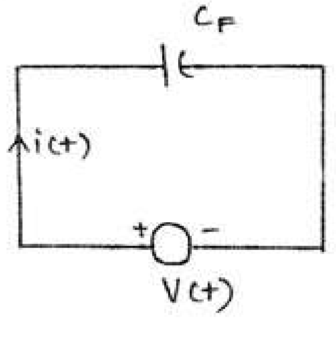
Where K is the constant of propotionality known as the spring constant.

At Equilibrium, according to Newton law.

$$F(t) = F_K$$

$$\boxed{F(t) = K\,x(t) = K \int u(t)\,dt} \quad \text{— ①}$$

Capacitance :-



$$i(t) = \frac{d\,q(t)}{dt}$$

$$q(t) = \int i(t)\,dt$$

$$v(t) = \frac{q(t)}{C} = \frac{1}{C}\int i(t)\,dt \quad \text{— ②}$$

Comparing Equation ① & ② They are analogous

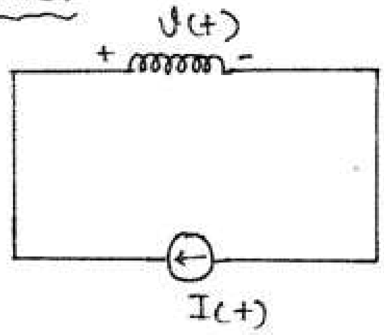
if $F(t) = V(t)$

$K = \frac{1}{C}$

$x(t) = q(t)$

$u(t) = i(t)$

Inductance:-



$$I(t) = \frac{1}{L}\int V(t)\,dt = \frac{1}{L}\,\phi(t) \quad \text{— ③} \qquad \because V(t) = \frac{d\,\phi(t)}{dt}$$

Equations ① and ③ are analogous if

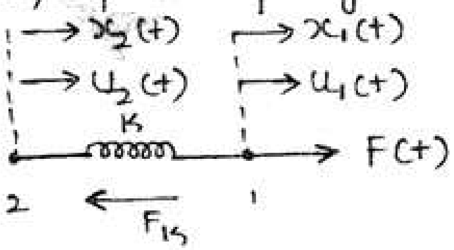
$F(t) = I(t)$

$K = \frac{1}{L}$

$x(t) = \phi(t)$

$u(t) = V(t)$

ii) When both Ends of the spring are free to move.



Counter force produced by the spring $F_K \propto (x_1(t) - x_2(t))$

$$F_K = K\left[x_1(t) - x_2(t)\right]$$

At equilibrium according to Newton's law.

$$F(t) = F_K$$

$$F(t) = K\left[x_1(t) - x_2(t)\right]$$

* If one End of the Spring is connected to the reference it has one displacement and if both Ends are free to move it has two displacements.

* Counter force produced by Spring is proportional to net displacement of the Spring

* When force is Compared With Voltage, Capacitance is the Electrical Analog. When force is Compared With Current, inductance is the Electrical Analog for the Mechanical Element Spring.

3) Dashpot or Friction :-

* The friction Exists in physical Systems whenever Mechanical Surfaces are operated in sliding contact.

* There are 3 types of Friction they are.

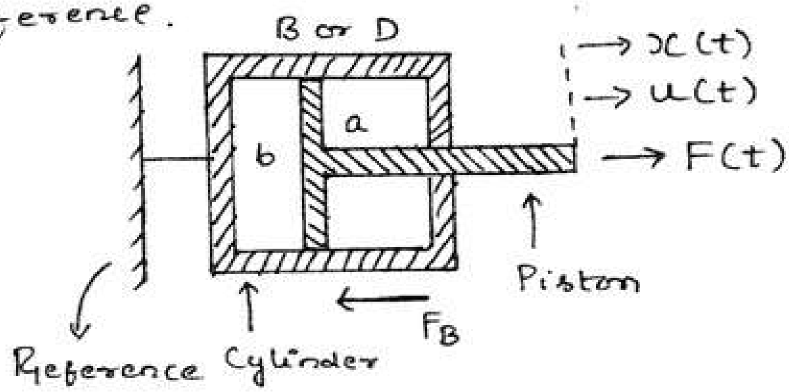
a) Coulomb friction force :- This is the force of sliding friction between dry Surfaces. Coulomb friction force is substantially Constant.

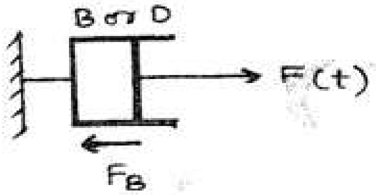
b) Viscous friction force :- It is the friction between moving Surfaces separated by a Viscous fluid or between a solid body and a fluid medium. it is Proportional to the Velocity. it is predominates

c) Stiction :- This is the force required to initiate motion between two Contacting Surfaces.

Viscous friction :-

Case i: When one end of the dashpot is connected to the reference.





The Counter force produced by the Dashpot is Propotional relative velocity between piston and the cylinder.

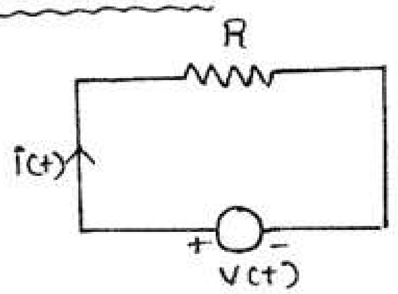
$$F_B \propto (u(t) - 0)$$

$$F_B = B\,u(t) = B\frac{dx(t)}{dt}$$

B is the constant of propotionality known as the viscous friction Co-efficient. At Equilibrium according to Newton law $\boxed{F(t) = F_B}$

$$\boxed{F(t) = B\,u(t) = B\frac{dx(t)}{dt}} \quad \text{—} ①$$

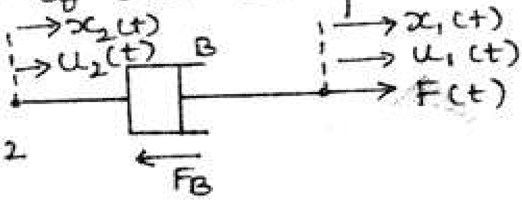
Resistance:-



$$v(t) = R \cdot i(t) = R \frac{dq(t)}{dt}.$$

Case ii:

When both Ends of the dashpot are free to move.



The Counter force produced by the dashpot is

$$F_B \propto (u_1(t) - u_2(t))$$

$$F_B = B\left[u_1(t) - u_2(t)\right]$$

$$F_B = B\left[\frac{d}{dt}x_1(t) - \frac{d}{dt}x_2(t)\right]$$

At Equilibrium according to Newton's law.

$$F(t) = F_B$$

$$F(t) = B\left[u_1(t) - u_2(t)\right] = B\frac{d}{dt}\left[x_1(t) - x_2(t)\right]$$

* When One End of the dashpot is Connected to the reference it has One displacement and if both Ends are free to move it has two displacement.
* Counter force produced by the dashpot is propotional first derivative of net displacement.
* When force is Compared with Voltage resistance is the Electrical analogy for dashpot and when

force is compared with current, Conductance is the Electrical analogy for the Mechanical Element dashpot.

$$F(t) = B\,u(t) = B\frac{d\,x(t)}{dt}$$

$$V(t) = R\,i(t) = R\frac{d\,q(t)}{dt}$$

$$i(t) = \frac{v(t)}{R} = G\,u(t) = G\frac{d\,\phi(t)}{dt}$$

Tabulation for Converting Mechanical System to Force-Voltage and Force-Current analogy.

Mechanical System	Electrical System	
	F-V Analogy	F-I Analogy
Force [F(t)]	Voltage [V(t)]	Current [I(t)]
Velocity [u(t)]	Current [I(t)]	Voltage [V(t)]
Displacement [x(t)]	Charge [Q(t)]	Magnetic Flux [$\phi(t)$]
Mass [M]	Inductance [L]	Capacitance [C]
Spring Constant [K]	Reciprocal of Capacitance $\left[\frac{1}{C}\right]$	Reciprocal of inductance $\left[\frac{1}{L}\right]$
Compliance $\left[\frac{1}{K}\right]$	Capacitance [C]	Inductance [L]
Dashpot Constant [B]	Resistance [R]	Conductance [G]

Things to remember

F-V Analogy:-

$$\Rightarrow \frac{dq(t)}{dt} = i(t)$$

$$\Rightarrow q(t) = \int i(t)\, dt$$

F-I Analogy:-

$$\Rightarrow \frac{d\phi(t)}{dt} = V(t)$$

$$\Rightarrow \phi(t) = \int V(t)\, dt.$$

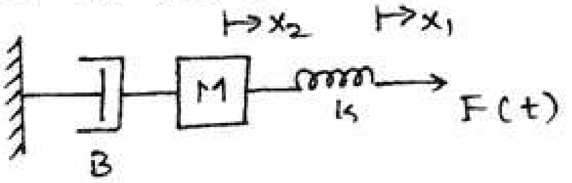
Note:-

* If the force directly acts on the mass, then the number of displacement is Equal to number of masses in the system, provided there is no direct series Connected of two springs or two dashpot or dashpot and a spring.

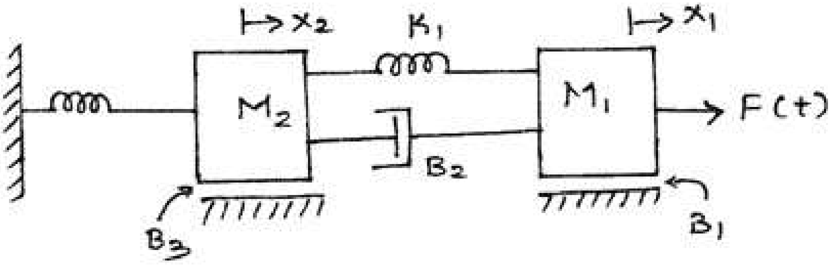
* If the force directly acts on the spring or the dashpot then the number of displacement in the Mechanical System is Equal to number of masses +1. Provided there is no direct series Connection of two spring or two dashpot and spring and a dashpot.

Problems to be solved in class

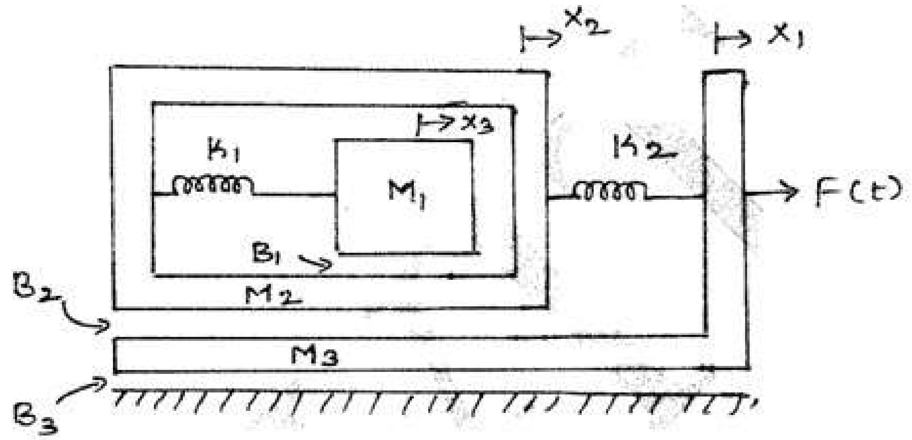
1)



2)



3)



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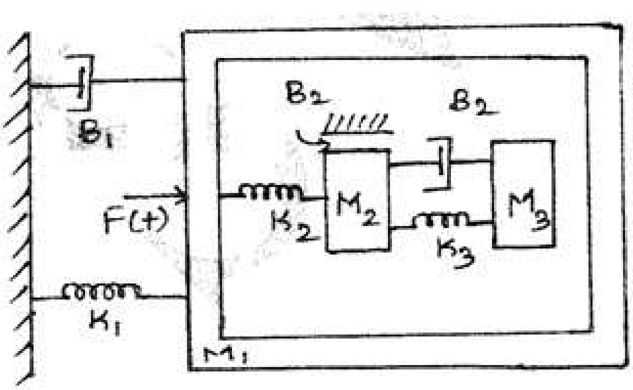

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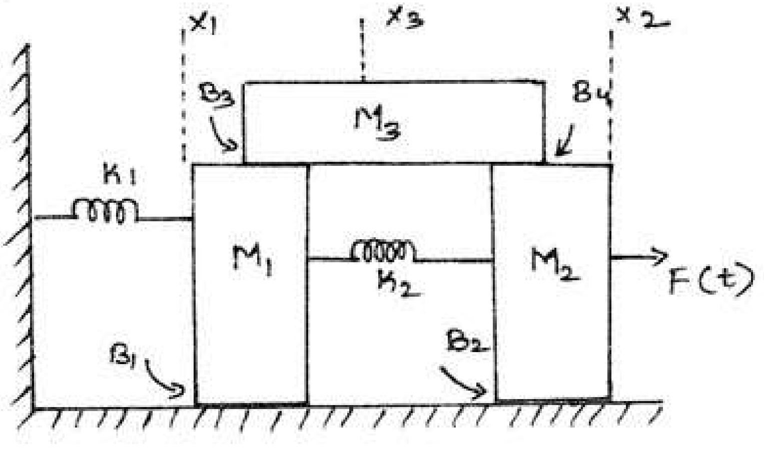
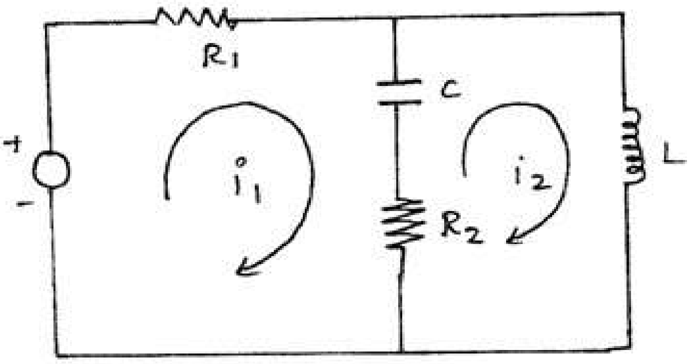
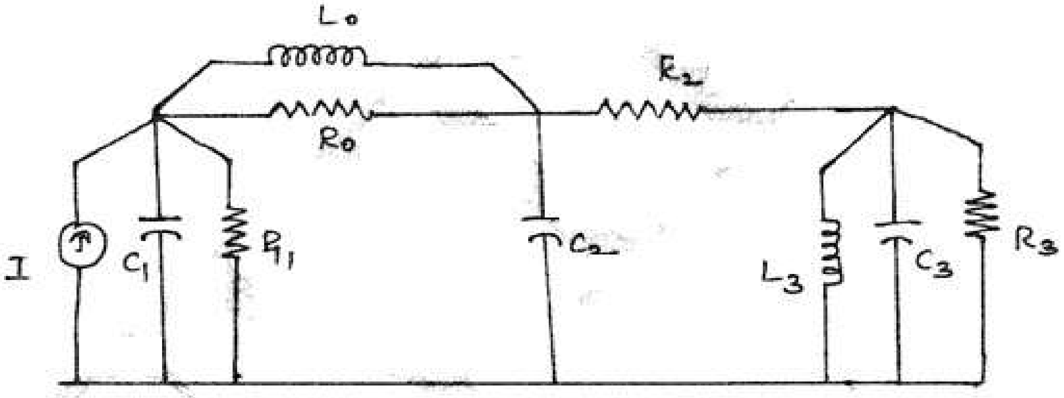


6) Draw the mechanical system for a given force Voltage. analogous Electric Circuit



7) Draw the Mechanical System for a given force Current analogous electric Circuit.

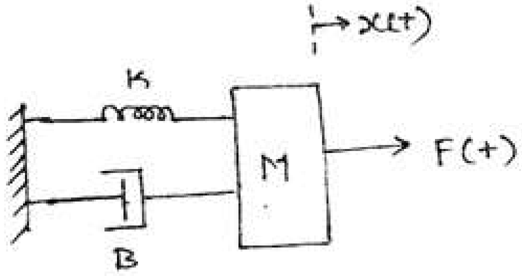


Problems on Translational Systems:-

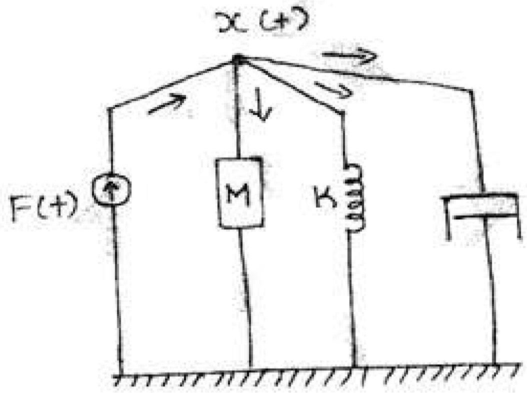
⇒ Write differential Equations for the mechanical Systems shown below and also Electrical Analogous Circuit based on force Voltage analogy or force Current analogy.

Dec 2009

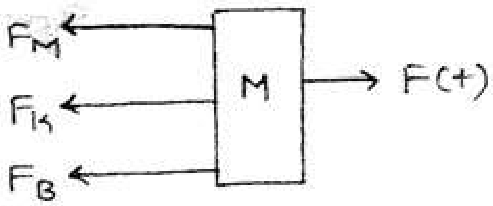
1)



Solution:- The mechanical network for a given mechanical system is as shown below



The free body diagram is as follows



The Equilibrium Equation of a System are

$$F(t) = F_M + F_B + F_K$$

$$F(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) \longrightarrow (1)$$

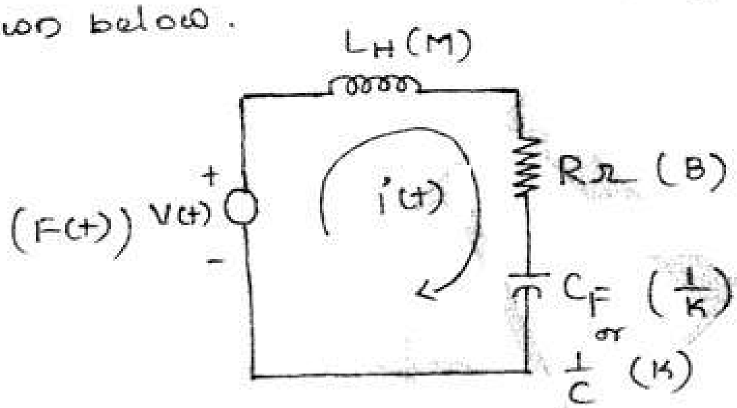
F-V Analogy :- Substituting the Electrical Analagous based on force voltage analogy in Equation ①

$$V(t) = L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t)$$

But $i(t) = \frac{dq(t)}{dt}$ & $\int i(t)\,dt = q(t)$

$$V(t) = L\frac{d}{dt}i(t) + R\,i(t) + \frac{1}{C}\int i(t)\,dt \rightarrow ②$$

The Electrical Circuit Satisfying Equation ② is as shown below.



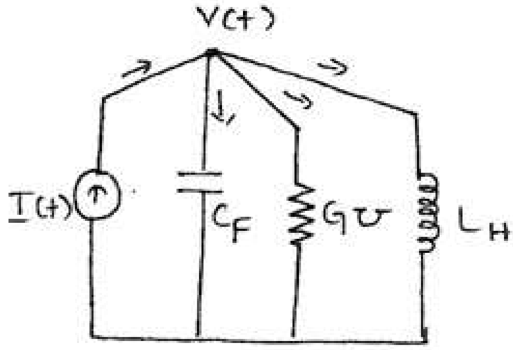
F-I Analogy :- Substituting the electrical analogous based on ~~force~~ Current analogy in Equation ① we get.

$$I(t) = C\frac{d^2\phi(t)}{dt^2} + G\frac{d\phi(t)}{dt} + \frac{1}{L}\phi(t)$$

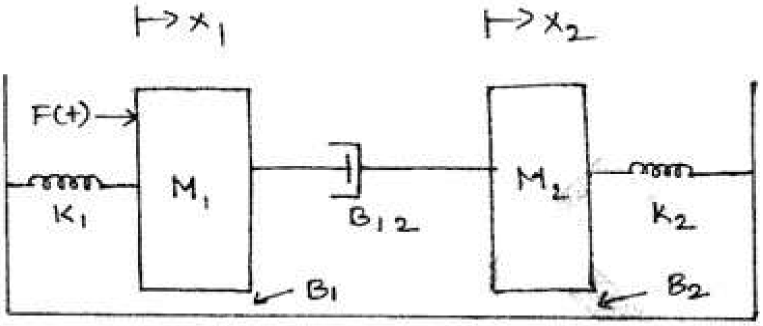
But $V(t) = \frac{d\phi(t)}{dt}$ & $\int V(t)\,dt = \phi(t)$

$$I(t) = C\frac{dV(t)}{dt} + G\,V(t) + \frac{1}{L}\int V(t)\,dt \quad — ③$$

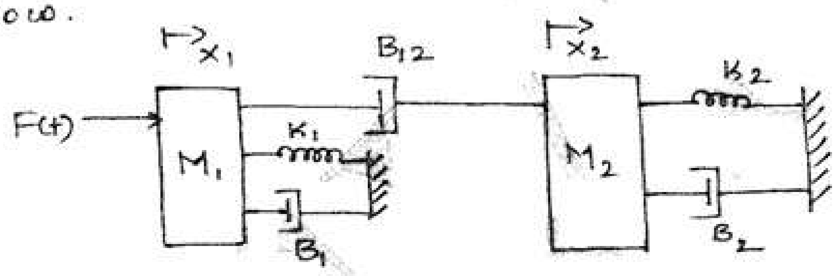
The electrical Circuit Satisfying Equation ③ is as shown below.



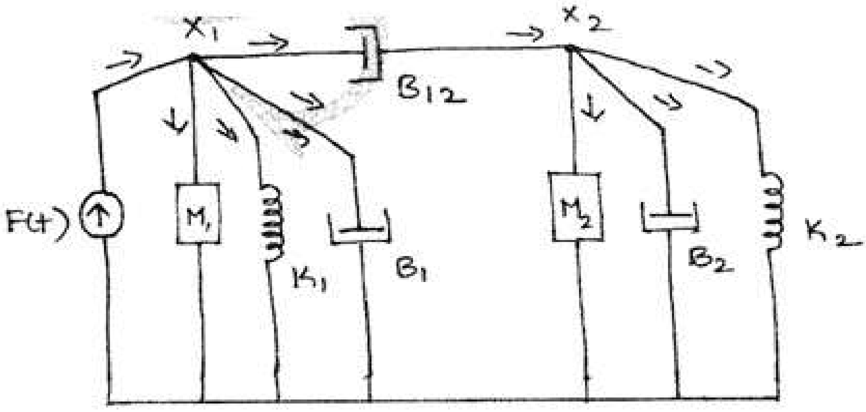
2)



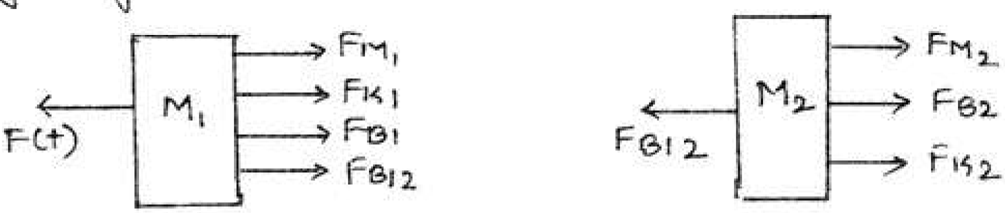
Solution: The mechanical system is redrawn as shown below.



The Mechanical network is as shown below.



Free body diagram :-



The Equilibrium Equations are given by.

At x_1

$$F(t) = F_{M_1} + F_{K_1} + F_{B_1} + F_{B_{12}}$$

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{d x_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) \longrightarrow ①$$

At x_2

$$F_{B_{12}} = F_{M_2} + F_{B_2} + F_{K_2}$$

$$B_{12} \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d x_2}{dt} + K_2 x_2 \longrightarrow ②$$

F-V Analogy:-

Substituting electrical analogous based on Force voltage analogy in Equation ① + ②

from ①

$$V(t) = L_1 \frac{d^2 q_1}{dt^2} + \frac{1}{C_1} q_1 + R_1 \frac{d q_1}{dt} + R_{12} \frac{d}{dt}(q_1 - q_2)$$

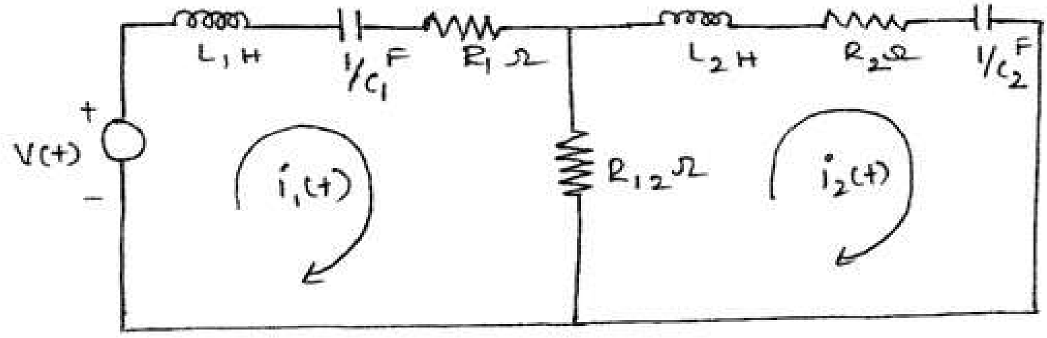
$$V(t) = L_1 \frac{d i_1}{dt} + \frac{1}{C} \int i_1 \, dt + R_1 i_1 + R_{12}(i_1 - i_2) \longrightarrow ③$$

from ②

$$R_{12} \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + R_2 \frac{d q_2}{dt} + \frac{1}{C_2} q_2$$

$$R_{12}(i_1 - i_2) = L_2 \frac{d i_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 \, dt \longrightarrow ④$$

The electrical circuit satisfying Equations ③ and ④ is as shown below.



F-I Analogy:- Substituting electrical analogous based on force current analogy in Equation ① & ② we get

from ①

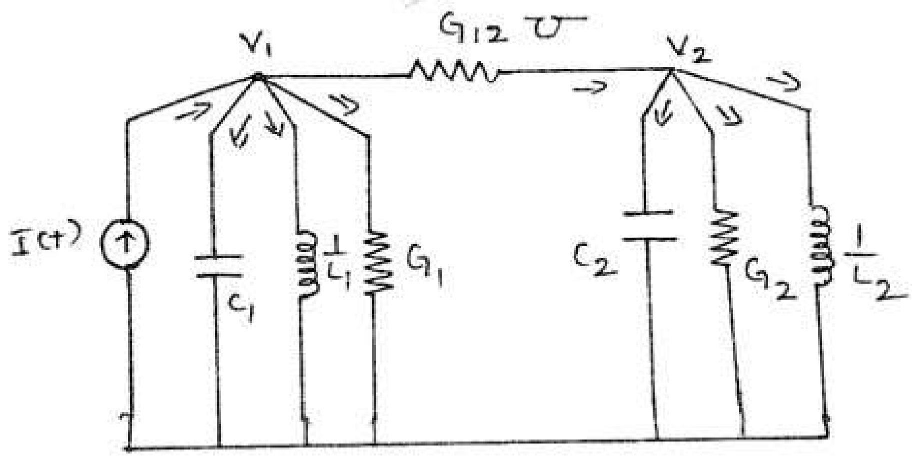
$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{L_1}\phi_1 + G_1 \frac{d}{dt}\phi_1 + G_{12}\frac{d}{dt}(\phi_1 - \phi_2)$$

$$I(t) = C_1 \frac{d}{dt} V_1 + \frac{1}{L_1}\int V_1\, dt + G_1 V_1 + G_{12}(V_1 - V_2) \rightarrow ⑤$$

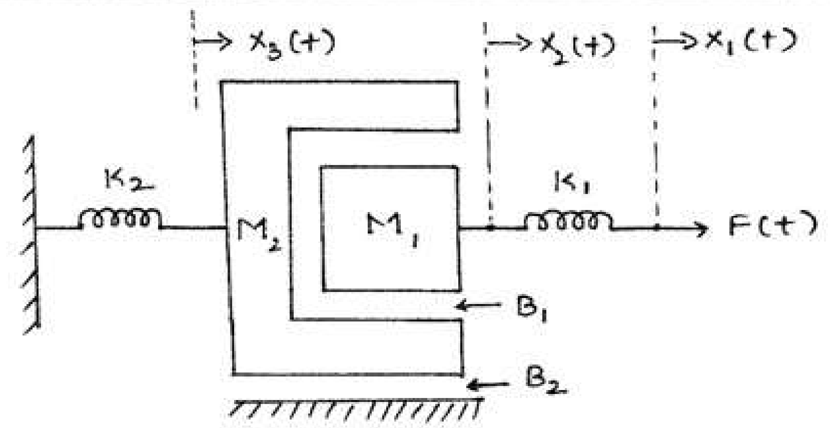
from ②

$$G_{12}\frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2}{dt^2}\phi_2 + G_2 \frac{d}{dt}\phi_2 + \frac{1}{L_2}\phi_2$$

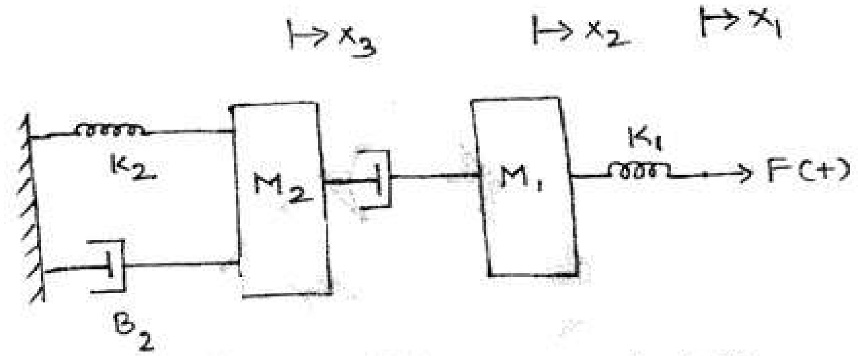
$$G_{12}(V_1 - V_2) = C_2 \frac{d}{dt} V_2 + G_2 V_2 + \frac{1}{L_2}\int V_2\, dt \rightarrow ⑥$$



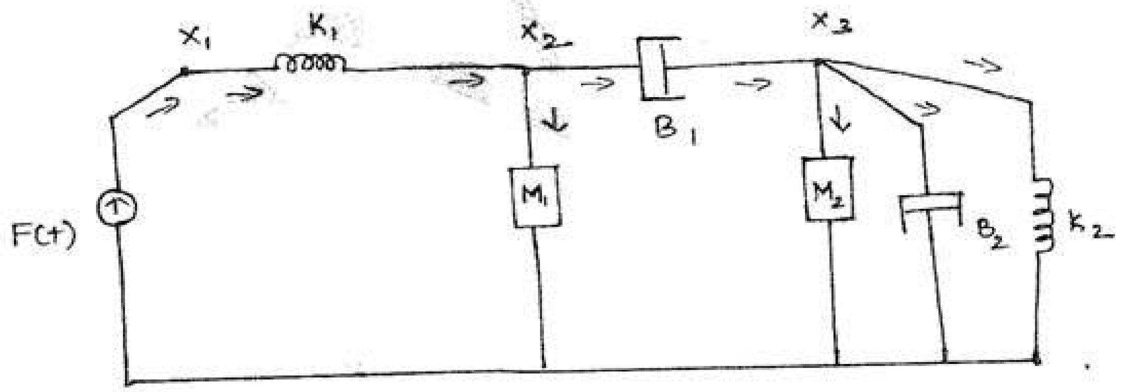
3)



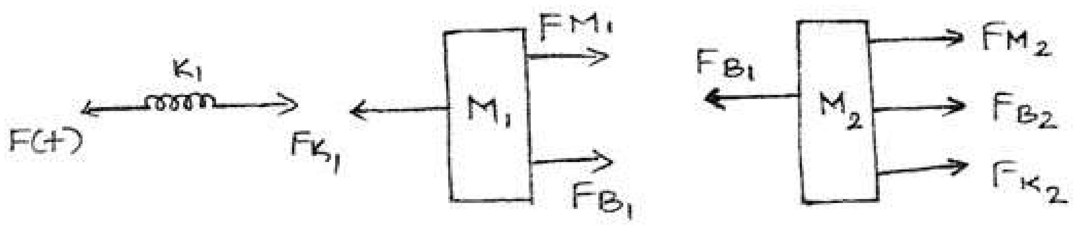
Solution:- The mechanical system is redrawn as shown below.



The Mechanical network is as shown below.



Free body diagram:-



Equilibrium Equations for a given system are.

At x_1; $F(t) = K_1(x_1 - x_2) \longrightarrow (1)$

At x_2: $K_1(x_1 - x_2) = M_1 \frac{d^2 x_2}{dt^2} + B_1 \frac{d}{dt}(x_2 - x_3) \longrightarrow (2)$

At x_3: $B_1 \frac{d}{dt}(x_2 - x_3) = M_2 \frac{d^2 x_3}{dt^2} + B_2 \frac{d x_3}{dt} + K_2 x_3 \longrightarrow (3)$

F-V Analogy:- By substituting electrical analogous based on force - voltage analogy in equations (1), (2) & (3)

From (1)

$$V(t) = \frac{1}{C_1}(q_1 - q_2)$$

$$V(t) = \frac{1}{C_1}\int (i_1 - i_2)\,dt \longrightarrow (4)$$

from (2)

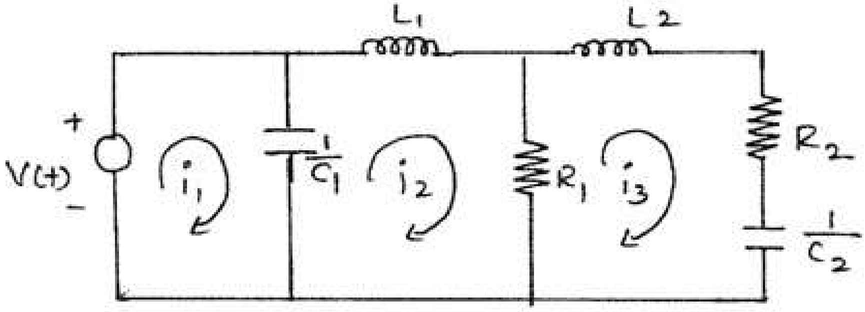
$$\frac{1}{C_1}(q_1 - q_2) = L_1 \frac{d^2 q_2}{dt^2} + R_1 \frac{d}{dt}(q_2 - q_3)$$

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = L_1 \frac{d}{dt} i_2 + R_1 (i_2 - i_2) \longrightarrow (5)$$

From (3)

$$R_1 \frac{d}{dt}(q_2 - q_3) = L_2 \frac{d^2 q_3}{dt^2} + R_2 \frac{d q_3}{dt} + \frac{1}{C_2} q_2$$

$$R_1 (i_2 - i_3) = L_2 \frac{d}{dt} i_3 + R_2 i_3 + \frac{1}{C_2}\int i_3\,dt \longrightarrow (6).$$



F-I Analogy :- By substituting electrical analogous based force current analogy in equation ①, ② and ③

from ①

$$I(t) = K_1(\phi_1 - \phi_2)$$

$$I(t) = \frac{1}{L_1}\int (V_1 - V_2)\,dt \quad \text{—— ⑦}$$

From ②

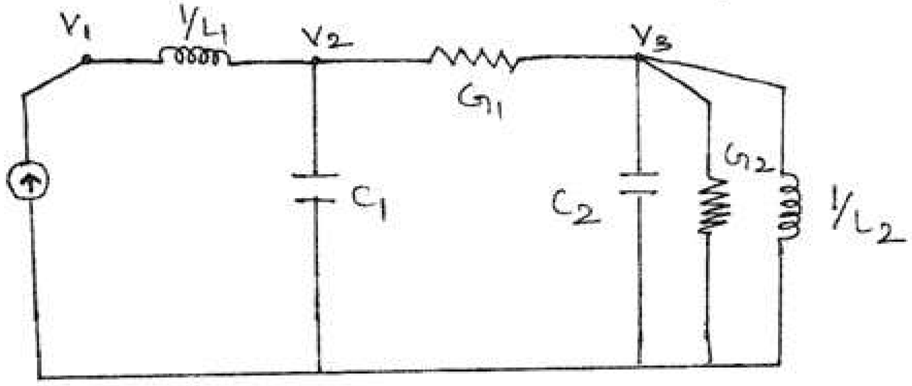
$$\frac{1}{L_1}(\phi_1 - \phi_2) = C_1\frac{d^2\phi_2}{dt^2} + G_1\frac{d}{dt}(\phi_2 - \phi_3)$$

$$\frac{1}{L_1}\int (V_1 - V_2)\,dt = C_1\frac{d}{dt}V_2 + G_1(V_2 - V_3) \quad \text{— ⑧}$$

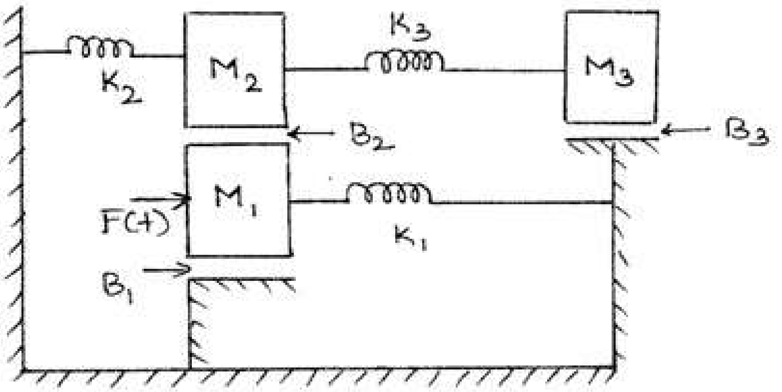
From ③

$$G_1\frac{d}{dt}(\phi_2 - \phi_3) = C_2\frac{d^2}{dt^2}\phi_3 + G_2\frac{d}{dt}\phi_3 + \frac{1}{L_2}\phi_3$$

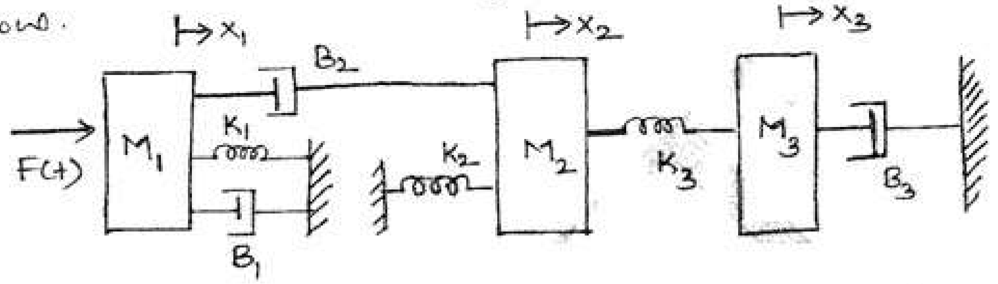
$$G_1(V_2 - V_3) = C_2\frac{d}{dt}V_3 + G_2V_3 + \frac{1}{L_2}\int V_3\,dt \rightarrow ⑨.$$



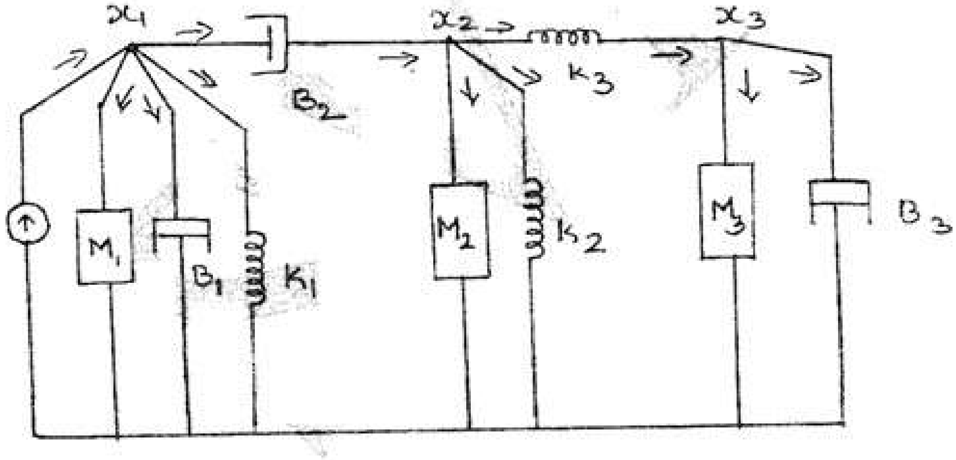
4)



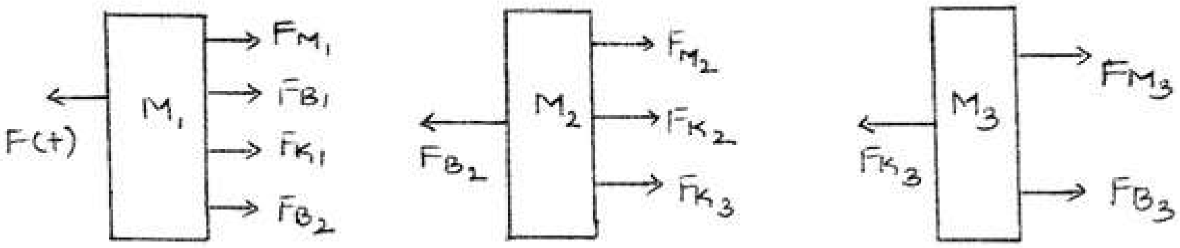
Solution: The mechanical System is redrawn as shown below.



The mechanical network is as shown below.



Free body diagram:



Equilibrium Equations are given by

At x_1 ; $F(t) = F_{M_1} + F_{B_1} + F_{K_1} + F_{B_2}$

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d x_1}{dt} + K_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) \longrightarrow (1)$$

At x_2 ; $F_{B_2} = F_{M_2} + F_{K_2} + F_{K_3}$

$$B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + K_3 (x_2 - x_3) \longrightarrow (2)$$

At x_3 ;

$F_{K_3} = F_{m_3} + F_{B_3}$

$$K_3 (x_2 - x_3) = M_3 \frac{d^2 x_3}{dt^2} + B_3 \frac{d x_3}{dt} \longrightarrow (3)$$

F-V Analogy:-

By substituting electrical analogous based on force voltage analogy in Equation (1), (2) and (3)

From 1:-

$$V(t) = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{d q_1}{dt} + \frac{1}{C_1} q_1 + R_2 \frac{d}{dt}(q_1 - q_2)$$

$$V(t) = L_1 \frac{d i_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 \, dt + R_2 (i_1 - i_2) \longrightarrow (4)$$

From 2:-

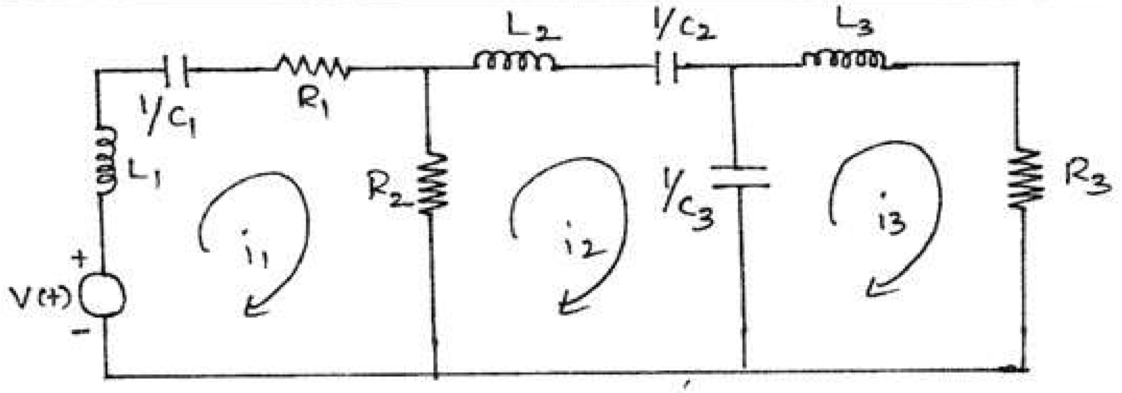
$$R_2 \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + \frac{1}{C_2} q_2 + \frac{1}{C_3} (q_2 - q_3)$$

$$R_2 (i_1 - i_2) = L_2 \frac{d i_2}{dt} + \frac{1}{C_2} \int i_2 \, dt + \frac{1}{C_3} \int (i_2 - i_3) \, dt \rightarrow (5)$$

From 3:-

$$\frac{1}{C_3} (q_2 - q_3) = L_3 \frac{d^2 q_3}{dt^2} + R_3 \frac{d q_3}{dt}$$

$$\frac{1}{C_3} \int (i_2 - i_3) \, dt = L_3 \frac{d i_3}{dt} + R_3 i_3 \longrightarrow (5)$$



F-I Analogy:-

By substituting Electrical Analogous based on the force Current Analogy in Equations ①, ② and ③

From ①

$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + \frac{1}{L_1}\phi_1 + G_1 \frac{d}{dt}\phi_1 + G_2 \frac{d}{dt}(\phi_1 - \phi_2)$$

$$I(t) = C_1 \frac{d V_1}{dt} + \frac{1}{L_1}\int V_1\, dt + G_1 V_1 + G_2 (V_1 - V_2) \rightarrow ⑦$$

From ②

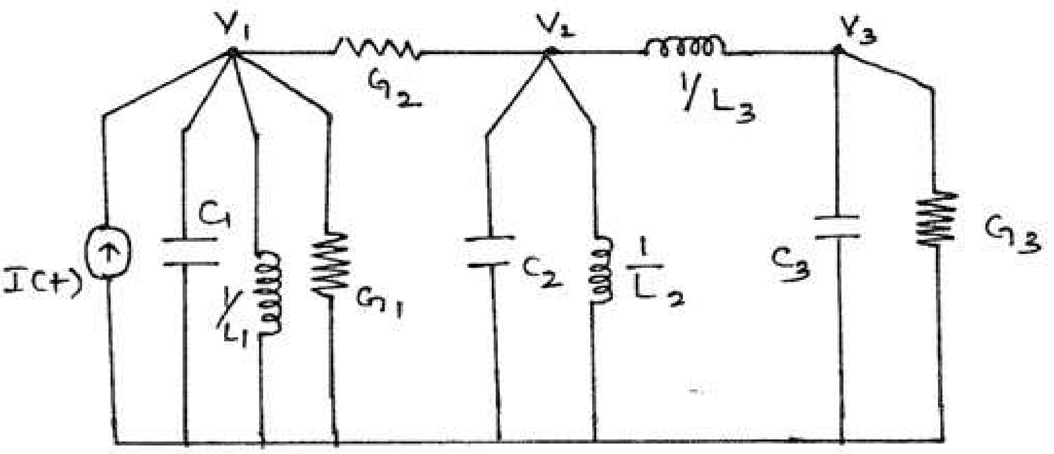
$$G_2 \frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2 \phi_2}{dt^2} + \frac{1}{L_2}\phi_2 + \frac{1}{L_3}(\phi_2 - \phi_3)$$

$$G_2 (V_1 - V_2) = C_2 \frac{d}{dt} V_2 + \frac{1}{L_2}\int V_2\, dt + \frac{1}{L_3}\int (V_2 - V_3)\, dt \rightarrow ⑧$$

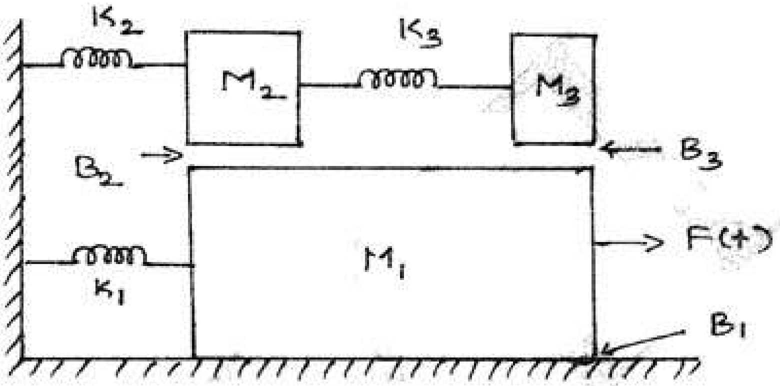
From ③

$$\frac{1}{L_3}(\phi_2 - \phi_3) = C_3 \frac{d^2}{dt^2}\phi_3 + G_3 \frac{d}{dt}\phi_3$$

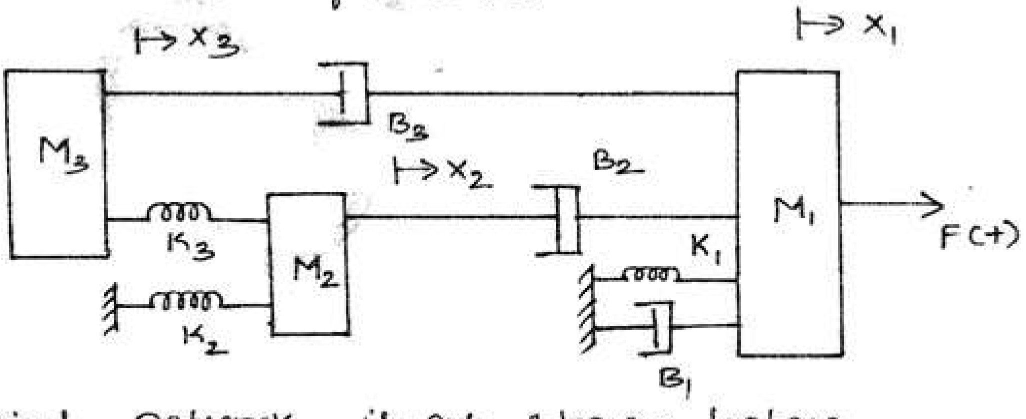
$$\frac{1}{L_3}\int (V_2 - V_3)\, dt = C_3 \frac{d}{dt} V_3 + G_3 V_3 \longrightarrow ⑨$$



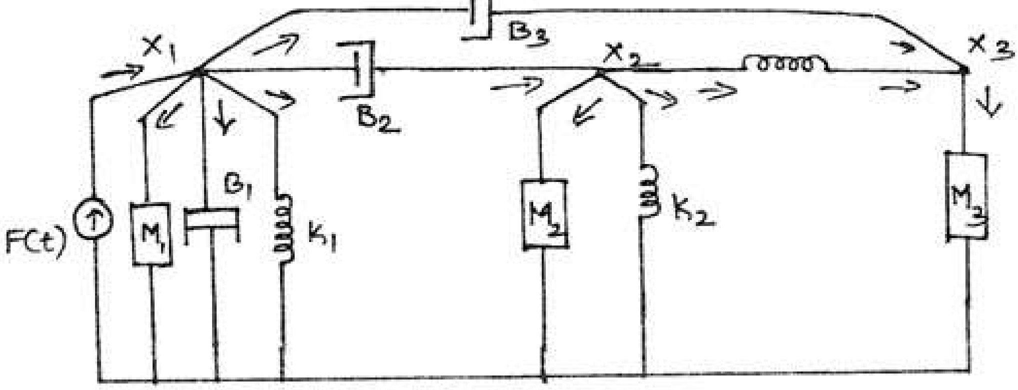
5)



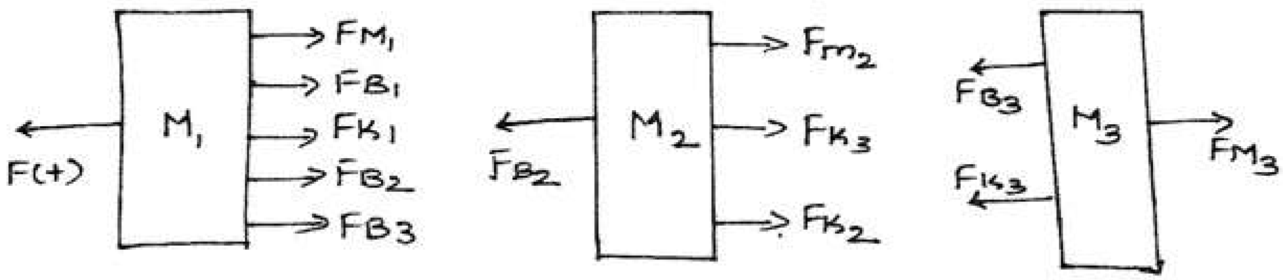
Solution:- Mechanical System is redrawn as shown below.



Mechanical network is as shown below.



Free body diagram :-



The Equilibrium Equations are given by

At x_1 ;

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_2 \frac{d}{dt}(x_1 - x_2) + B_3 \frac{d}{dt}(x_1 - x_3) \rightarrow (1)$$

At x_2 :-

$$B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + K_3 (x_2 - x_3) \longrightarrow (2)$$

At x_3 :-

$$K_3 (x_2 - x_3) + B_3 \frac{d}{dt}(x_1 - x_3) = M_3 \frac{d^2 x_3}{dt^2} \longrightarrow (3)$$

F-V Analogy :-

By substituting electrical analogous based on force voltage analogy in Equation (1), (2) and (3).

From (1)

$$V(t) = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{1}{C_1} q_1 + R_2 \frac{d}{dt}(q_1 - q_2) + R_3 \frac{d}{dt}(q_1 - q_3)$$

$$V(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + R_2 (i_1 - i_2) + R_3 (i_1 - i_3) \rightarrow (4)$$

From x_2 :-

$$R_2 \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2}{dt^2} q_2 + \frac{1}{C_2} q_2 + \frac{1}{C_3}(q_2 - q_3)$$

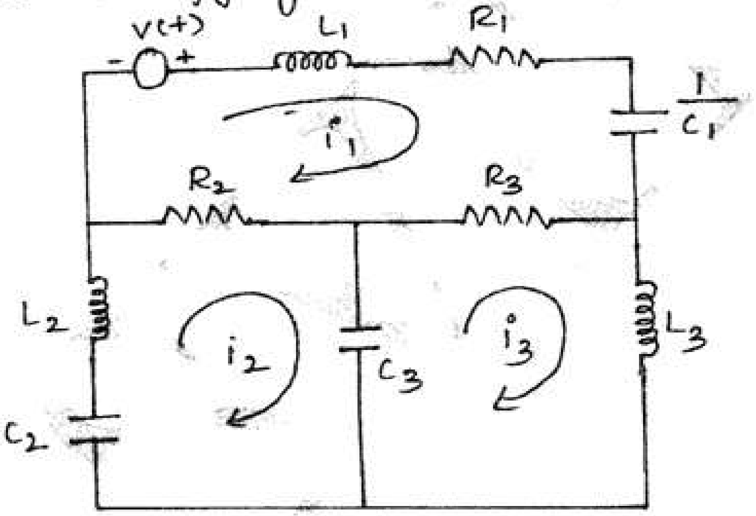
$$R_2 (i_1 - i_2) = L_2 \frac{d}{dt} i_2 + \frac{1}{C_2}\int i_2 \, dt + \frac{1}{C_3}\int (i_2 - i_3) dt \quad - (5)$$

From x_3 :-

$$\frac{1}{C_3}(q_2 - q_3) + R_3 \frac{d}{dt}(q_1 - q_3) = L_3 \frac{d^2}{dt^2} q_3$$

$$\frac{1}{C_3}\int (i_2 - i_3) dt + R_3 (i_1 - i_3) = L_3 \frac{d}{dt} i_3 \longrightarrow (6)$$

The Circuit Satisfying Equation (4), (5) and (6)



F-I Analogy :-

By Substituting Electrical Analogous based on F-I analogy to Equation (1), (2) and (3)

From (1)

$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + G_1 \frac{d \phi_1}{dt} + \frac{1}{L_1}\phi_1 + G_2 \frac{d}{dt}(\phi_1 - \phi_2) + G_3 \frac{d}{dt}(\phi_1 - \phi_3)$$

$$I(t) = C_1 \frac{d}{dt} V_1 + G_1 V_1 + \frac{1}{L_1}\int i_1 \, dt + G_2 (V_1 - V_2) + G_3 (V_1 - V_3) \rightarrow (7)$$

From x_2:-

$$G_2 \frac{d}{dt}(\phi_1-\phi_2) = C_2 \frac{d^2}{dt^2}\phi_2 + \frac{1}{L_2}\phi_2 + \frac{1}{L_3}(\phi_2-\phi_3)$$

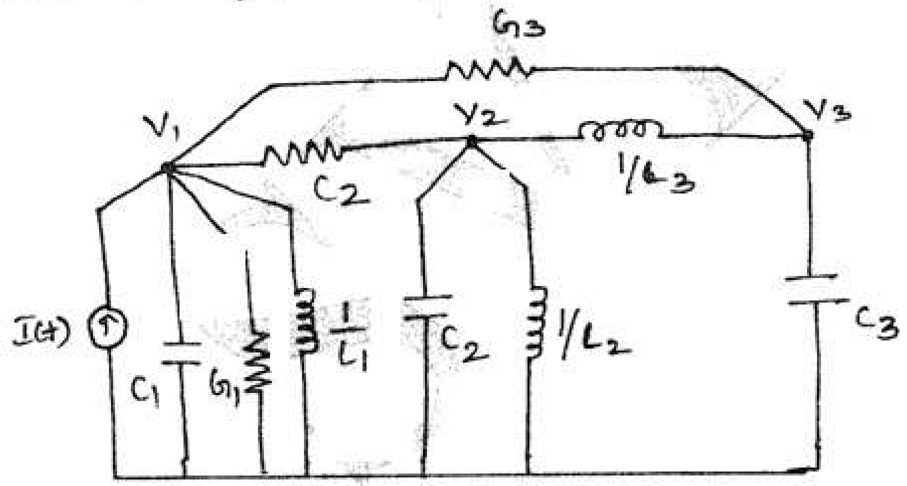
$$G_2(V_1-V_2) = C_2 \frac{d V_2}{dt} + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_3}\int (V_2-V_3)\,dt \rightarrow (8)$$

From x_3:-

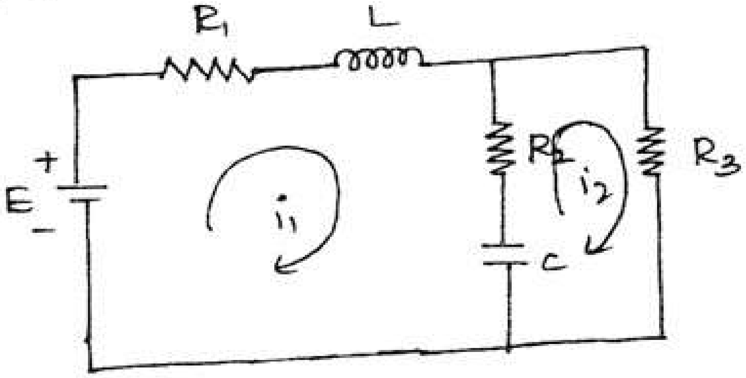
$$\frac{1}{L_3}(\phi_2-\phi_3) + G_3 \frac{d}{dt}(\phi_1-\phi_3) = C_3 \frac{d^2}{dt^2}\phi_3$$

$$\frac{1}{L_3}\int (V_2-V_3)\,dt + G_3(V_1-V_3) = C_3 \frac{d}{dt}V_3 \longrightarrow (9)$$

Circuit Satisfies Equation (7), (8) and (9)



⇒ Draw the force voltage analogous Mechanical System for the Electrical Circuit shown in the figure.



Solution By applying KVL to the given Circuit

from loop i_1 :-

$$E = R_1 i_1 + L \frac{d}{dt} i_1 + R_2 (i_1 - i_2) + \frac{1}{C}\int (i_1 - i_2)\, dt$$

$$E = R_1 \frac{dq_1}{dt} + L \frac{d^2 q_1}{dt^2} + R_2 \frac{d}{dt}(q_1 - q_2) + \frac{1}{C}(q_1 - q_2) \rightarrow ①$$

from loop i_2 :-

$$R_3 i_2 + \frac{1}{C}\int (i_2 - i_1)\, dt + R_2 (i_2 - i_1) = 0$$

$$R_3 \frac{dq_2}{dt} + \frac{1}{C}(q_2 - q_1) + R_2 \frac{d}{dt}(q_2 - q_1) = 0 \longrightarrow ②$$

By substituting the Mechanical Elements in Equation ① & ② based on F-V Analogy.

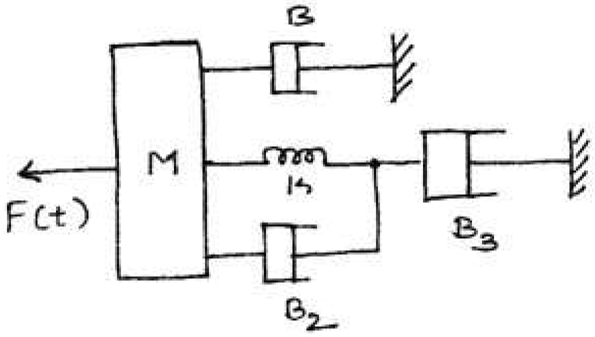
From 1 :-

$$F(t) = B_1 \frac{dx_1}{dt} + M \frac{d^2 x_1}{dt^2} + B_2 \frac{d(x_1 - x_2)}{dt} + K(q_1 - q_2) \rightarrow ③$$

From 2 :-

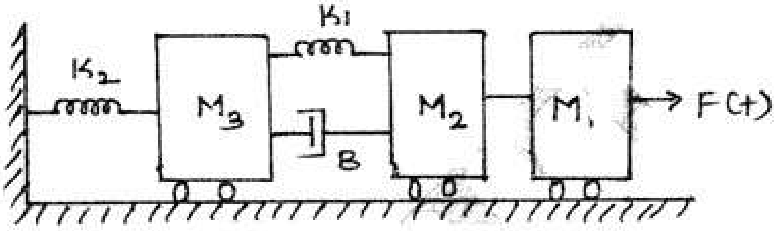
$$B_3 \frac{d x_2}{dt} + K(x_2 - x_1) + B_2 \frac{d}{dt}(x_2 - x_1) = 0 \rightarrow ④$$

The mechanical System satisfies Equation (3) & (4)



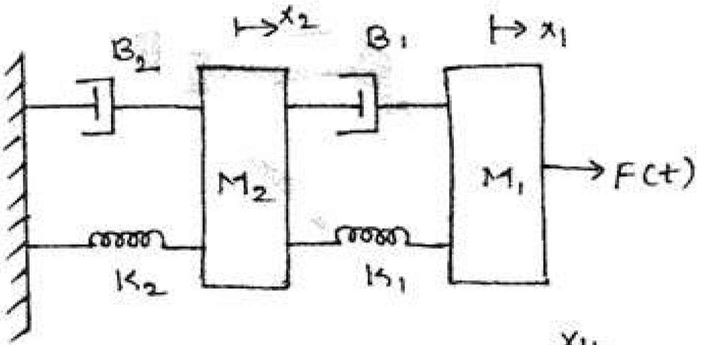
Practice Problems on Translational Systems:-

1)

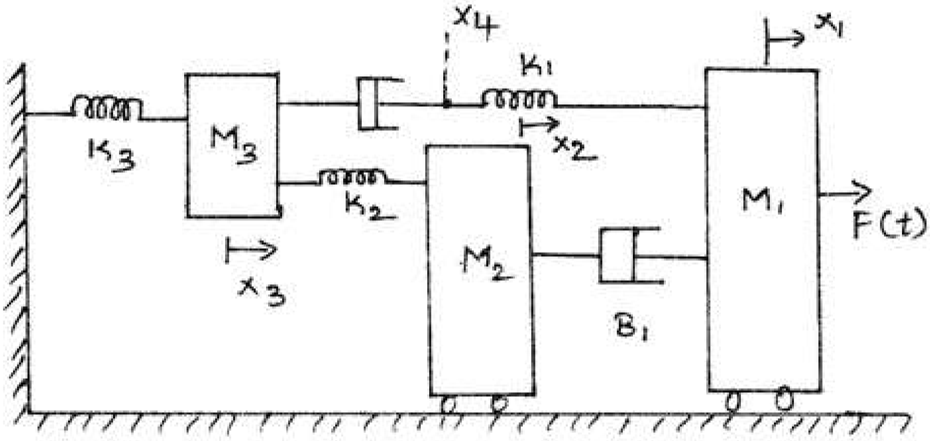


for the mechanical System shown in the figure above. write the differential Equations and draw the Electrical analogous based on Force - Voltage analogy and force-current Analogy.

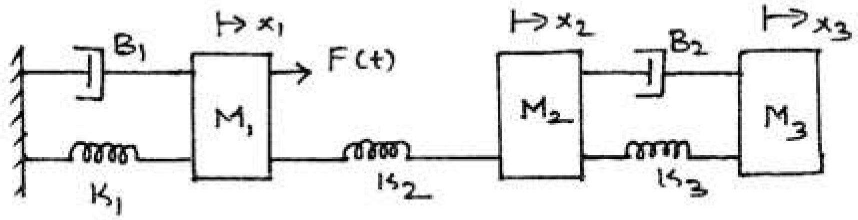
2)



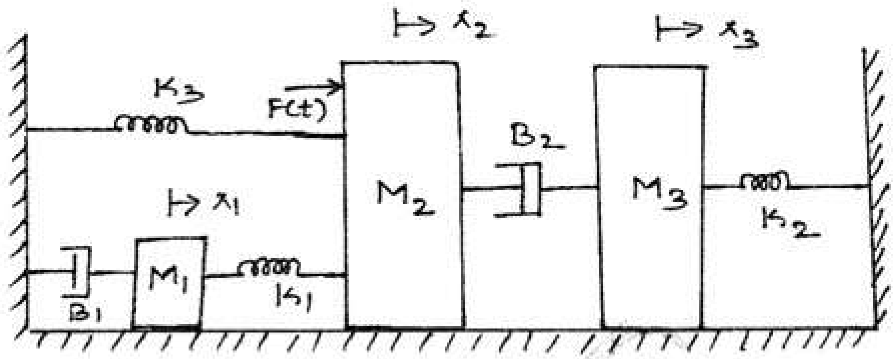
3)



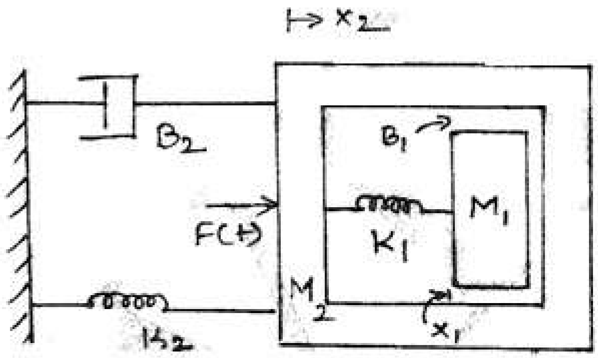
4)


x1
x2
x3
B1
F(t)
B2
M1
M2
M3
K1
K2
K3

5)


x2
x3
K3
F(t)
B2
M2
M3
K2
x1
M1
K1
B1

6)

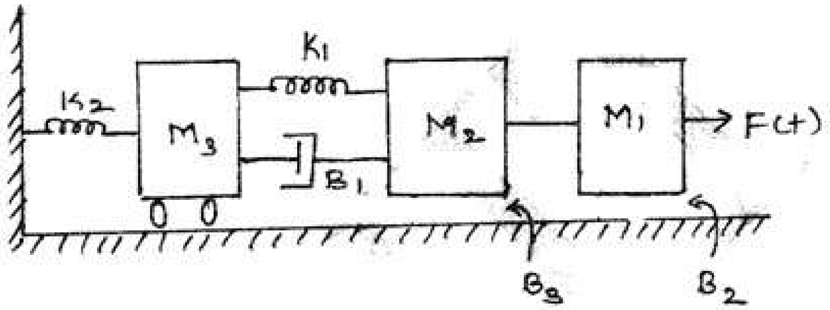

x2
B2
B1
M1
F(t)
K1
M2
x1
K2

Assignment on Translational Systems.

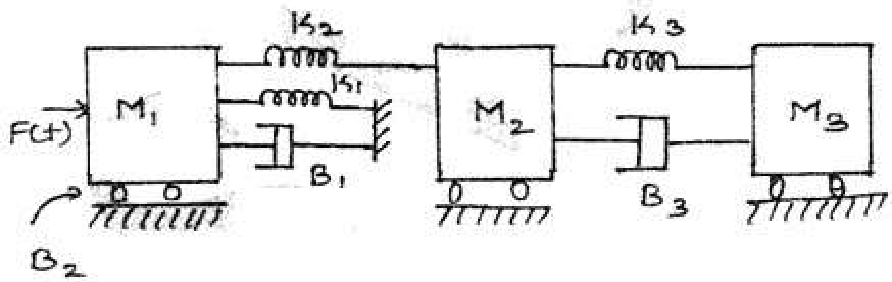
1) Define Control System. With an Example Explain the types of Control System.

2) Give the comparisions between Openloop Control System and closed loop Control System.

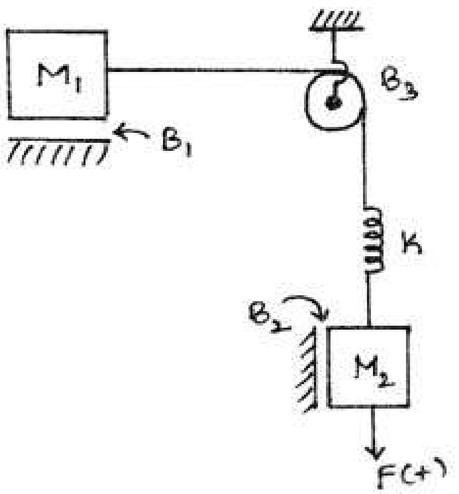
3) For a given mechanical System write the Electrical. analogous circuit based force voltage and force. Current analogy.



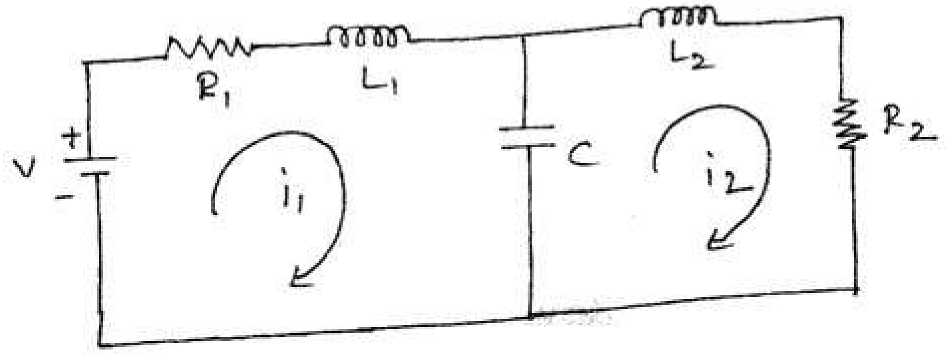
4)



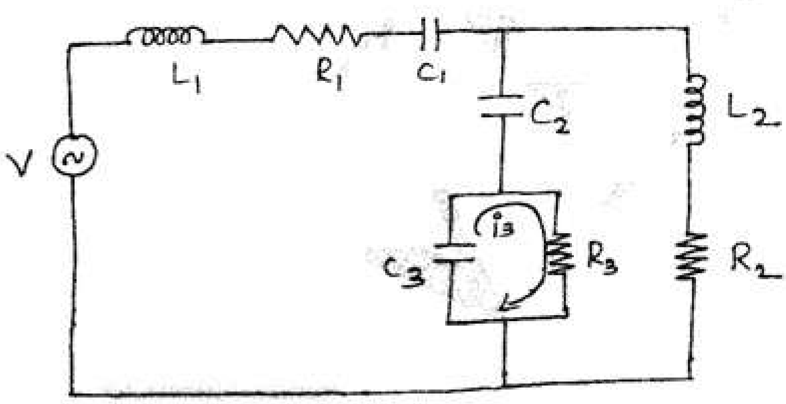
5)



6) Draw the force voltage analogous mechanical system for the electrical circuit shown in figure writing the loop equation for the electric circuit then transforming them to there mechanical analog



7)



Rotational Systems:-

The basic mechanical Elements of a rotational System are

a) Moment of Inertia (J)

b) Rotational Spring (K)

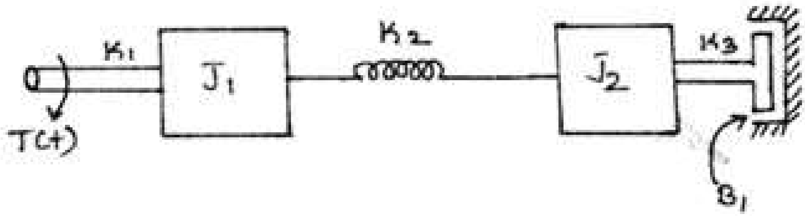
c) Rotational Dashpot (B)

* Tabulation for Converting Rotational Systems to Electrical analogous (Torque → voltage and Torque → current)

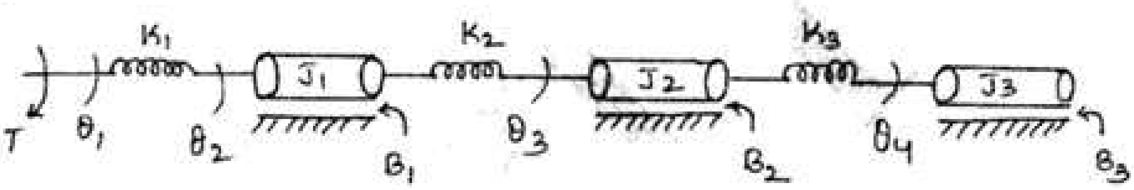
Mechanical elements	Electrical Analogous	
	(T-V Analogy)	(T-I Analogy)
Torque [T(t)]	Voltage [V(t)]	Current [I(t)]
Angular Velocity [ω(t)]	Current [I(t)]	Voltage [V(t)]
Angular Displacement [θ(t)]	Charge [q(t)]	Magnetic Flux [φ(t)]
Moment of Inertia [J]	Inductance [L]	Capacitance [C]
Rotational Spring [K]	Reciprocal of capacitance [$\frac{1}{C}$]	Reciprocal of Inductance [$\frac{1}{L}$]
Rotational Dashpot [B]	Resistance [R]	Conductance [G]

Problems to be solved in the class

1) For the Mechanical System shown in the figure write the differential Equations of the system also draw the torque - voltage and torque - current Electrical analogous.



2)



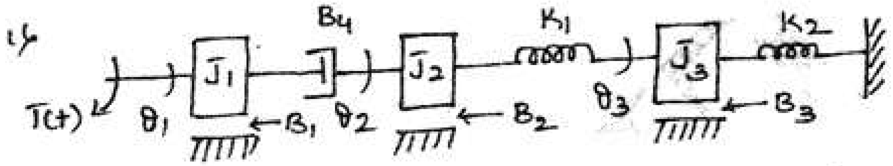
Note:- Angular Velocity (ω) $= \frac{d}{dt}\theta(t)$

Angular Acceleration (α) $= \frac{d^2}{dt^2}\theta(t)$

Problems on Rotational Systems:-

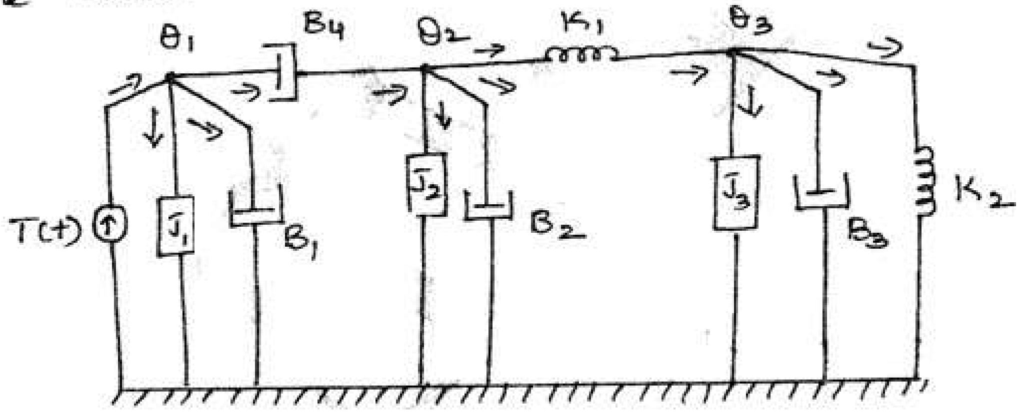
⇒ For the mechanical System shown in figure. Write the differential Equations of the System also draw the torque Voltage and torque current Electrical Analogous.

1)



Solution:-

The mechanical network for a given system



The Equilibrium Equations of the mechanical System are given by

At θ_1 = $T(t) = J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d}{dt}\theta_1 + B_4\frac{d}{dt}(\theta_1-\theta_2) \longrightarrow (1)$

At θ_2 ; $B_4\frac{d}{dt}(\theta_1-\theta_2) = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d}{dt}\theta_2 + K_1(\theta_2-\theta_3) \rightarrow (2)$

At θ_3 ; $K_1(\theta_2-\theta_3) = J_3\frac{d^2}{dt^2}\theta_3 + B_3\frac{d}{dt}\theta_3 + K_2\theta_3 \longrightarrow (3)$

T-V Analogy

By substituting electrical analogous based on torque-voltage analogy in Equation ①, ② and ③.

From ①,

$$v(t) = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{d q_1}{dt} + R_4 \frac{d}{dt}(q_1 - q_2)$$

$$V(t) = L_1 \frac{d i_1}{dt} + R_1 i_1 + R_4 (i_1 - i_2) \quad \text{— ④}$$

From ②,

$$R_4 \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2 q_2}{dt^2} + R_2 \frac{d}{dt} q_2 + \frac{1}{C_1}(q_2 - q_3)$$

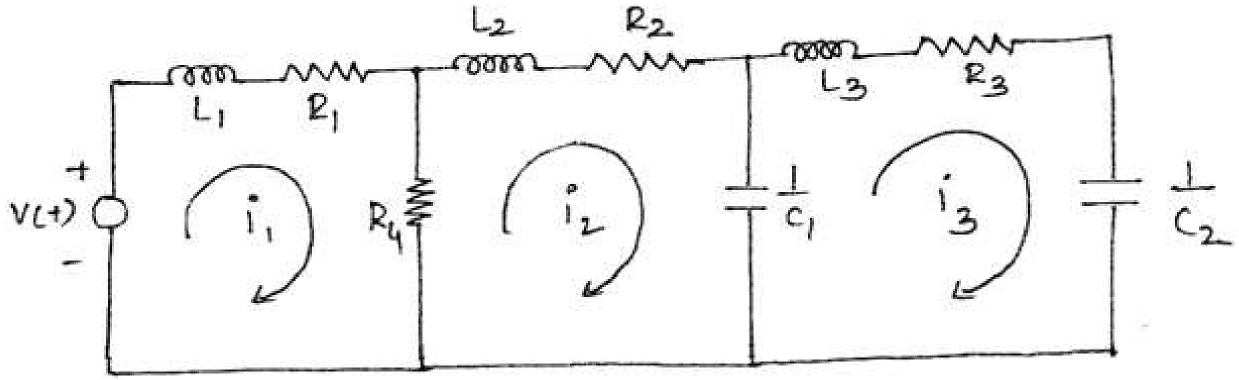
$$R_4 (i_1 - i_2) = L_2 \frac{d}{dt} i_2 + R_2 i_2 + \frac{1}{C_1} \int (i_2 - i_3)\, dt \quad \text{— ⑤}$$

From ③,

$$\frac{1}{C_1}(q_2 - q_3) = L_3 \frac{d^2 q_3}{dt^2} + R_3 \frac{d q_3}{dt} + \frac{1}{C_2} q_2$$

$$\frac{1}{C_1} \int (i_2 - i_3)\, dt = L_3 \frac{d i_3}{dt} + R_3 i_3 + \frac{1}{C_2} \int i_3\, dt \quad \text{— ⑥}$$

Electrical Circuit Satisfies Equation ④, ⑤ and ⑥



T-I Analogy:-

By substituting electrical analogous based on torque current analogy in Equation ①, ② and ③

From ①

$$I(t) = C_1 \frac{d^2 \phi_1}{dt^2} + G_1 \frac{d \phi_1}{dt} + G_4 \frac{d}{dt}(\phi_1 - \phi_2)$$

$$I(t) = C_1 \frac{dV_1}{dt} + G_1 V_1 + G_4 (V_1 - V_2) \quad \text{—— (7)}$$

From ②

$$G_4 \frac{d}{dt}(\phi_1 - \phi_2) = C_2 \frac{d^2 \phi_2}{dt^2} + G_2 \frac{d}{dt}\phi_2 + \frac{1}{L_1}(\phi_2 - \phi_3)$$

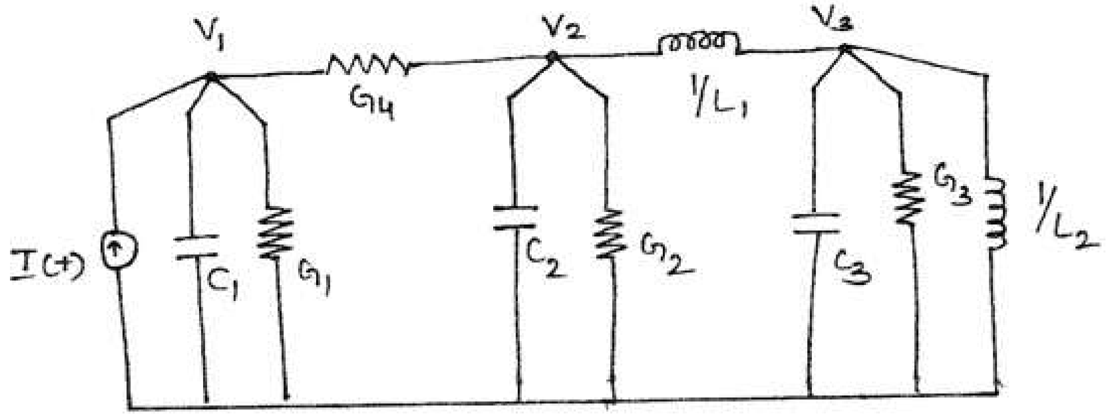
$$G_4 (V_1 - V_2) = C_2 \frac{d V_2}{dt} + G_2 V_2 + \frac{1}{L_1}\int (V_2 - V_3)\,dt \quad \text{— (8)}$$

From ③

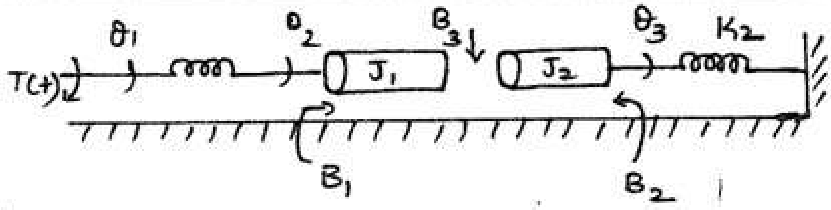
$$\frac{1}{L_1}(\phi_2 - \phi_3) = C_3 \frac{d^2 \phi_3}{dt^2} + G_3 \frac{d \phi_3}{dt} + \frac{1}{L_2}\phi_3$$

$$\frac{1}{L_1}\int (V_2 - V_3)\,dt = C_3 \frac{d V_3}{dt} + G_3 V_3 + \frac{1}{L_2}\int V_3\,dt \quad \text{— (9)}$$

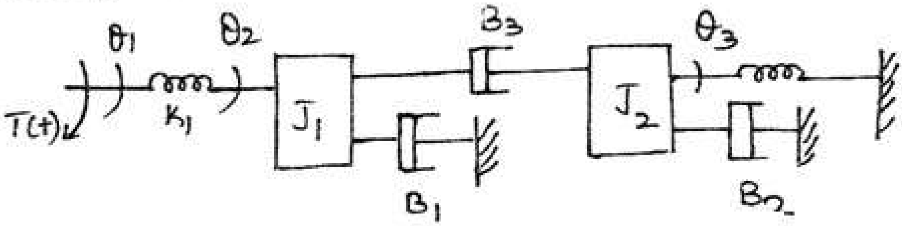
Electrical Circuit satisfying Equation 7, 8 and 9



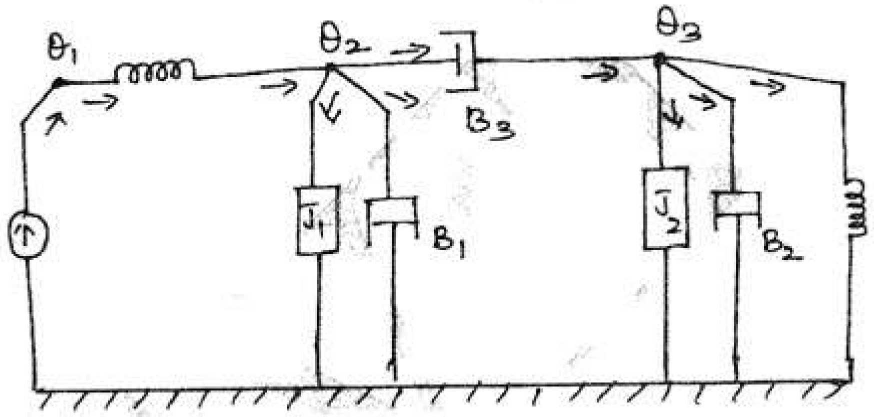
2)



Solution:- The Mechanical system is redrawn as shown below.



The Mechanical network is as drawn below.



The Equilibrium Equations are given by

At θ_1 ; $T(t) = K_1(\theta_1 - \theta_2)$ —— ①

At θ_2 ; $K_1(\theta_1 - \theta_2) = J_1 \frac{d^2\theta_2}{dt^2} + B_1 \frac{d\theta_2}{dt} + B_3 \frac{d}{dt}(\theta_2 - \theta_3) \rightarrow ②$

At θ_3 ; $B_3 \frac{d}{dt}(\theta_2 - \theta_3) = J_2 \frac{d^2\theta_3}{dt^2} + B_2 \frac{d\theta_3}{dt} + K_2\theta_3 \longrightarrow ③$

T-V Analogy:-

By substituting Electrical analogous based on torque. Voltage analogy in Equations ①, ② and ③.

From ①

$$V(t) = \frac{1}{C_1}(q_1 - q_2)$$

$$V(t) = \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad — ④$$

From ②

$$\frac{1}{C_1}(q_1 - q_2) = L_1 \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_2}{dt} + R_3 \frac{d}{dt}(q_2 - q_3)$$

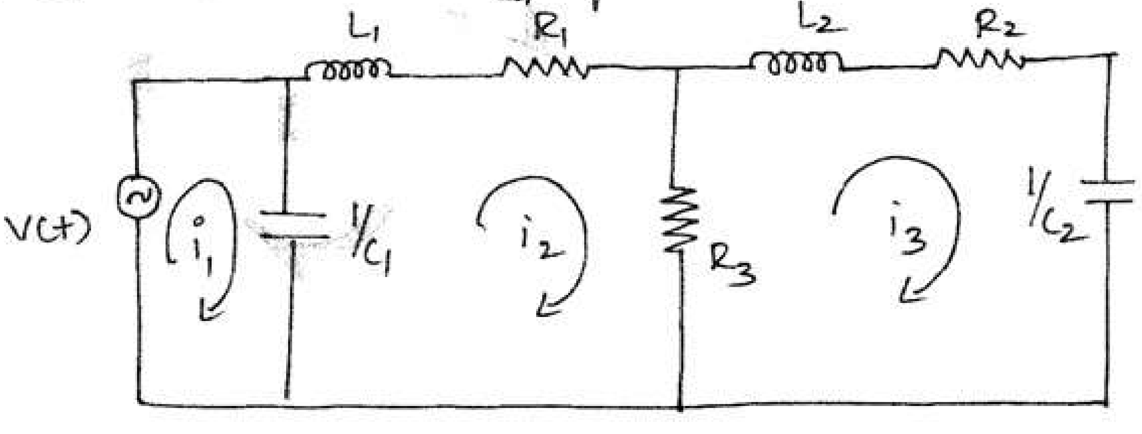
$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = L_1 \frac{di_2}{dt} + R_1 i_2 + R_3 (i_2 - i_3) \quad — ⑤$$

From ③

$$R_3 \frac{d}{dt}(q_2 - q_3) = L_1 \frac{d^2}{dt^2} q_3 + R_2 \frac{dq_3}{dt} + \frac{1}{C_2} q_3$$

$$R_3 (i_2 - i_3) = L_1 \frac{di_3}{dt} + R_2 i_3 + \frac{1}{C_2}\int i_3\,dt \rightarrow ⑥$$

Electrical Circuit satisfying Equation ④, ⑤ and ⑥.



T-I Analogy:-

By substituting Electrical Analogous based on torque current analogy in Equation ①, ② and ③.

From ①

$$I(t) = \frac{1}{L_1}(\phi_1 - \phi_2)$$

$$I(t) = \frac{1}{L_1}\int (V_1 - V_2)\, dt \quad \text{—} ⑦$$

From ②

$$\frac{1}{L_1}(\phi_1 - \phi_2) = C_1 \frac{d^2}{dt^2}\phi_2 + G_1 \frac{d}{dt}\phi_2 + G_3 \frac{d}{dt}(\phi_2 - \phi_3)$$

$$\frac{1}{L_1}\int (V_1 - V_2)\, dt = C_1 \frac{d}{dt}V_2 + G_1 V_2 + G_3 (V_2 - V_3) \longrightarrow ⑧$$

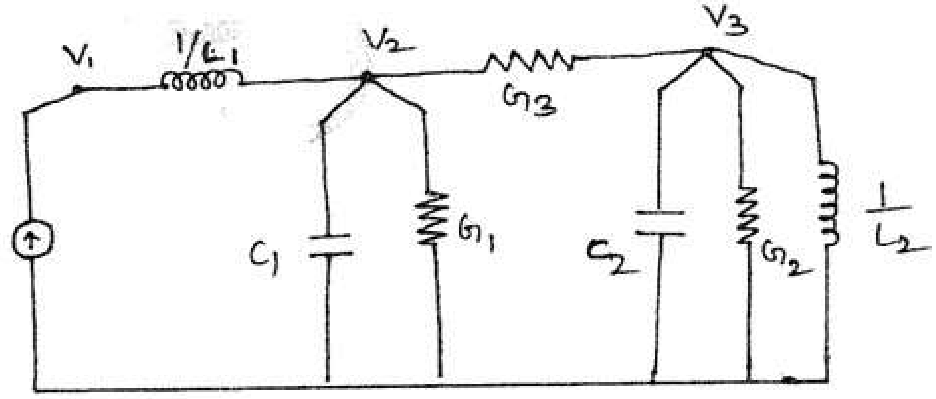
From ③

$$G_3 \frac{d}{dt}(\phi_2 - \phi_3) = C_2 \frac{d^2}{dt^2}\phi_3 + G_2 \frac{d}{dt}\phi_3 + \frac{1}{L_2}\phi_3$$

or

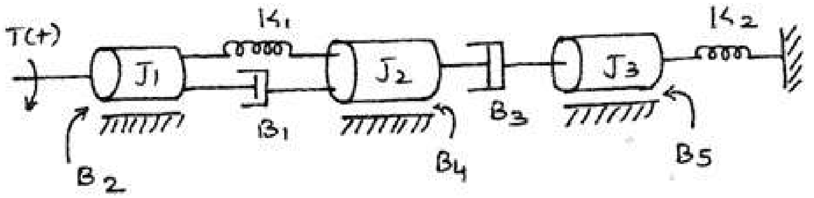
$$G_3 (V_2 - V_3) = C_2 \frac{d}{dt}V_3 + G_2 V_3 + \frac{1}{L_2}\int V_3\, dt \longrightarrow ⑨$$

Electrical Circuit Satisfies Equation ⑦, ⑧ and ⑨.



Assignment on Rotational Systems.

1) Write the differential Equations describing the behavior of the mechanical system shown in the figure. Also draw an analogous Electrical Circuit. (T-V and T-I)



Module no 1
Questions

Q-1: Explain with examples open loop and closed loop control systems. List merits and demerits of both. **Jun. 2014, 10 Marks**

Q-2: Compare open loop and closed loop control systems and give one practical example of each. **Jun. 2013, 6 Marks**

Q-3: Compare linear and non-linear control systems. **Dec. 2012, 4 Marks**

Q-4: Define control system. Draw the basic block diagram of a control loop giving all the relevant details. **Dec. 2012, 4 Marks**

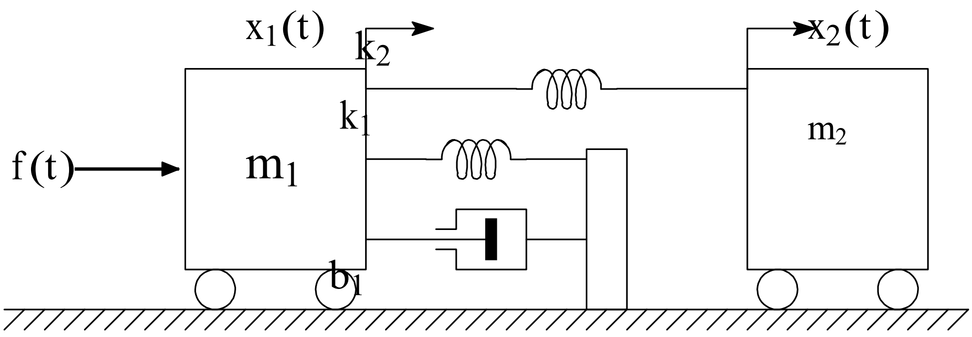
Q-5: Derive the electrical analogous quantities for the mechanical quantities using force- voltage analogy. **Dec. 2010, 5 Marks**

Q-6: Derive the mathematical model for an armature controlled DC motor. **Dec. 2010, 5 Marks**

Q-7: For what purpose feedback is used in control system? Mention the effects of feedback on (i) stability (ii) overall gain (iii) disturbance and (iv) sensitivity of control systems. **Jul. 2005, 10 Marks**

Q-8: For the system shown in Figure.1 write mechanical network and obtain its mathematical mod **Jul. 2013, 6 Marks**

Figure 1:



Q-9: For the system shown in Figure.2 write its mechanical network and obtain mathematical model and electrical analogous based on force-current analogy. **Jul. 2013, 8 Marks**

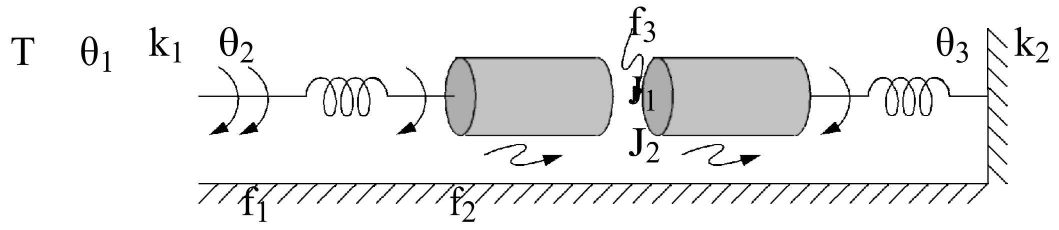


Figure 2: .

Q-10: Draw the electrical network based on torque-current analogy give all the performance equations for Figure. 3 **Jul. 2014, 10 Marks**

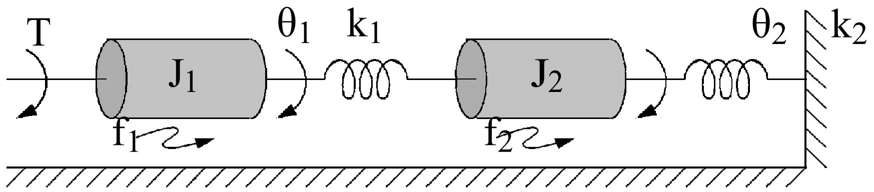


Figure 3: .

Q-11: For the rotational system shown in Figure.4. (i) Draw the mechanical network. (ii) Write the differential equations. (iii) Obtain torque to voltage analogy. **Dec. 2012, 08 Marks**

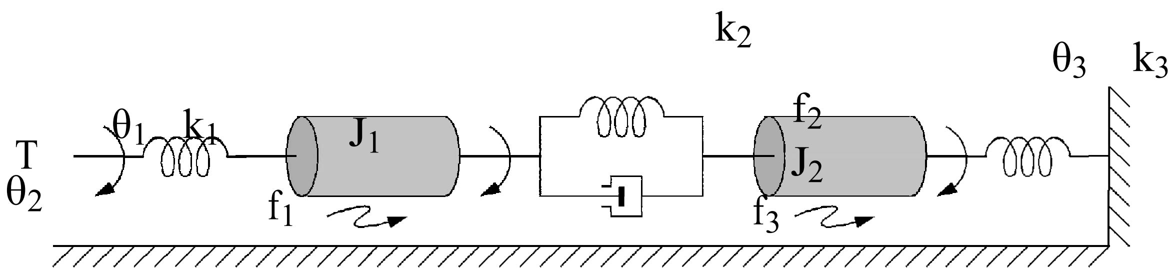


Figure 4:

Q-12: For the mechanical system shown in Figure.5.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on force-voltage analogy and write the analogous electrical equations.

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

Q-13: For the mechanical system shown in Figure.6.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on force-voltage analogy and write the analogous electrical equations.

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

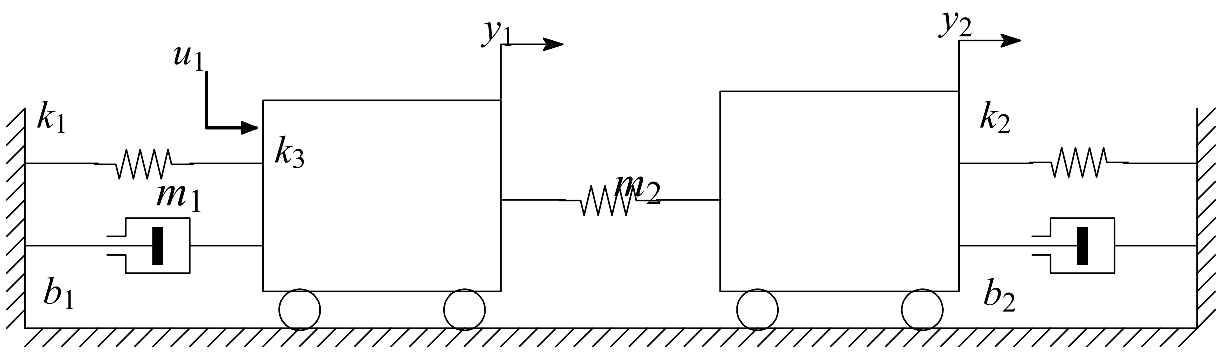


Figure 5: Mechanical system: u_i is force (N), y_i is displacement (m), m_i is mass (Kg), k_i is spring constant (N/m), b_i is viscous friction coefficient (N-s/m).

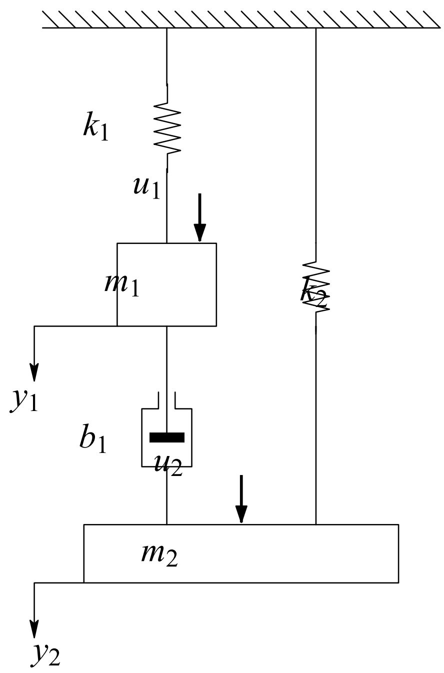


Figure 6: Mechanical system: u_i is force (N), y_i is displacement (m), m_i is mass (Kg), k_i is spring constant (N/m), b_i is viscous friction coefficient (N-s/m).

Q-14: For the mechanical system shown in Figure.7.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on force-voltage analogy and write the analogous electrical equations.

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

Q-15: For the mechanical system shown in Figure.8.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on force-voltage analogy and write the analogous electrical equations.

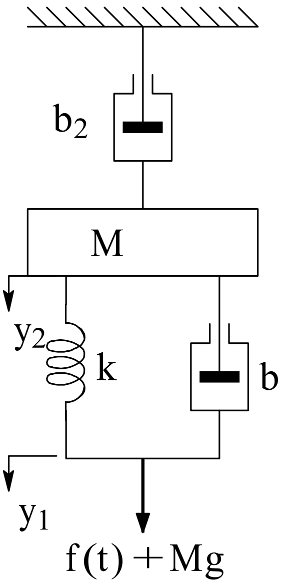


Figure 7: Mechanical system: f(t) is force (N), y_i is displacement (m), m_i is mass (Kg), k_i is spring constant (N/m), b_i is viscous friction coefficient (N-s/m), g is acceleration due to gravity (N-s^2/m).

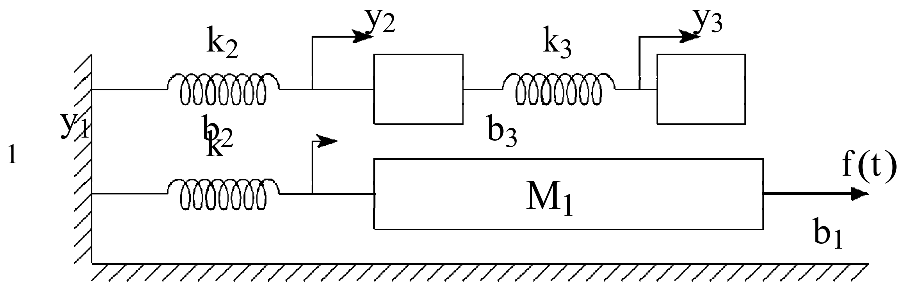


Figure 8: Mechanical system: f(t) is force (N), y_i is displacement (m), m_i is mass (Kg), k_i is spring constant (N/m), b_i is viscous friction coefficient (N-s/m)).

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

Q-16: For the mechanical system shown in Figure.9.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on force-voltage analogy and write the analogous electrical equations.

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

Q-17: For the mechanical system shown in Figure.10.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on force-voltage analogy and write the analogous electrical equations.

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

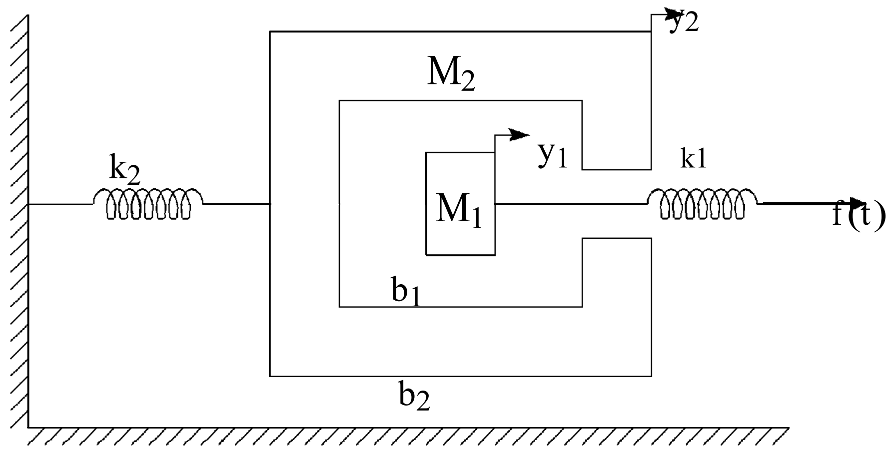


Figure 9: Mechanical system: f(t) is force (N), y_i is displacement (m), m_i is mass (Kg), k_i is spring constant (N/m), b_i is viscous friction coefficient (N-s/m).

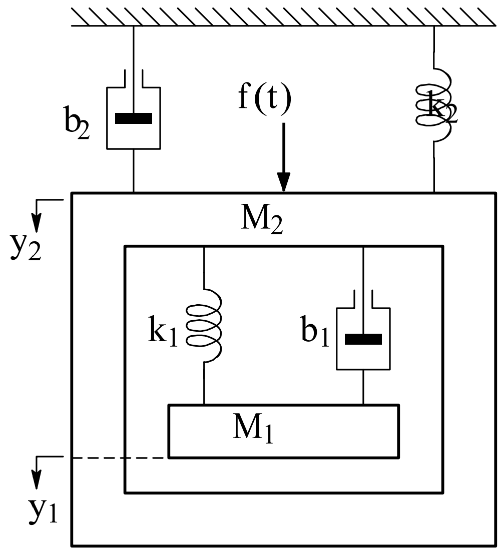


Figure 10: Mechanical system: f(t) is force (N), y_i is displacement (m), m_i is mass (Kg), k_i is spring constant (N/m), b_i is viscous friction coefficient (N-s/m).

Q-18: For the mechanical system shown in Figure.11.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on torque-voltage analogy and write the analo- gous electrical equations.

(iii) Draw the electrical network based on torque-current analogy and write the analo- gous electrical equations.

Q-19: For the mechanical system shown in Figure.12.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on torque-voltage analogy and write the analo- gous electrical equations.

(iii) Draw the electrical network based on torque-current analogy and write the analo- gous electrical equations.

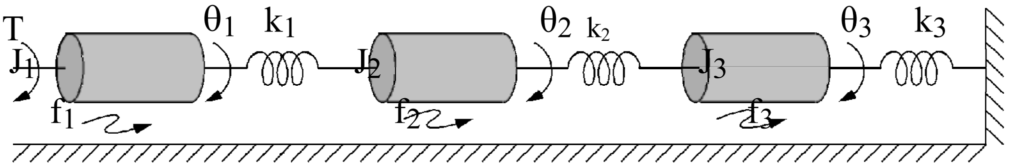


Figure 11: Mechanical system: T is torque (Nm), y_i is displacement (m), J_i is moment of inertia (Kgm^2), k_i is torsional spring constant (Nm/rad), f_i is viscous friction coefficient (Nm-s/rad).

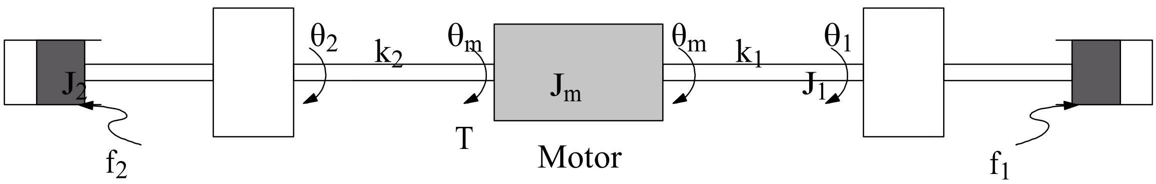


Figure 12: Mechanical system: T is torque (Nm), y_i is displacement (m), J_i is moment of inertia (Kgm^2), k_i is torsional spring constant (Nm/rad), f_i is viscous friction coefficient (Nm-s/rad).

Q-20: Figure.13 shows a motor-load system coupled through a gear train with gear ratio $n = N_1/N_2$. The motor torque is $T_m(t)$ and $T_L(t)$ represents a load torque.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on torque-voltage analogy and write the analo- gous electrical equations.

(iii) Draw the electrical network based on torque-current analogy and write the analo- gous electrical equations.

Q-21: Figure.14 shows the diagram of a print-wheel system with belts and pulleys. The belts are modeled as linear springs with spring constants k_1 and k_2.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on torque-voltage analogy and write the analo- gous electrical equations.

(iii) Draw the electrical network based on torque-current analogy and write the analo- gous electrical equations.

Q-22: For the mechanical system shown in Figure.15.

(i) Draw the free-body diagram and mechanical network. Write the differential equa- tions describing behaviour of the system.

(ii) Draw the electrical network based on torque-voltage analogy and write the analo- gous electrical equations.

(iii) Draw the electrical network based on torque-current analogy and write the analo- gous electrical equations.

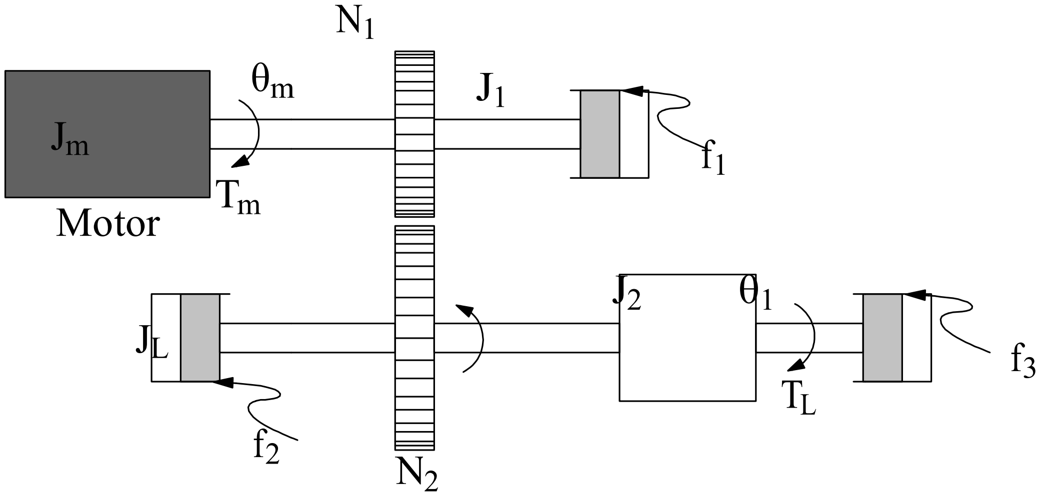


Figure 13: Mechanical system: T_m is motor torque (Nm), y_i is displacement (m), J_i is moment of inertia (Kgm^2), k_i is torsional spring constant (Nm/rad), f_i is viscous friction coefficient (Nm-s/rad).

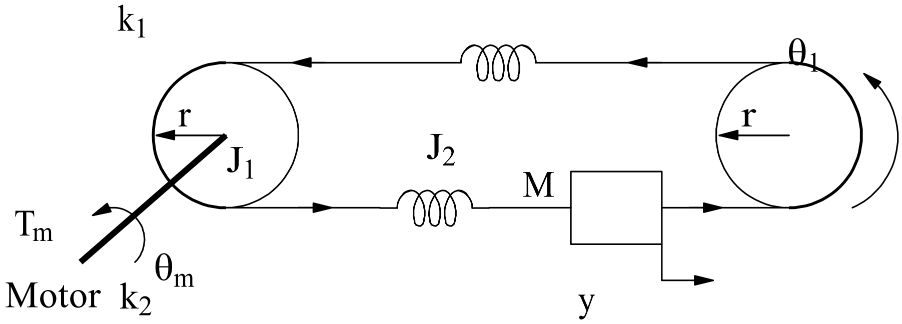


Figure 14: Mechanical system: T_m is motor torque (Nm), y_i is displacement (m), J_i is moment of inertia (Kgm^2), k_i is torsional spring constant (Nm/rad), y is displacement (m).

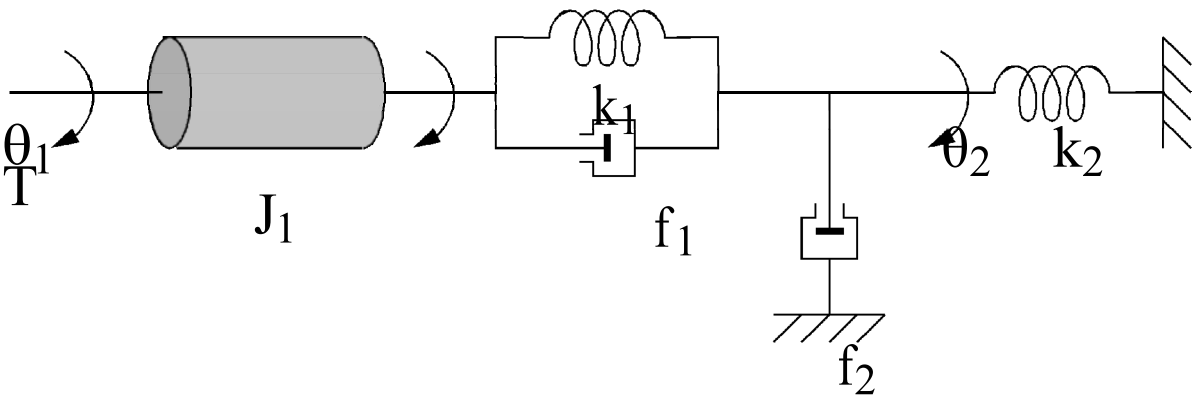


Figure 15: Mechanical system: T is torque (Nm), y_i is displacement (m), J_i is moment of inertia M(Kgm^2), k_i is torsional spring constant (Nm/rad), f_i is viscous friction coefficient (Nm-s/rad).

Q:23. For the mechanicrrl system shorrn in Fig.Ql(b) the analogotts clectrical netu'ork bascd onF-V analogy.

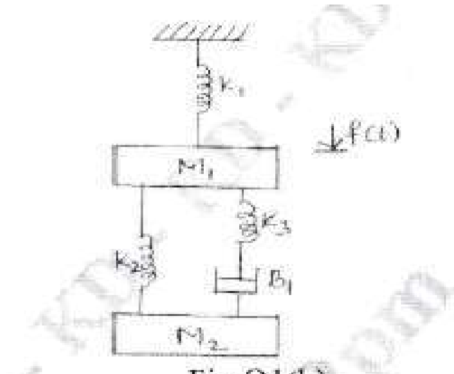


Fig.Q1(b)

Q:24. For the mechanical system shown in Fig.Ql(b):
i) Draw the mechanical network.
ii) Obtain equations of motion.
iii) Draw an electrical network based on force current analogy.

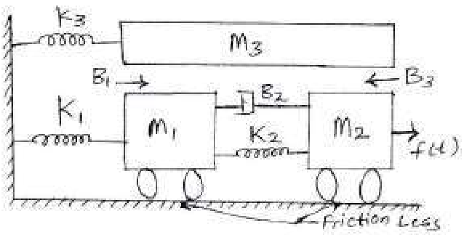


Fig.Q1(b)

Q:25. Write the differential eil\iailons fbr the mechanical s)steni shoun in Fig.QI(b) and ohtain F-V and F-I analogous e lectrica i networks. (05 Nlarks)I

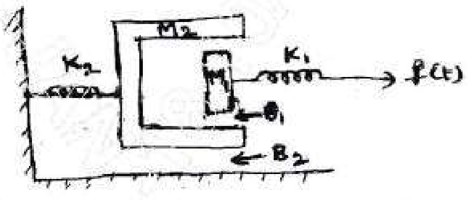


Fig.Q1(b)

Q:26.

b. For the mechanical system shown in Fig.Q.1(b), write i) The mechanical network ii) the equations of motion and iii) the force-current analogous electrical network. **(08 Marks)**

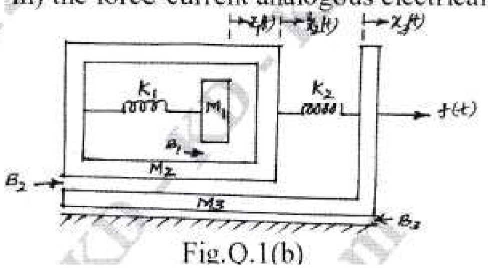


Fig.Q.1(b)

Q:27

Find the transfer function $\frac{I(s)}{Ui(s)}$ for the circuit shown in Fig.Q.1(b) and K is the gain of an ideal amplifier. **(06 Marks)**

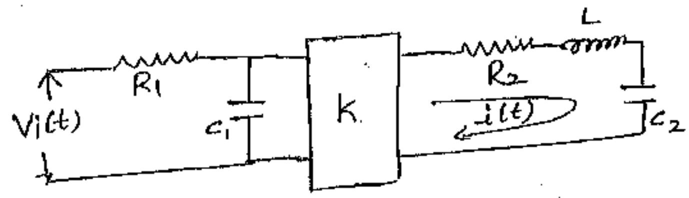


Fig.Q.1(b)

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CONTROL SYSTEMS NOTES (18EC43)
(As per Choice based Credit System (CBCS) Scheme)
IVTH SEMESTER

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING
KLE COLLEGE OF ENGINEERING AND TECHNOLOGY CHIKODI

"Don't see others doing better than you, beat your own records every day, because success is a fight between you and yourself"

CONTENTS

MODULE-2

Syllabus:

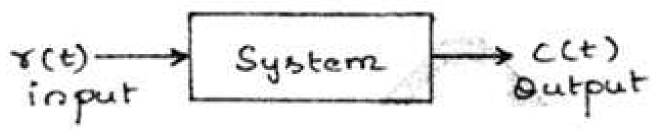
- Transfer functions
- Block diagrams and
- signal flow graphs
- Block diagram algebra and
- Signal Flow graphs.

Transfer function :-

* Transfer function gives the mathematical Equivalent model for a System.

Defination I :-

The Transfer function is defined as the ratio of laplace Transform of the Output to the Laplace transform of the Input Considering all Initial Condition Equals to zero;



For LTI Systems :-

$$\text{Transfer function} = \frac{L.T[\text{output}]}{L.T[\text{input}]} = \frac{L[c(t)]}{L[r(t)]}$$

$$T.F = \left.\frac{C(S)}{R(S)}\right|_{I.C=0}$$

LTI Systems :-

A System which has been made up of R, L, C Element are always LTI System because R, L and C have linear characteristics and there value does not change with time.

Defination 2 :-

The Transfer function of LTI System is defined as laplace transform of Impulse response with all Initial Conditions is Equal to zero.

$$\text{i.e,}\quad T.F = LT[\text{Impulse Response}]\Big|_{\text{all initial Condition}=0}$$

→ Impulse response is also called as System response, natural response, free force response, Input response.

Properties of Transfer function (T.F) :-

* The transfer function of a system is the laplace transform of it's impulse response for zero initial conditions.
* The transfer function can be determined from system input - output pair by taking ratio of laplace of output to laplace of input.
* The System differential Equation can be obtained from transfer function by replacing S- variable with linear differential operator D, defined by $D = \frac{d}{dt}$
* The transfer function is independent of the inputs to the system.
* The system poles/zeros can be found out from transfer function
* Stability can be determined from the characteristic Equation
* The transfer-function is defined only for linear time invariant functions. it is not defined non-linear system.

Points to remember

* Laplace transforms pairs:-

SL. NO	F(t)	F(S) = L [F(t)]
1)	$\delta(t)$ impulse response at $t=0$	1
2)	u(t) unit step at $t=0$	$1/s$
3)	$u(t-T)$ unit step at $t=T$	$\frac{1}{s}e^{-sT}$
4)	t	$1/s^2$
5)	$t^2/2$	$1/s^3$
6)	t^n	$\frac{n!}{s^{n+1}}$
7)	e^{-at}	$\frac{1}{s+a}$
8)	e^{at}	$\frac{1}{s-a}$
9)	$t \cdot e^{-at}$	$\frac{1}{(s+a)^2}$
10)	$t \cdot e^{at}$	$\frac{1}{(s-a)^2}$
11)	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
12)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
13)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
14)	$e^{-at}\sin \omega t$	$\omega/(s+\alpha)^2+\omega^2$
15)	$e^{-at}\cos \omega t$	$(s+\alpha)/(s+\alpha)^2+\omega^2$

16	$\sinh \alpha t$	$\dfrac{\alpha}{s^2-\alpha^2}$
17	$\cosh \alpha t$	$\dfrac{s}{s^2-\alpha^2}$
18	$\dfrac{1}{a^2}\left(at-1+e^{-at}\right)$	$\dfrac{1}{s^2(s+a)}$

* Laplace transform of Resistance 'R' $L_T[R] = R$
* Laplace transform of Capacitance 'C' $L_T[C] = \frac{1}{sC}$
* Laplace transform of Inductance 'L' $L_T[L] = sL$

* $L[f(t)] = F(s)$.

* $L\left[\frac{d}{dt}x(t)\right] = \left[s.X(s) - x(0)\right] = s.X(s) \quad \because x(0) = 0$

* $L\left[\frac{d^2}{dt^2}x(t)\right] = \left[s^2X(s) - s\,x(0) - x'(0)\right] = s^2X(s) \quad \because x(0) = 0 = x'(0)$

* $L\left[t\,f(t)\right] = \frac{d}{ds}F(s)$.

* Laplace transform of linear Combination.

$$L\left[a f_1(t) + b f_2(t)\right] = a F_1(s) + b F_2(s)$$

Where $f_1(t)$, $f_2(t)$ are functions of time and $a + b$ are constants.

* Scale change. $f\left[\frac{t}{a}\right] \Rightarrow a F(as) ; a > 0$

* Real translation $f(t - t_0) \Rightarrow e^{-st_0} F(s)$

* Complex translation $e^{-at} f(t) \Rightarrow F(s+a)$

* Multiplication by t $t^n f(t) \Rightarrow (-1)^n \frac{d^n F(s)}{d s^n}$

* Multiplication by $\frac{1}{t}$ $\frac{1}{t} f(t) \Rightarrow \int_s^\infty F(s) ds$

* initial Value theorem $\lim_{t \to 0} f(t) = \lim_{s \to \infty} s F(s)$

* Final value theorem $\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$

* IF the laplace transform of $f(t)$ is $F(s)$, then

(i) $L \frac{d}{dt} f(t) = \left[s F(s) - F(0)\right]$ $\because F(0) = 0$.

(ii) $L \frac{d^2 f(t)}{dt^2} = \left[s^2 F(s) - s F(0) - F'(0)\right]$

(iii) $L \frac{d^3 f(t)}{dt^3} = \left[s^3 F(s) - s^2 F(0) - sF'(0) - F''(0)\right]$

Where $f(0), f'(0), f''(0)$ are the values of $f(t)$, $\frac{d}{dt} f(t)$, $\frac{d^2 f(t)}{dt^2}$ ----- at $t = 0$

* If the Laplace transform of $f(t)$ is $F(s)$, then

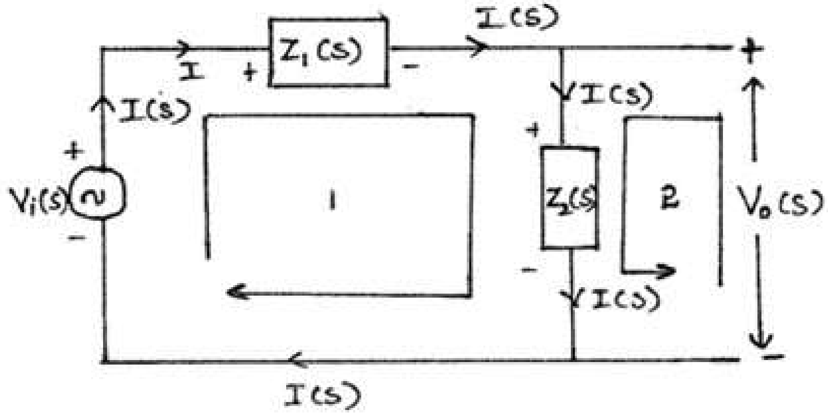
(i) $L\int f(t) = \left[\dfrac{F(s)}{s} + \dfrac{f^{-1}(0)}{s}\right]$

(ii) $L\iint f(t) = \left[\dfrac{F(s)}{s^2} + \dfrac{f^{-1}(0)}{s^2} + \dfrac{f^{2}(0)}{s}\right]$

(iii) $L\iiint f(t) = \left[\dfrac{F(s)}{s^3} + \dfrac{f^{-1}(0)}{s^3} + \dfrac{f^{-2}(0)}{s^2} + \dfrac{f^{-3}(0)}{s}\right]$

Where $f^{-1}(0)$, $f^{-2}(0)$, $f^{-3}(0)$ are the values of $\int f(t)$, $\iint f(t)$, $\iiint f(t)$ .... at $t = (0)$.

Transfer Function Of Electrical Networks:-



By applying KVL to loop 1

$$+V_i(s) - Z_1(s)I(s) - Z_2(s)I(s) = 0$$

$$V_i(s) = Z_1(s)I(s) + Z_2(s)I(s)$$

$$V_i(s) = I(s)\left[Z_1(s) + Z_2(s)\right]$$

$$I(s) = \frac{V_i(s)}{Z_1(s) + Z_2(s)} \longrightarrow (1)$$

By applying KVL to loop 2.

$$+V_o(s) - Z_2(s)I(s) = 0$$

$$V_o(s) = Z_2(s)I(s) \longrightarrow (2)$$

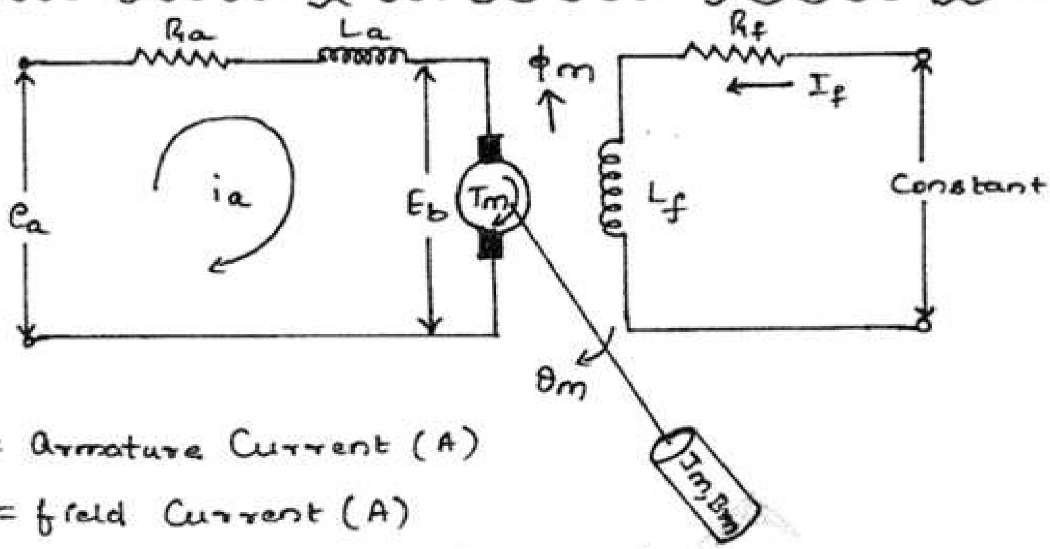
By substituting Equation (1) in (2) We get

$$V_o(s) = Z_2(s) \cdot \frac{V_i(s)}{Z_1(s) + Z_2(s)}$$

∴ Transfer function is given by

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}}$$

Transfer function of an Armature Controlled DC motor.



i_a = Armature Current (A)

i_f = field Current (A)

e_a = Applied Armature Voltage (V)

e_b = back emf (Volts)

T_m = torque developed by motor (Nm)

θ = Angular displacement of motor-shaft (rad).

J = Equivalent moment of inertia of motor and load referred to motor shaft ($Kg\text{-}m^2$)

B = Equivalent viscous friction co-efficient of motor and load referred to motor shaft $\left(\frac{Nm}{rad/s}\right)$

* Flux is directly proportional to current through field winding

$$\phi_m = K_f I_f = \text{Constant} \longrightarrow (1)$$

* Torque produced is proportional to product of flux and armature current

$$T_m = K_m \phi_m I_a$$

By substituting the value of ϕ_m in (1)

$$\therefore T_m = K_m K_f I_f I_a \longrightarrow (2)$$

* Back Emf "E_b" is directly proportional to shaft Velocity "W_m", as flux ϕ_m is constant.

* We know that velocity $W_m = \frac{d}{dt}\theta_m(t) = W_m(s) = s\,\theta_m(s)$

$$\text{Back Emf } E_b(s) = K_b W_m(s) = K_b s\,\theta_m(s) \rightarrow 3$$

* By applying KVL to armature circuit we get

$$e_a = E_b + I_a(R_a) + L_a \frac{d\,i_a}{dt}$$

By taking laplace transform

$$E_a(s) = E_b(s) + I_a(s)\cdot R_a + L_a \cdot s\, I_a(s)$$

$$E_a(s) = E_b(s) + I_a(s)\left[R_a + s L_a\right]$$

$$\boxed{I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + sL_a}} \rightarrow 4$$

* By substituting Equation 4 in 2

$$T_m = K_m K_f I_f \left\{\frac{E_a(s) - E_b(s)}{R_a - sL_a}\right\} \quad - 5$$

* The differential Equation is given by.

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\,\theta_m}{dt} = J_m s^2\,\theta_m(s) + B_m s\,\theta_m(s)$$

$$\therefore\ T_m = \left\{J_m s^2 + B_m s\right\}\theta_m(s) \quad —— 6$$

Substituting Equation 5 in 6

$$K_m K_f I_f \left\{\frac{E_a(s) - E_b(s)}{R_a - sL_a}\right\} = \left\{J_m s^2 + B_m s\right\}\theta_m(s)$$

$$\frac{K_m K_f I_f E_a(s) - K_m K_f I_f E_b(s)}{R_a - sL_a} = \left\{J_m s^2 + B_m s\right\} \theta_m(s) \longrightarrow ⑦$$

By substituting ③ in ⑦

$$\frac{K_m K_f I_f E_a(s) - K_m K_f I_f K_b s \theta_m(s)}{R_a - sL_a} = \left\{J_m s^2 + B_m s\right\} \theta_m(s)$$

$$\frac{K_m K_f I_f E_a(s)}{R_a - sL_a} - \frac{K_m K_f I_f K_b s \theta_m(s)}{R_a - sL_a} = \left\{J_m s^2 + B_m s\right\} \theta_m(s)$$

$$\frac{K_m K_f I_f E_a(s)}{R_a - sL_a} = \frac{K_m K_f I_f K_b s \theta_m(s)}{R_a - sL_a} + (J_m s^2 + B_m s)\, \theta_m(s)$$

$$\frac{K_m K_f I_f}{R_a - sL_a} E_a(s) = \left[\frac{K_m K_f I_f K_b s}{R_a - sL_a} + J_m s^2 + B_m s\right] \theta_m(s)$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m K_f I_f}{R_a + sL_a}}{\dfrac{K_m K_f I_f K_b s}{R_a + sL_a} + J_m s^2 + B_m s}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m K_f I_f}{R_a + sL_a}}{\dfrac{K_m K_f I_f K_b s + J_m s^2 (R_a + sL_a) + B_m s(R_a}{R_a + sL_a}}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_m K_f I_f}{K_m K_f I_f K_b s + (R_a + sL_a)(J_m s^2 + B_m s)}$$

$$= \frac{K_m K_f I_f}{(R_a + sL_a)(J_m s^2 + B_m s)\left[1 + \frac{K_m K_f I_f K_b s}{(R_a + sL_a)(J_m s^2 + B_m s)}\right]}$$

$$= \frac{\dfrac{K_m K_f I_f}{(R_a + sL_a)(J_m s^2 + B_m s)}}{1 + \dfrac{K_m K_f I_f K_b s}{(R_a + sL_a)(J_m s^2 + B_m s)}}$$

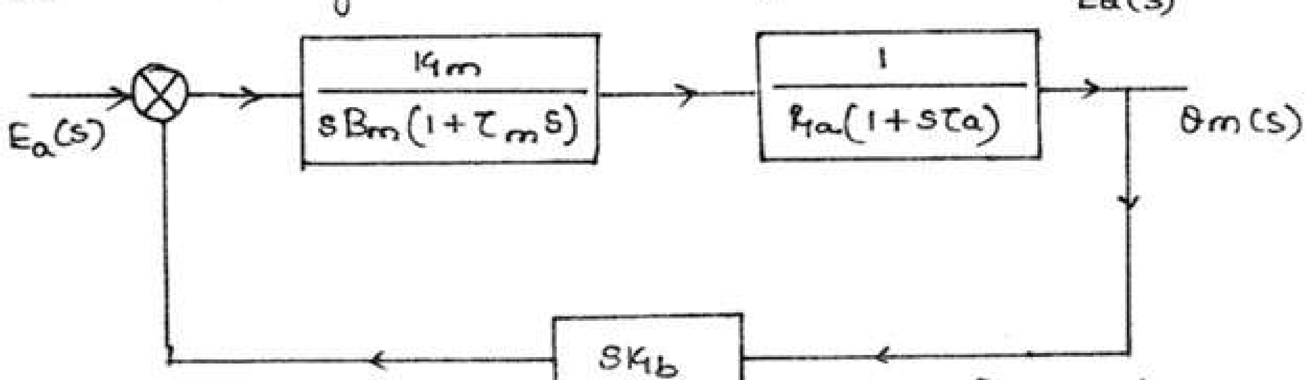
$$= \frac{\dfrac{K_m K_f I_f}{s B_m \left(s \, {J_m}/{B_m} + 1\right) R_a \left(1 + {sL_a}/{R_a}\right)}}{1 + \dfrac{K_m K_f I_f K_b s}{s B_m \left(s \, {J_m}/{B_m} + 1\right) R_a \left(1 + {sL_a}/{R_a}\right)}}$$

By substituting $K_m K_f I_f = K_m$,

$\frac{J_m}{B_m} = \tau_m$, $\frac{L_a}{R_a} = \tau_a$ we get

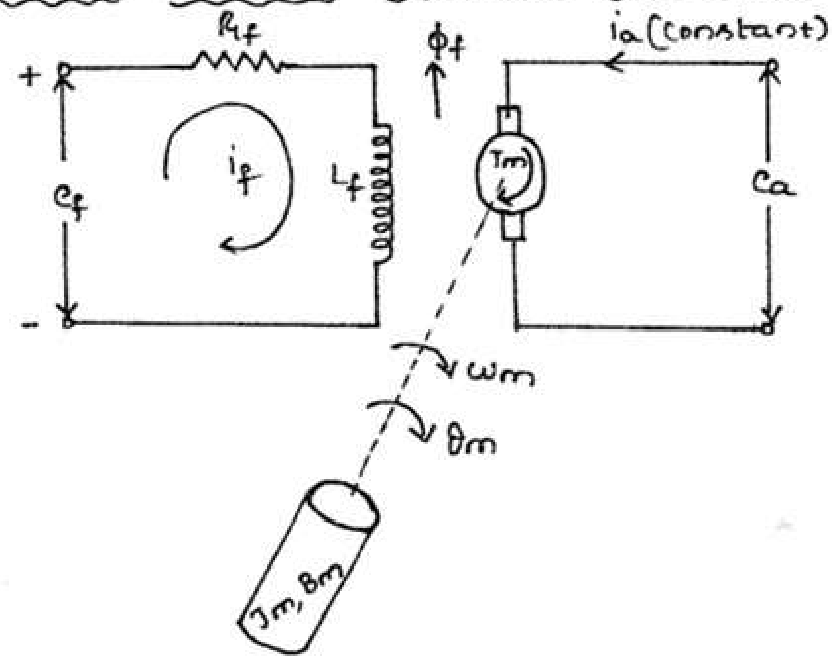
$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{s R_a B_m (1 + s\tau_m)(1 + s\tau_a)}}{1 + \dfrac{K_m \cdot s K_b}{s R_a B_m (1 + s\tau_m)(1 + s\tau_a)}}}$$

The block diagram below satisfies the T.F $\frac{\theta_m(s)}{E_a(s)}$



Block diagram of Armature Controlled D.C moto

Transfer function of field Controlled D.C. motor.



$R_f \rightarrow$ field Winding resistance (Ω).

$L_f \rightarrow$ field Winding inductance (H)

$e \rightarrow$ field Control Voltage (V).

$i_f \rightarrow$ field Current (A).

$T_m \rightarrow$ torque developed by motor (Nm).

$J_m \rightarrow$ Equivalent moment of inertia of motor (Kg-m^2)

$B_m \rightarrow$ Equivalent Viscous friction Coefficient of motor (Nm/rad/s)

$\theta \rightarrow$ Angular displacement.

* Input to the motor is Constant Armature Current
* Magnetic flux ϕ_f is proportional to field Current I_f

$$\therefore \phi_f = K_f I_f \longrightarrow ①$$

* Torque T_m is Proportional to product of flux and armature Current

$$T_m \propto \phi_f I_a$$

$$\therefore T_m = K'\phi I_a = K' K_f I_f I_a$$

$$T_m = K_m K_f I_f \longrightarrow (2)$$

Where $K_m = K' I_a$ = Constant

By applying KVL to field current

$$L_f \frac{di_f}{dt} + R_f I_f = e_f \longrightarrow (3)$$

By taking laplace transform

$$E_f(s) = L_f(s) I_f(s) + R_f I_f(s)$$

$$E_f(s) = \left[L_f(s) + R_f\right] I_f(s)$$

$$I_f(s) = \frac{E_f(s)}{sL_f + R_f} \longrightarrow (4)$$

By applying laplace transforms for Equation (2)

$$T_m(s) = K_m K_f I_f(s)$$

By substituting the value of $I_f(s)$

$$T_m(s) = K_m K_f \frac{E_f(s)}{sL_f + R_f}$$

$$T_m(s) = \frac{K_m K_f \; E_f(s)}{(sL_f + R_f)} \longrightarrow (5)$$

The differential Equation is given by

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \longrightarrow (6)$$

By taking laplace transform for Equation ⑥

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\, \theta_m(s)$$

$$T_m(s) = (J_m s^2 + B_m s)\,\theta_m(s) \quad — ⑦$$

By substituting Equation ⑦ in ⑤ we get

$$(s^2 J_m + s B_m)\,\theta_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)}$$

Here $E_f(s)$ is the input and $\theta_m(s)$ is the Output.

The transfer function is given by $= \dfrac{\theta_m(s)}{E_f(s)}$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{(s^2 J_m + s B_m)(sL_f + R_f)}$$

$$= \frac{K_m K_f}{B_m\left[\frac{s^2 J_m}{B_m} + s\right] R_f\left[\frac{sL_f}{R_f} + 1\right]}$$

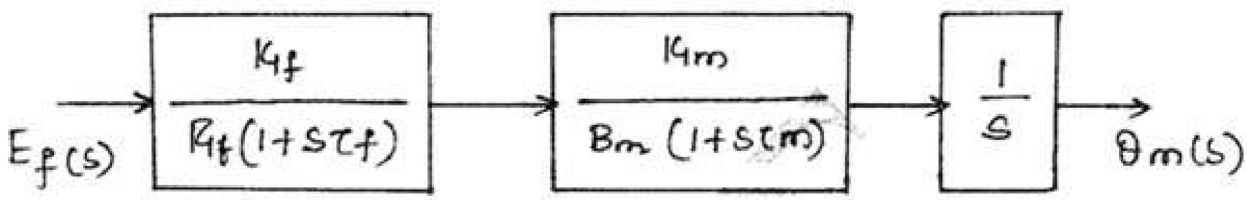
$$= \frac{K_m K_f}{s B_m R_f\left[s\frac{J_m}{B_m} + 1\right]\left[s\frac{L_f}{R_f} + 1\right]}$$

Substitute $\dfrac{J_m}{B_m} = \tau_m$ and $\dfrac{L_f}{R_f} = \tau_f$

$$\therefore \frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{s R_f B_m [1 + s\tau_m][1 + s\tau_f]}$$

$$T.F = \frac{\theta_m(s)}{E_f(s)} = \frac{K_f}{R_f[1+s\tau_f]} \cdot \frac{K_m}{B_m[1+s\tau_m]} \cdot \frac{1}{s}$$

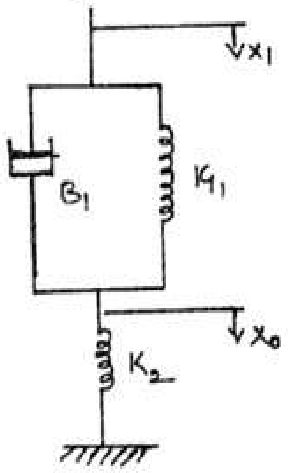
The block diagram satisfies the transfer function $\frac{\theta_m(s)}{E_f(s)}$



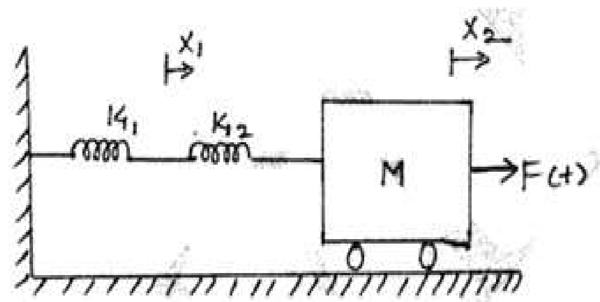
Block diagram of field Controlled DC motor.

Problems to be solved in the class.

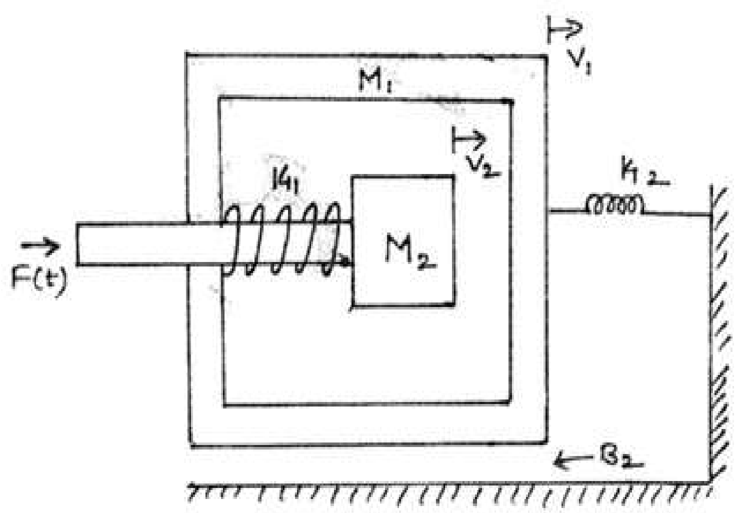
1) Obtain the transfer function for the system shown in the figure.



2)

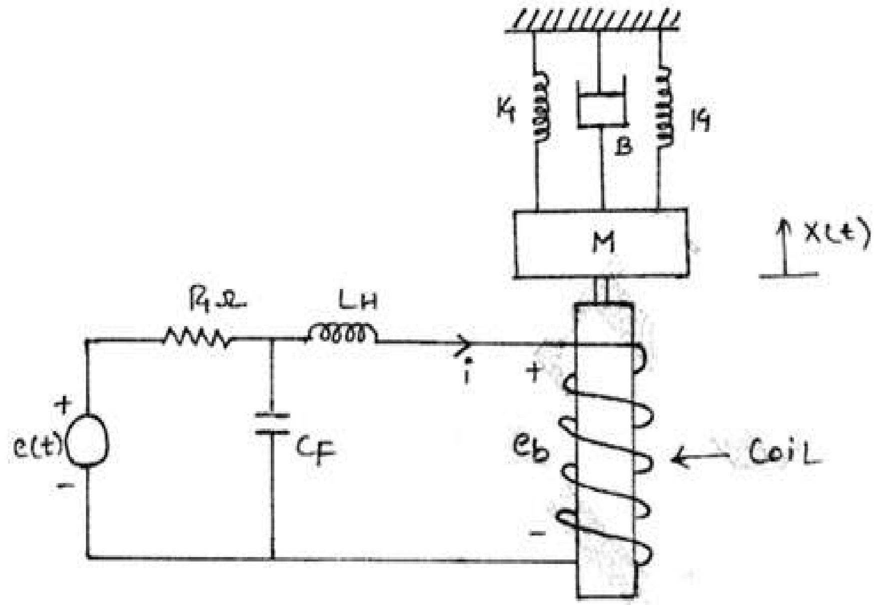


3)

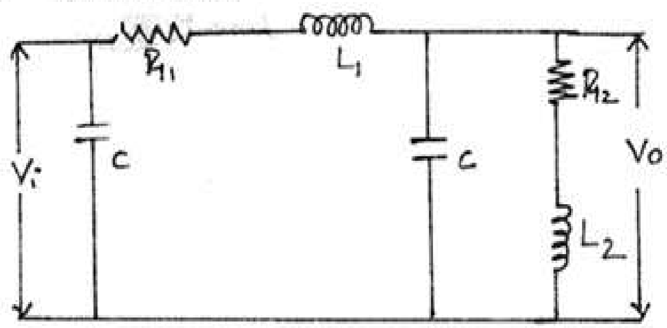


Derive the transfer function $\frac{V_2(S)}{F(S)}$ for the Mechanical System shown in the figure above.

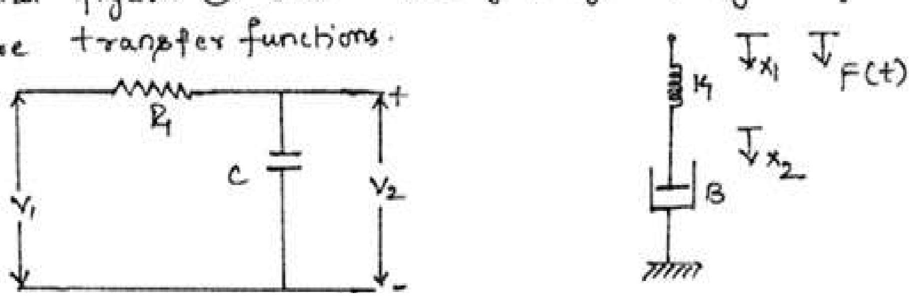
4) Find the transfer function $\frac{X(s)}{E(s)}$ for the Electromechanical System shown in figure, the coil has a back emf $e_b = K_1 \frac{dx}{dt}$ and the coil current produces a force $f_c = K_2 i$ on the mass M where K_1 & K_2 are constants.



5) Determine the transfer function for the Electrical network shown in the figure Assuming zero initial condition.

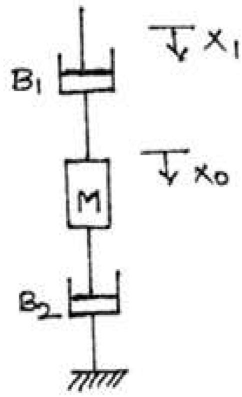


6) Show that the two Systems shown in the figure ⓐ and figure ⓑ are Analogous Systems by comparing there transfer functions.



Problems On Transfer function :-

1) Obtain the transfer function for the system shown in the figure.



Solution :- The Equilibrium Equation for a given Mechanical System is given by.

$$B_1 \frac{d}{dt}(x_1 - x_0) = M \frac{d^2}{dt^2} x_0 + B_2 \frac{d}{dt} x_0$$

$$B_1 \frac{dx_1}{dt} - B_1 \frac{dx_0}{dt} = M \frac{d^2 x_0}{dt^2} + B_2 \frac{d x_0}{dt} \longrightarrow (1)$$

By Taking laplace transform for Equation (1) we get.

$$B_1 S X_1(S) - B_1 S X_0(S) = M S^2 X_0(S) + B_2 S X_0(S)$$

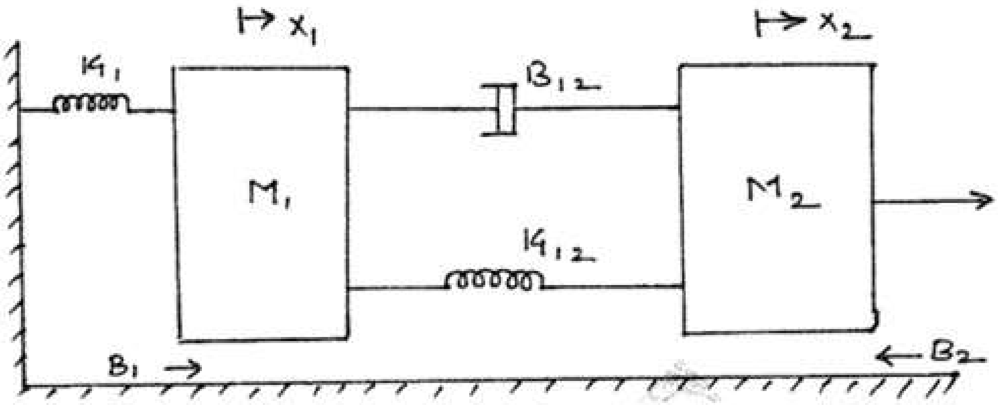
$$B_1 S X_1(S) = M S^2 X_0(S) + B_2 S X_0(S) + B_1 S X_0(S)$$

$$B_1 S X_1(S) = (M S^2 + B_2 S + B_1 S) X_0(S)$$

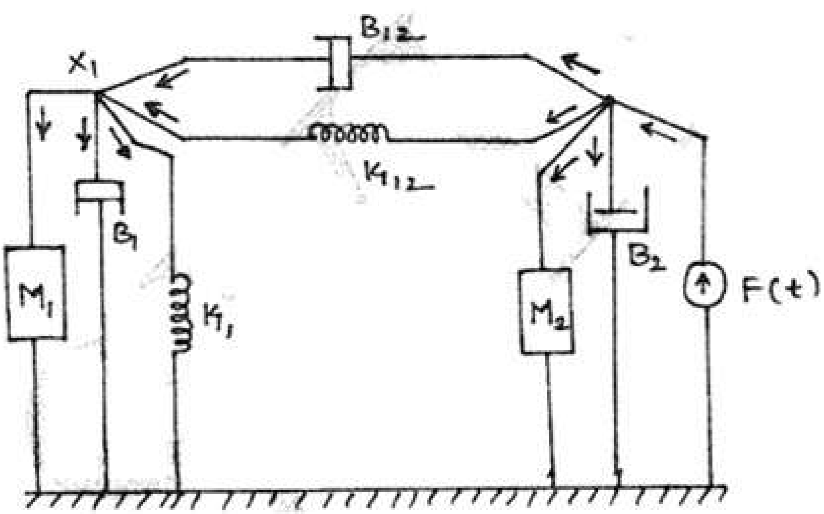
$$\frac{X_0(S)}{X_1(S)} = \frac{B_1 S}{M S^2 + B_2 S + B_1 S} = \frac{B_1}{MS + B_2 + B_1}$$

$$\boxed{\frac{X_0(S)}{X_1(S)} = \frac{B_1}{MS + B_2 + B_1}}$$

2) For a Given Mechanical System write the System Equation and Obtain Transfer function of the System.



Solution:- The Mechanical network for a given mechanical System is as shown below.



The Equilibrium Equations for a Mechanical network are.

at x_2;

$$F(t) = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{d x_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_{12}(x_2 - x_1) \rightarrow (1)$$

at x_1;

$$B_{12} \frac{d}{dt}(x_2 - x_1) + K_{12}(x_2 - x_1) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d x_1}{dt} + K_1 x_1 \rightarrow (2)$$

By taking laplace transforms for Equation (1) and (2)

For x_2 :-

$$F(S) = M_2 S^2 X_2(S) + B_2 S X_2(S) + B_{12} S (X_2(S) - X_1(S)) + K_{12}(X_2(S) - X_1(S))$$

$$F(S) = \left[M_2 S^2 + B_2 S\right] X_2(S) + \left[X_2(S) - X_1(S)\right]\left[S B_{12} + K_{12}\right]$$

$$\boxed{F(S) = (M_2 S^2 + B_2 S) X_2(S) + (S B_{12} + K_{12}) X_2(S) - (S B_{12} + K_{12}) X_1(S)} \rightarrow ③$$

For x_1 :-

$$B_{12} S (X_2(S) - X_1(S)) + K_{12}(X_2(S) - X_1(S)) = M_1 S^2 X_1(S) + B_1 S X_1(S) + K_1 X_1(S)$$

$$X_2(S)\left[B_{12} S + K_{12}\right] = M_1 S^2 X_1(S) + B_1 S X_1(S) + K_{12} X_1(S) + K_1 X_1(S) + B_{12} S X_1(S)$$

$$\boxed{X_2(S) = \frac{X_1(S)\left[M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S\right]}{\left[B_{12} S + K_{12}\right]}} \rightarrow ④$$

By substituting Equation ④ in ③ for $X_2(S)$

$$F(S) = \frac{X_1(S)\left[M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S\right]\left[M_2 S^2 + B_2 S\right]}{\left[B_{12} S + K_{12}\right]} +$$

$$\frac{X_1(S)\left[M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S\right]\left[S B_{12} + K_{12}\right]}{\left[B_{12} S + K_{12}\right]} +$$

$$X_1(S)\left[S B_{12} + K_{12}\right]$$

By taking $X_1(S)$ Common

$$F(S) = X_1(S)\left[\frac{\left[M_1 S^2 + B_1 S + K_1 + K_{12} + B_{12} S\right]\left[M_2 S^2 + B_2 S\right]}{\left[B_{12} S + K_{12}\right]} + \right.$$

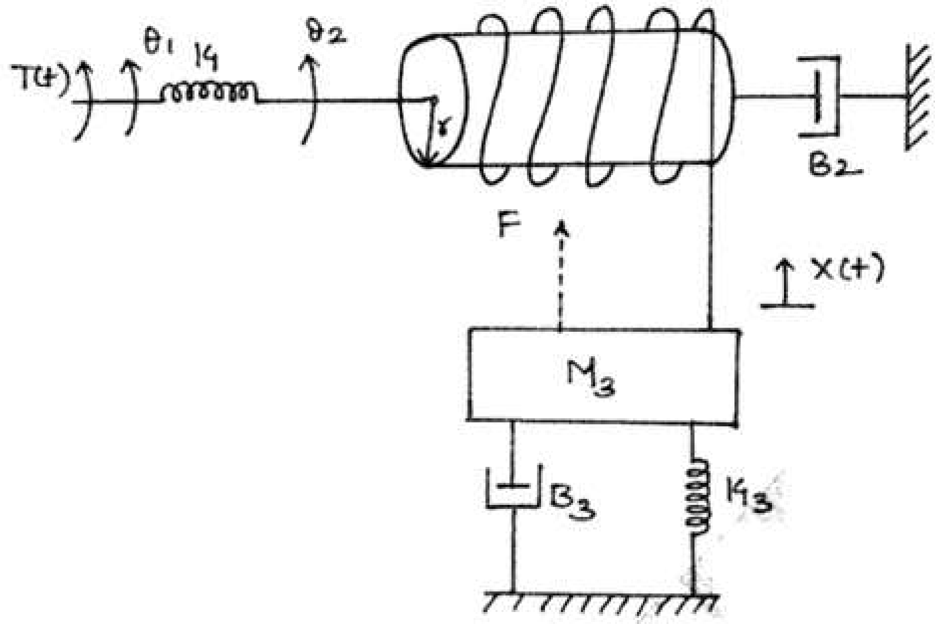
$$\left[\frac{\left[M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right]\left[SB_{12}+K_{12}\right]}{\left[SB_{12}+K_{12}\right]}-\left[SB_{12}+K_{12}\right]\right]$$

$$\frac{F(S)}{X_1(S)}=\left[\frac{\left(M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right)\left[\left(SB_{12}+K_{12}\right)+\left(M_2S^2+B_2S\right)\right]}{\left[SB_{12}+K_{12}\right]}-\left[SB_{12}+K_{12}\right]\right]$$

$$\frac{F(S)}{X_1(S)}=\frac{\left[\left(M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right)\left(SB_{12}+K_{12}\right)+\left(M_2S^2+B_2S\right)\left(M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right)-\left(SB_{12}+K_{12}\right)^2\right]}{SB_{12}+K_{12}}$$

$$\boxed{\therefore\ \frac{X_1(S)}{F(S)}=\frac{SB_{12}+K_{12}}{\left[\left(M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right)\left(SB_{12}+K_{12}\right)+\left(M_2S^2+B_2S\right)\left(M_1S^2+SB_1+K_1+SB_{12}+K_{12}\right)-\left(SB_{12}+K_{12}\right)^2\right]}}$$

③ In the Mechanical System shown, write the differential Equation of performance for this System.



Solution:- Equilibrium Equations of the Mechanical System are given by.

$$\text{At } \theta_1 = T(t) = K_1(\theta_1 - \theta_2) \quad \text{— ①}$$

$$\text{At } \theta_2 = K_1(\theta_1 - \theta_2) = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_1 \quad \text{— ②}$$

Where $T_1 = F \cdot r \longrightarrow$ ③

Where F is the force acting on the translational System at

$$x(t) : F = M_3 \frac{d^2 x(t)}{dt^2} + B_3 \frac{d\,x(t)}{dt} + K_3 x(t) \quad \text{— ④}$$

4) Find the transfer function for the system shown in the figure ① and ②. hence show that they are analogous to Each Other.

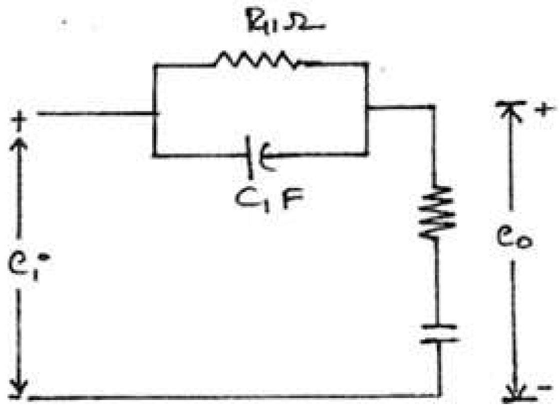


Fig ①

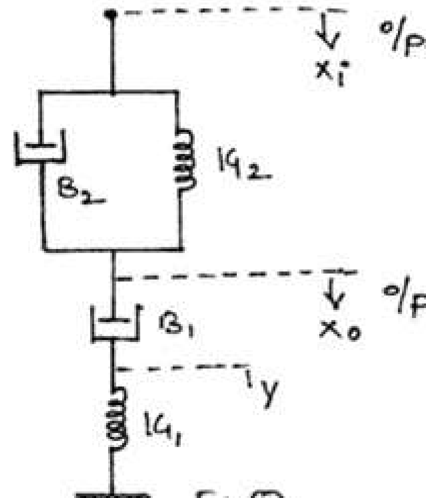


Fig ②

Solution:-

$x_i = i/p$, $x_o = o/p$.

The Equilibrium Equation of the mechanical System is given by.

$$\text{at } x_o: \; B_2 \frac{d}{dt}(x_i - x_o) + K_2 (x_i - x_o) = B_1 \frac{d}{dt}(x_o - y) \quad — ①$$

$$\text{at } y: \; B_1 \frac{d}{dt}(x_o - y) = K_1 y \quad — ②$$

By taking laplace transform Assuming zero initial Condition.

$$\text{From ①}: \; B_2 s\left[X_i(s) - X_o(s)\right] + K_2\left[X_i(s) - X_o(s)\right] = B_1 s\left[X_o(s) - Y(s)\right]$$

or

$$(B_2 s + K_2) X_i(s) = (B_1 s + B_2 s + K_2) X_o(s) - B_1 s Y(s) \quad — ③$$

$$\text{From ②}; \; B_1 s\left(X_o(s) - Y(s)\right) = K_1 Y(s)$$

or

$$B_1 s X_o(s) = (B_1 s + K_1) Y(s)$$

$$\text{or} \quad Y(S) = \frac{B_1 S}{B_1 S + K_1} \cdot X_0(S) \quad \text{—} \ (4)$$

By substituting (4) in (3)

$$(B_2 S + K_2)\, X_i(S) = (B_1 S + B_2 S + K_2)\, X_0(S) - \frac{(B_1 (S)^2 X_0(S))}{B_1 S + K_1}$$

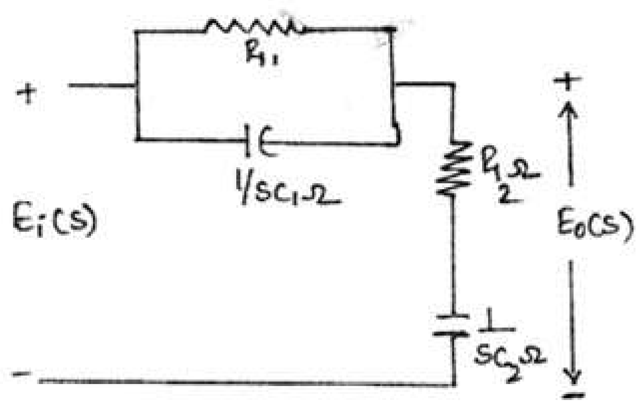
$$(B_2 S + K_2)\, X_i(S) = X_0(S) \left[\frac{(B_1 S + K_1)(B_1 S + B_2 S + K_2) - (B_1 S)^2}{(B_1 S + K_1)} \right]$$

$$\frac{X_0(S)}{X_i(S)} = \frac{(B_1 S + K_1)(B_2 S + K_2)}{(B_1 S + K_1)(B_1 S + B_2 S + K_2) - (B_1 S)^2}$$

$$= \frac{(B_1 S + K_1)(B_2 S + K_2)}{\cancel{(B_1 S)^2} + B_1 B_2 S^2 + B_1 S K_2 + B_1 S K_1 + B_2 S K_1 + K_1 K_2 - \cancel{(B_1 S)^2}}$$

$$\boxed{\frac{X_0(S)}{X_i(S)} = \frac{(B_1 S + K_1)(B_2 S + K_2)}{S^2 B_1 B_2 + S(B_1 K_2 + B_1 K_1 + B_2 K_1) + K_1 K_2}}$$

The transform Network for the Electrical Network is as shown below.



Note :-

$V(t) = R\, i(t)$

$V(S) = R\, i(S)$

$V(t) = L \frac{d\, i(t)}{dt}$

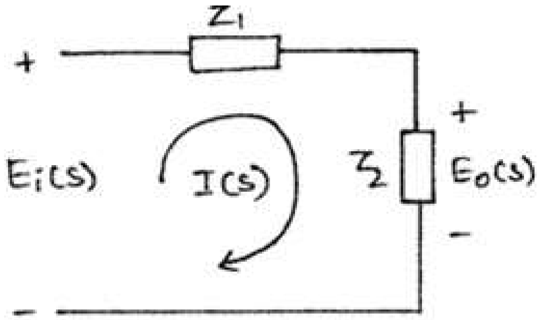
$V(S) = L\, S\, I_L(S)$

$= SL\, I_L(S)$

$V(t) = \frac{1}{C}\int i\,dt$

$V(S) = \frac{1}{C} \frac{I(S)}{S}$

$= \frac{1}{SC} I(S)$



$$Z_1 = \frac{R_1 \times \frac{1}{sC_1}}{R_1 \times \frac{1}{sC_1}} = \frac{R_1}{R_1 s C_1 + 1}$$

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{R_2 s C_2}{sC_2} + 1$$

$$I(s) = \frac{E_i(s)}{Z_1 + Z_2}$$

$$E_o(s) = Z_2 I(s)$$

$$= \frac{Z_2}{Z_1 + Z_2} \cdot E_i(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_1}{Z_1 + Z_2}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(\frac{R_2 s C_2 + 1}{sC_2}\right)}{\frac{R_1}{R_1 s C_1 + 1} + \frac{R_2 s C_2 + 1}{sC_2}} = \frac{(R_1 s C_1 + 1)(R_2 s C_2 + 1)}{R_1 s C_2 + (R_1 s C_1 + 1)(R_2 s C_2 + 1)}$$

$$= \frac{C_1\left(R_1 s + \frac{1}{C_1}\right) C_2\left(R_2 s + \frac{1}{C_2}\right)}{R_1 s C_2 + R_1 R_2 s^2 C_1 C_2 + R_1 s C_1 + R_2 s C_2 + 1}$$

$$= \frac{C_1 C_2 \left(R_1 s + \frac{1}{C_1}\right)\left(R_2 s + \frac{1}{C_2}\right)}{C_1 C_2 \left[R_1 R_2 s^2 + \frac{R_1 s}{C_1} + \frac{R_1 s}{C_2} + \frac{R_2 s}{C_1} + \frac{1}{C_1 C_2}\right]}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{\left(R_1 s + \frac{1}{C_1}\right)\left(R_2 s + \frac{1}{C_2}\right)}{R_1 R_2 s^2 + s\left(\frac{R_1}{C_2} + \frac{R_1}{C_1} + \frac{R_2}{C_1}\right) + \frac{1}{C_1 C_2}}} \rightarrow (6)$$

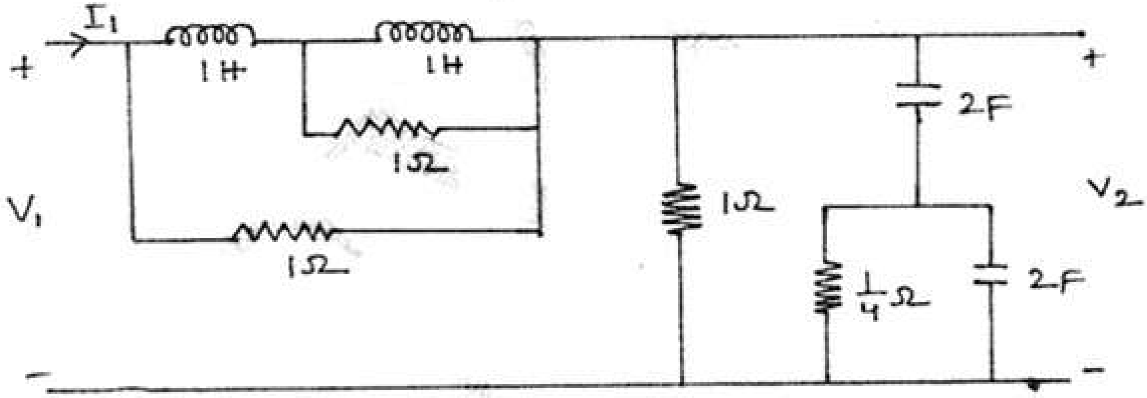
Equation (5) and (6) are mathematically simillar hence. the two System will be Analogous.

$$R_1 = B_1 \quad , \quad R_2 = B_2$$

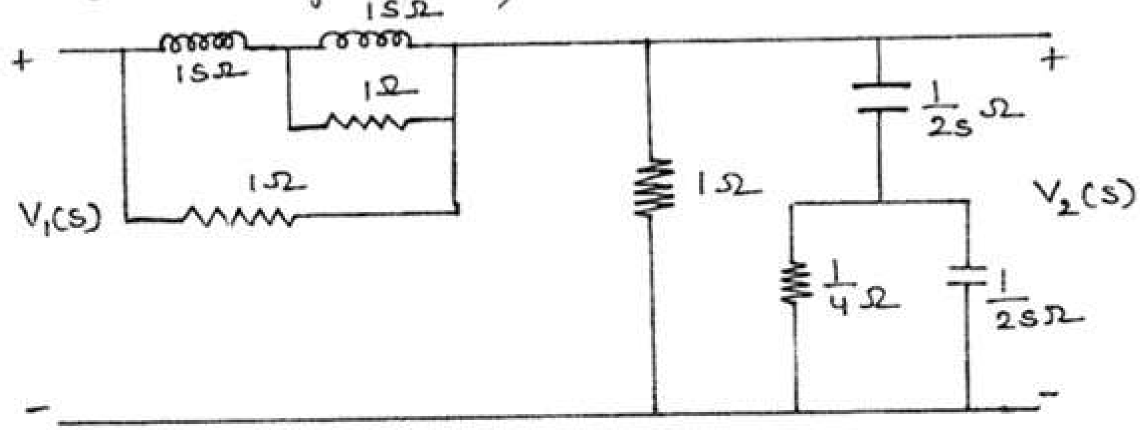
$$C_1 = \frac{1}{K_1} \quad , \quad C_2 = \frac{1}{K_2}$$

5) For the two port network shown in figure obtain the transfer function.

(i) $\frac{V_2(s)}{V_i(s)}$ (ii) $\frac{V_1(s)}{I_1(s)}$

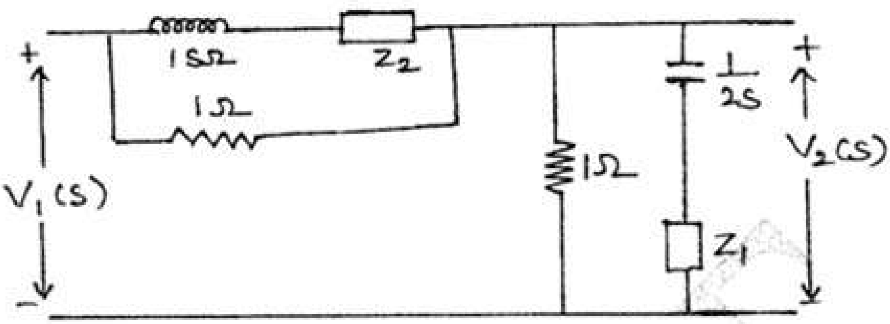


Solution :- Transformer network is as shown below.



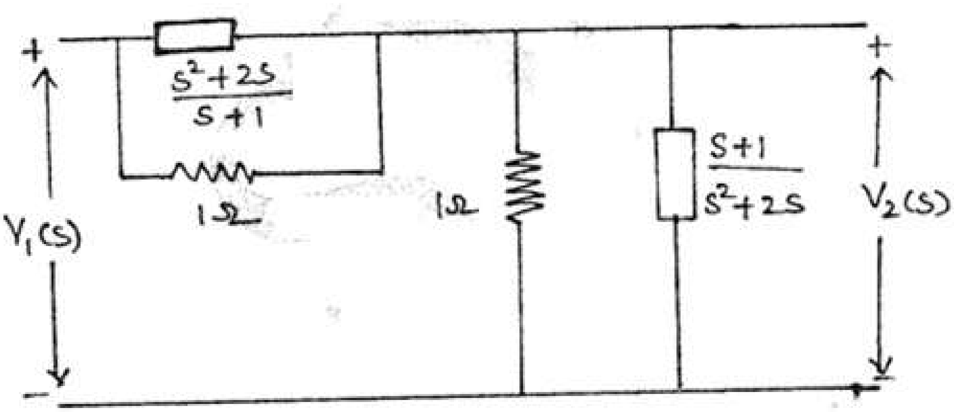
$$Z_1 = \frac{\frac{1}{4} * \frac{1}{2s}}{\frac{1}{4} * \frac{1}{2s}} = \frac{1}{2s+4}$$

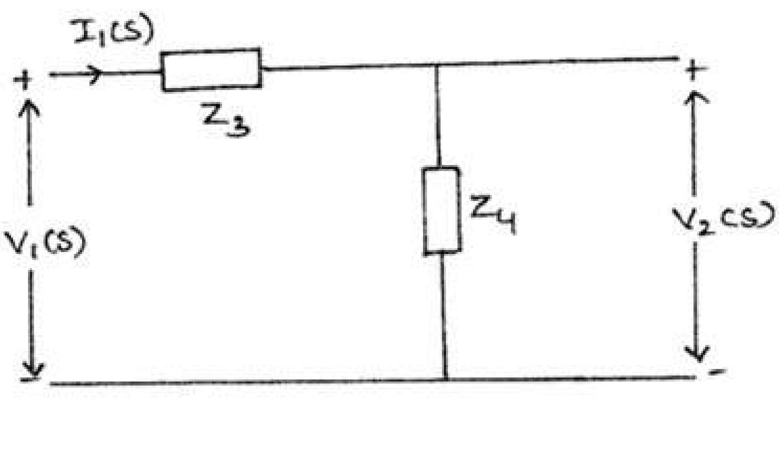
$$Z_2 = \frac{s \times 1}{s+1} = \frac{s}{s+1}$$



$$s + Z_2 = s + \frac{s}{s+1} = \frac{s^2+2s}{s+1}$$

$$Z_1 + \frac{1}{2s} = \frac{1}{2s+4} + \frac{1}{2s} = \frac{2s+2s+4}{2s(2s+4)} = \frac{4s+4}{4s^2+8s} = \frac{s+1}{s^2+2s}$$





$$Z_3 = \frac{\left(\frac{s^2+2s}{s+1}\right)\times 1}{\frac{s^2+2s}{s+1}+1} = \frac{s^2+2s}{s^2+3s+1}$$

$$Z_4 = \frac{1\times\frac{(s+1)}{s^2+2s}}{1+\frac{s+1}{s^2+2s}}$$

$$Z_4 = \frac{s+1}{s^2+2s+1+s} = \frac{s+1}{s^2+3s+1}$$

$$I_1(s) = \frac{V_1(s)}{Z_3+Z_4}$$

$$V_2(s) = I_1(s) + Z_4$$

$$V_2(s) = \frac{V_1(s)}{Z_3+Z_4}\cdot Z_4$$

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_4}{Z_3+Z_4}$$

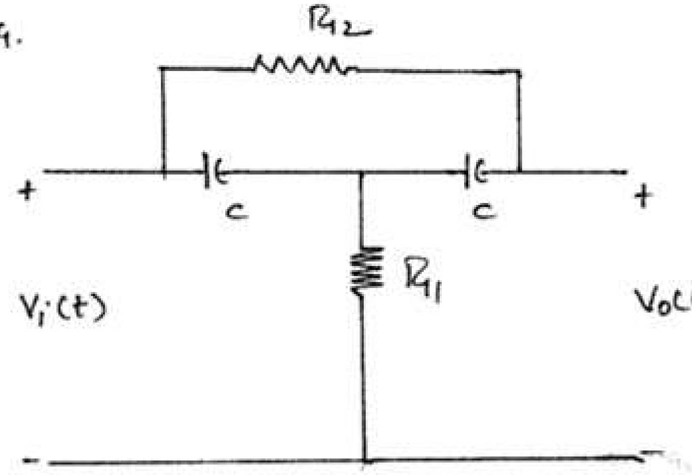
& $$\frac{V_1(s)}{I_1(s)} = Z_3+Z_4$$

$$Z_3+Z_4 = \frac{s^2+2s}{s^2+3s+1} + \frac{s+1}{s^2+3s+1}$$

$$Z_3+Z_4 = 1$$

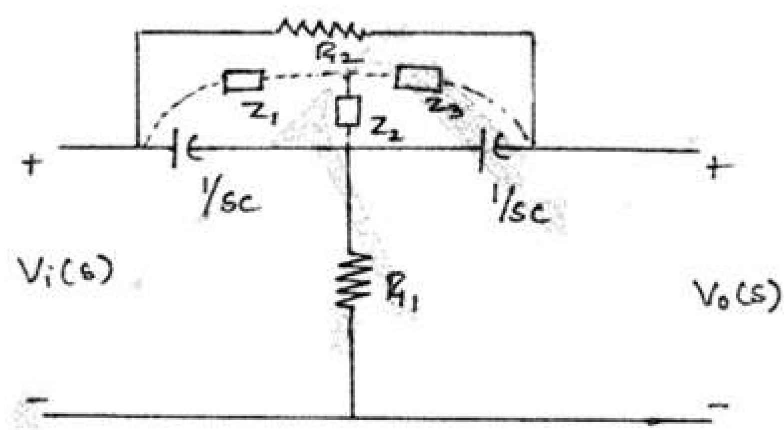
$$\therefore \boxed{\frac{V_2(s)}{V_1(s)} = \frac{Z_4}{1} = \frac{s+1}{s^2+3s+1}} \quad \& \quad \boxed{\frac{V_1(s)}{I_1(s)} = 1}$$

6) The circuit of bridge-T Network is shown in figure determine the transfer function $\frac{V_o(s)}{V_i(s)}$ of the network.

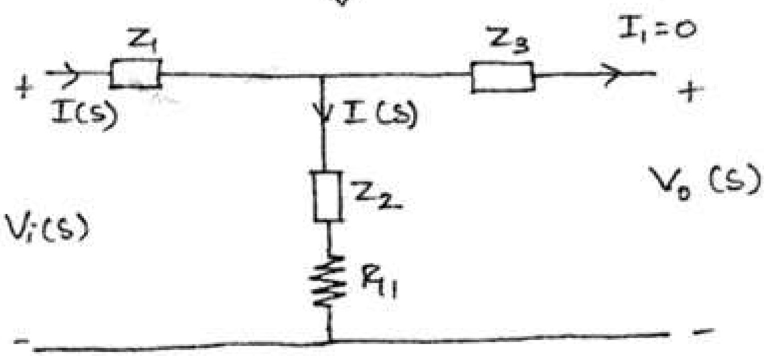


Solution :-

Transform network is as shown below.



$\Downarrow$



$$Z_1 = \frac{R_2 \times \frac{1}{sC}}{R_2 + \frac{1}{sC} + \frac{1}{sC}} = \frac{R_2}{R_2 sC + 2}$$

$$Z_3 = Z_1$$

$$Z_2 = \frac{\frac{1}{sC} \times \frac{1}{sC}}{R_2 + \frac{1}{sC} + \frac{1}{sC}} = \frac{1}{sC(R_2 sC + 2)}$$

$$I(s) = \frac{V_i(s)}{Z_1 + Z_2 + R_1}$$

$$V_o(s) = I(s)\,(Z_2 + R_1)$$

$$V_o(s) = \frac{V_i(s)}{Z_1 + Z_2 + R_1} \cdot Z_2 + R_1$$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2 + R_1}{Z_1 + Z_2 + R_1}$$

$$Z_1 + Z_2 = \frac{R_2}{2 + R_2 sC} + \frac{1}{sC(2 + R_2 sC)}$$

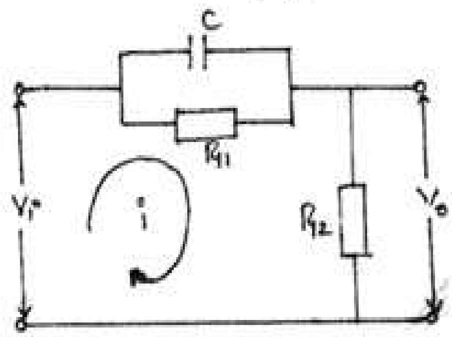
$$= \frac{R_2 sC + 1}{sC(2 + R_2 sC)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC(2 + R_2 sC)} + R_1}{\frac{(R_2 sC + 1)}{sC(2 + R_2 sC)} + R_1}$$

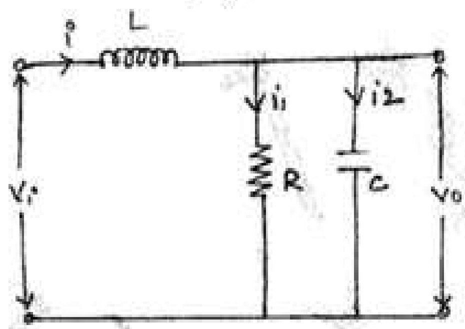
Transfer Function: $\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1 + R_1 sC(2 + R_2 sC)}{R_2 sC + 1 + R_1 sC(2 + R_2 sC)}}$

Practice Problems on Transfer Function

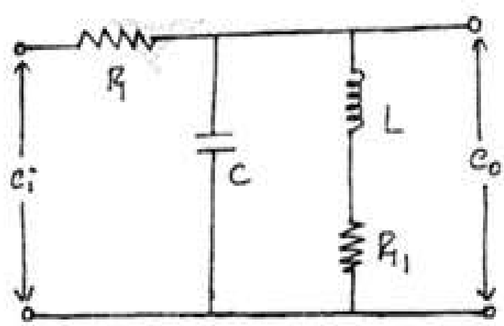
① Derive the transfer function $\frac{V_o(s)}{V_i(s)}$ for the circuit shown in the figure.



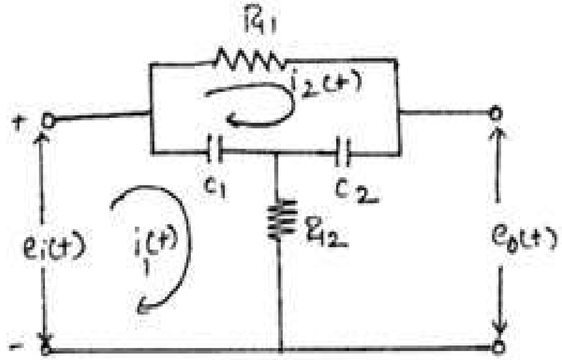
② Find the transfer function of the electrical network shown in the figure.



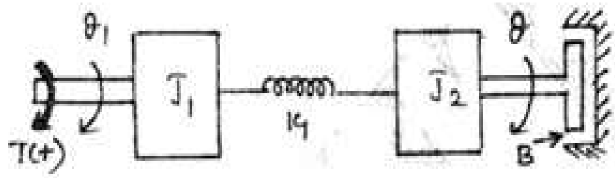
③ Derive the transfer function of the network shown in the figure



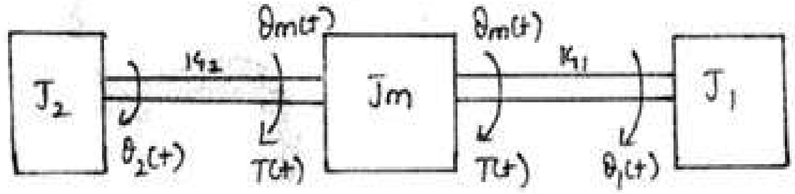
4) Find the transfer function of the electrical network shown in the figure



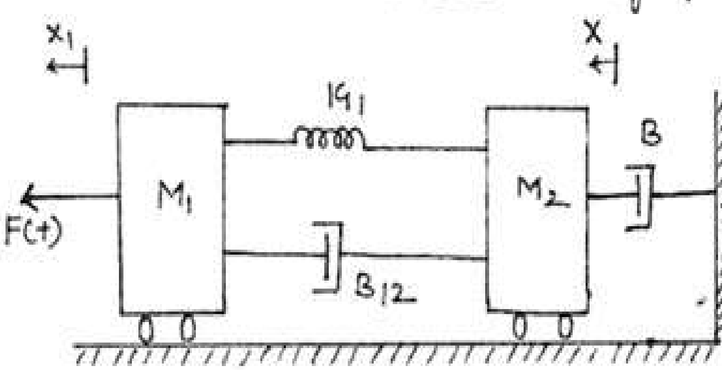
5) Obtain the transfer function of the mechanical System shown in the figure.



6) Find the transfer function $\frac{\theta_1(s)}{T(s)}$ for the system shown in the figure.

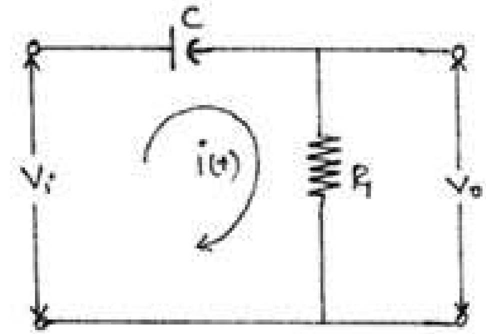


7) Write the differential Equations governing the Mechanical translational System shown in the figure. and determine the transfer function $\frac{X(s)}{F(s)}$

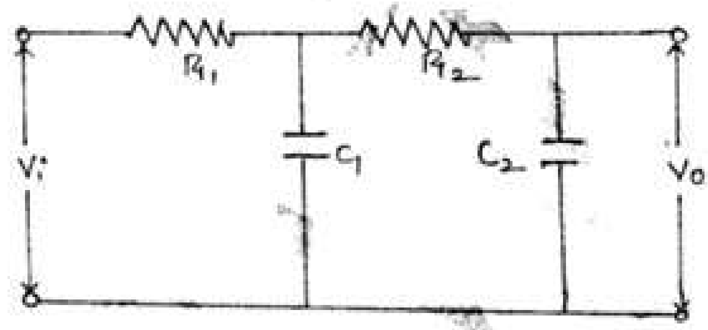


Assignment Problems On Transfer Function

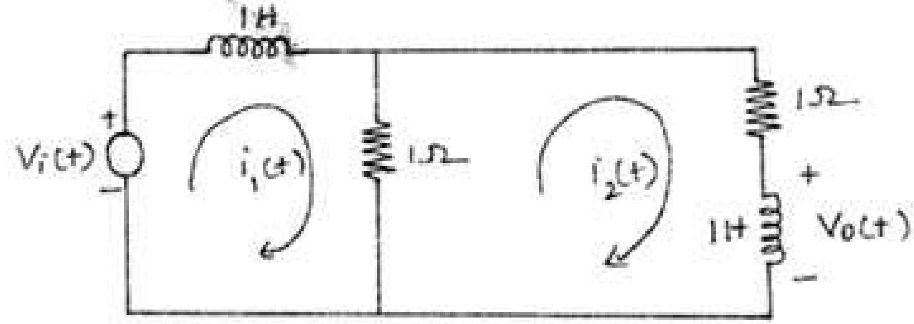
① Derive the transfer function $\frac{V_o(S)}{V_i(S)}$ for the Circuit shown in the figure.



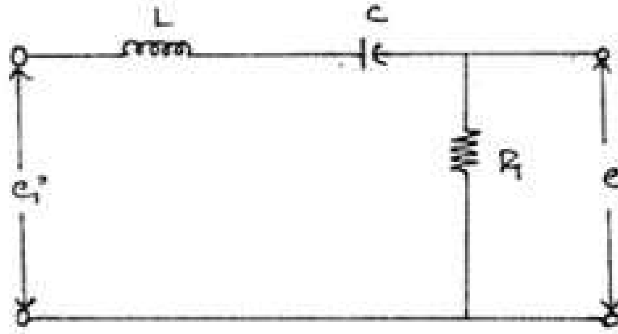
② Find the transfer function $\frac{V_o(S)}{V_i(S)}$ for the network shown in the figure.



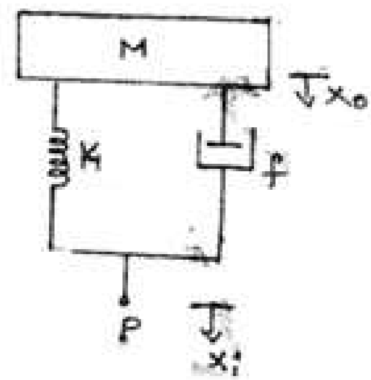
③ Find the input - output relationship of the Electrical network shown in the figure



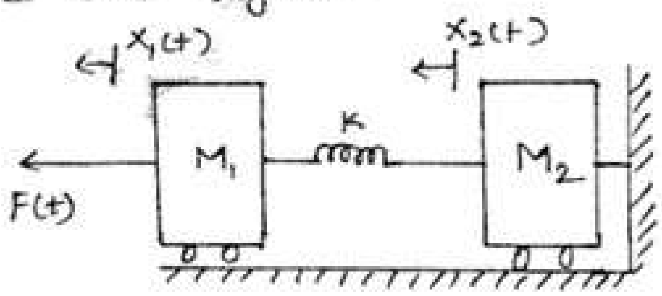
4) Derive the transfer function of the network shown in the figure.



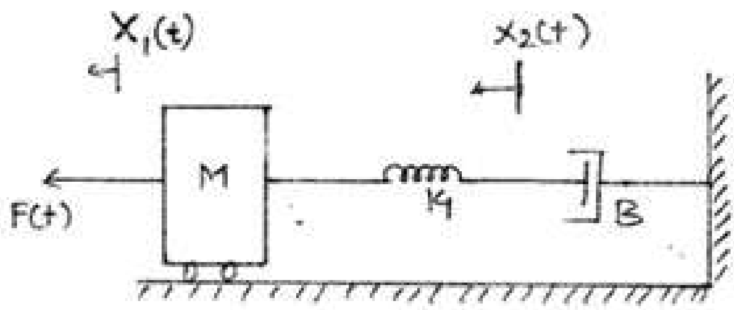
5) Find the transfer function $\frac{X_o(s)}{X_i(s)}$ for the system shown in the figure



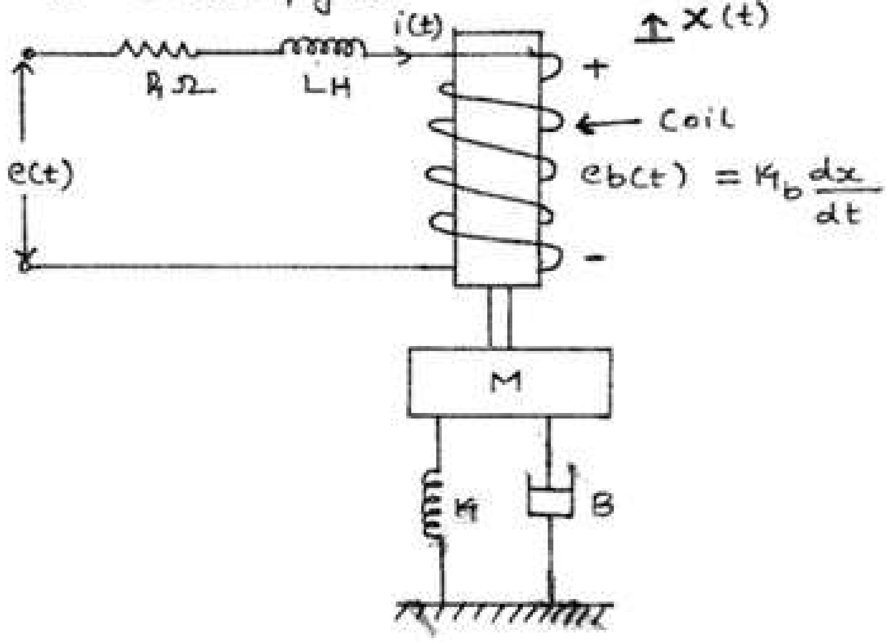
6) Find the transfer function $\frac{X_1(s)}{F(s)}$ for the system shown in the figure.



7)

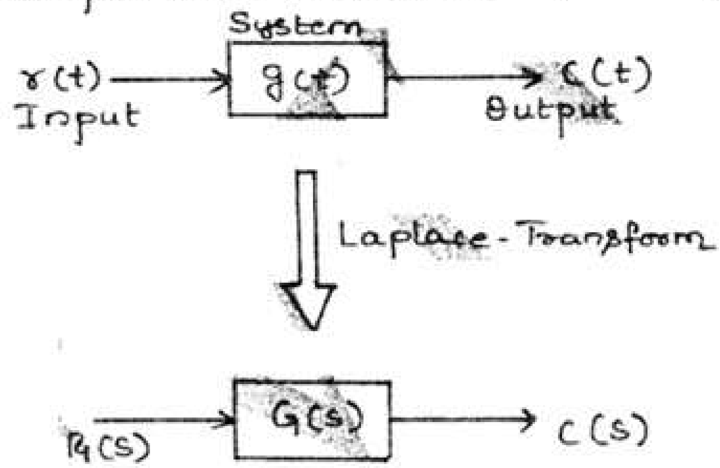


8) Find the transfer function $\frac{X(s)}{E(s)}$ for the Electromechanical System shown in the figure.



Block Diagram Algebra

* In this section we discuss development of block diagram for the systems.
* A system consists of number of components, the function of each component is represented by a block.
* All the blocks are interconnected by lines with arrows indicating the flow of signals from the output of one block to the input of another.
* Such a block diagram gives an overall idea of the inter-relationships that exist among various components.

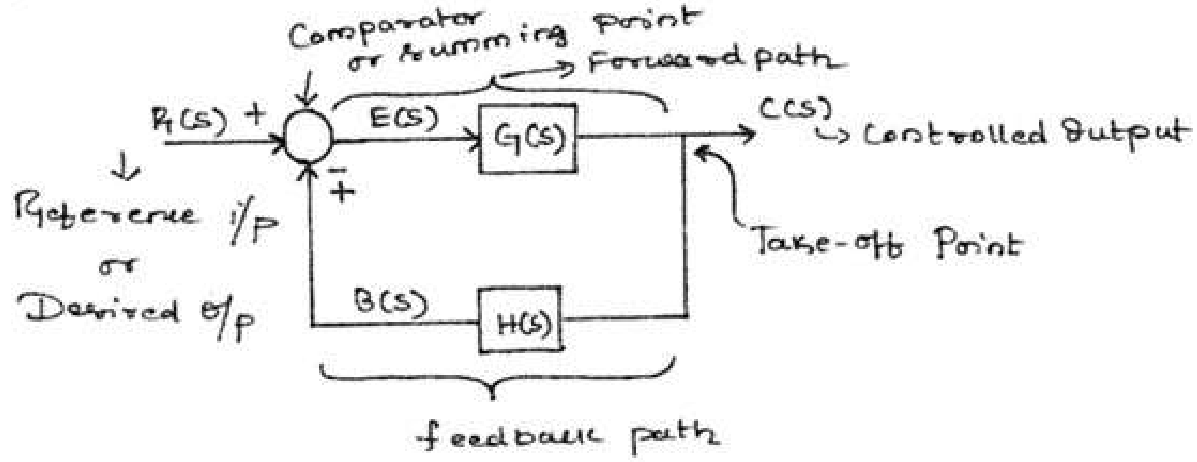


* Let us consider a system with transfer function G(S)
* The system can be represented by a block as shown in the figure above.
* The input signal into the block is R(S) which is the Laplace transform of input signal r(t)
* The output signal of the block is C(S) which is the Laplace transform of the output signal c(t).
* The flow of signal is unidirectional from the input to the output. The output C(S) is equal to the convolution of the input signal and transfer function G(S)

i.e, $\boxed{C(S) = G(S)\,R(S)}$

⇒ Block Diagram Transformation.

1) Feedback Control System (Eliminating feedback loop).



* The Error signal E(s) is given by

$$E(s) = R(s) - B(s) \quad \text{—— ①}$$

W.K.T $\quad G(s) = \dfrac{C(s)}{E(s)}$

$$\text{or} \quad E(s) = \frac{C(s)}{G(s)}$$

&

$$H(s) = \frac{B(s)}{C(s)}$$

or

$$B(s) = H(s) \cdot C(s)$$

By substituting the value of E(s) + B(s) in ①

$$\frac{C(s)}{G(s)} = R(s) - H(s) \cdot C(s)$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

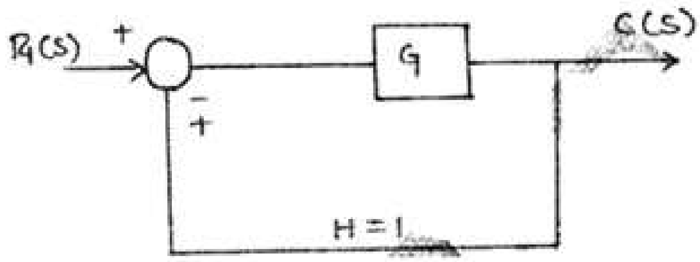
$$C(s) + G(s)H(s)C(s) = G(s)R(s)$$

$$C(s)\left[1 + G(s)H(s)\right] = G(s)R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{G}{1 + GH}}$$

$\frac{C(s)}{R(s)}$ is known as the overall transfer function or closed loop transfer function.

If H = 1, then it is said to be unity feedback control system, shown in the figure.

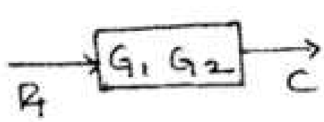


$$\boxed{\frac{C(s)}{R(s)} = \frac{C}{R} = \frac{G}{1 \pm G}}$$

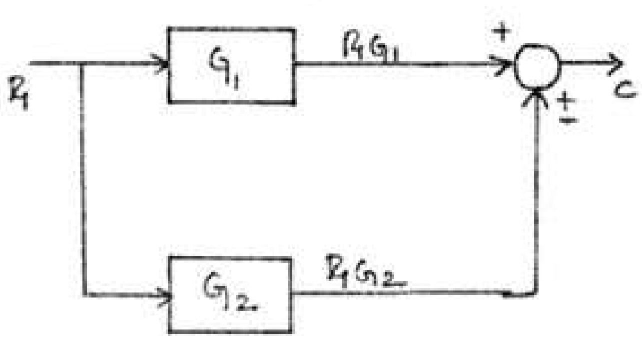
2) Blocks in cascade

$$C = R G_1 G_2$$

$$\frac{C}{R} = G_1 G_2$$

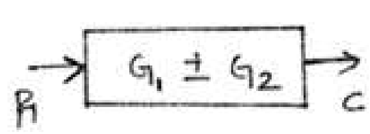


3) Blocks in parallel.

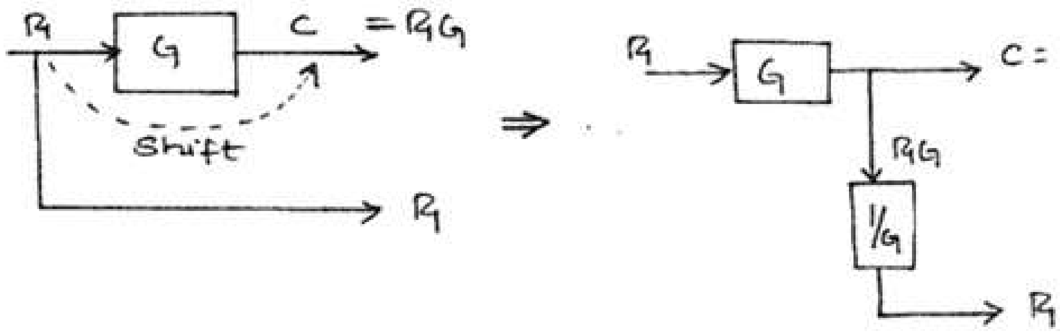


$$C = RG_1 \pm RG_2 = R(G_1 \pm G_2)$$

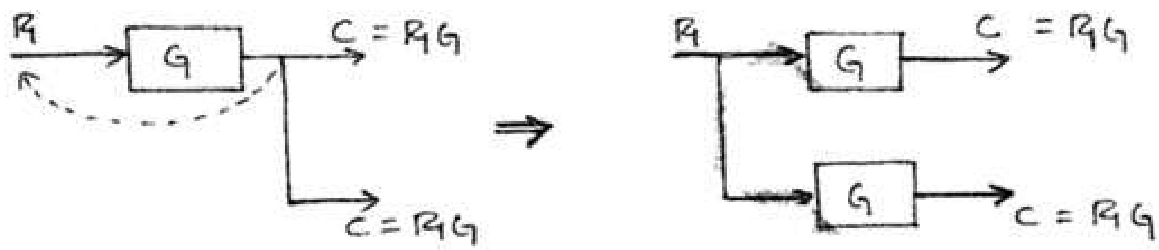
$$\frac{C}{R} = G_1 \pm G_2$$



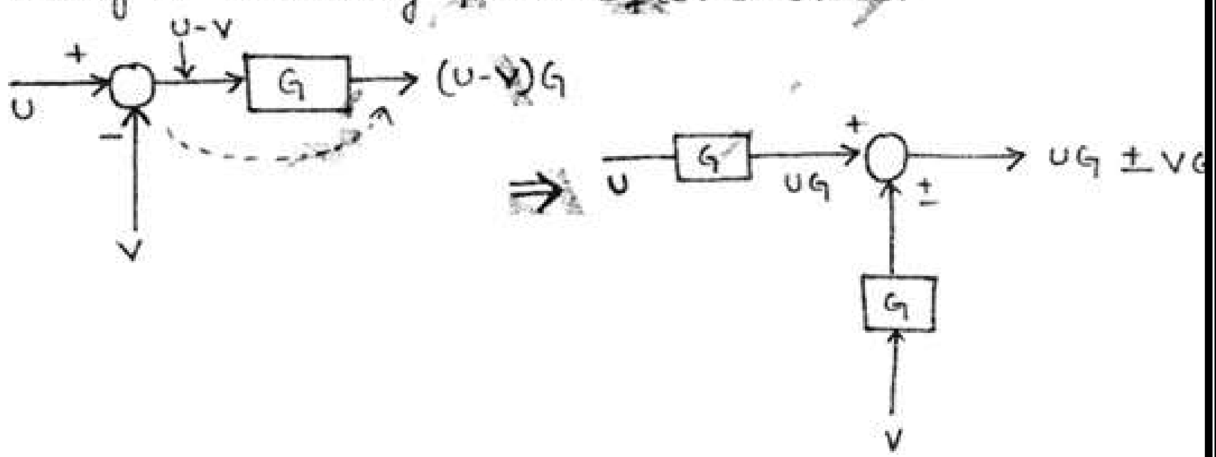
4) Shifting a take off point after a block.



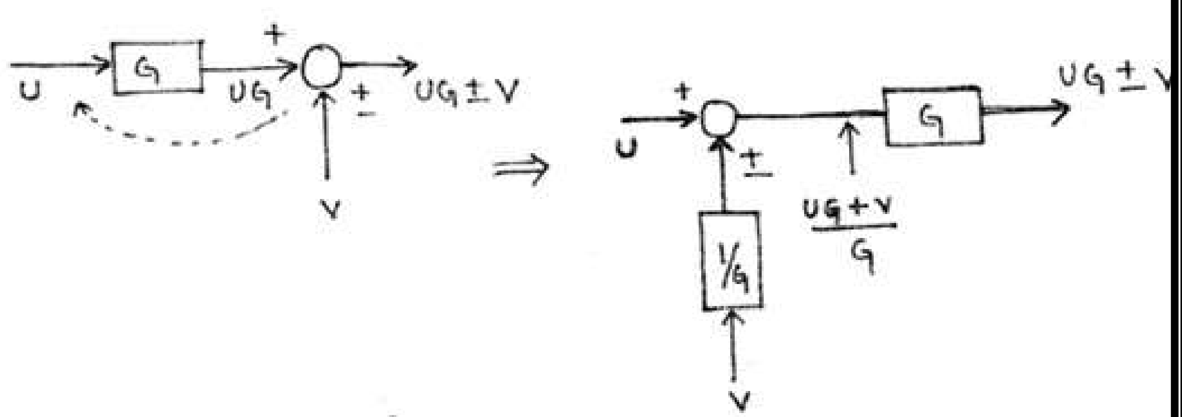
5) Shifting a take-off point before the block.



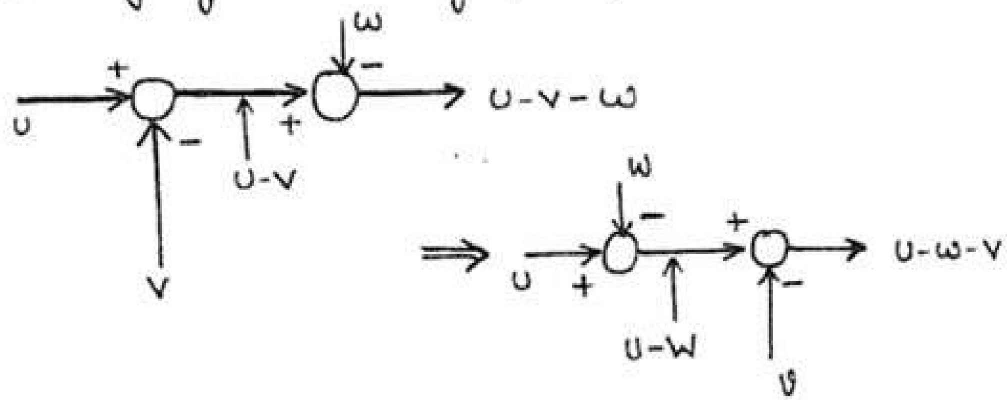
6) Moving a Summing Point after a block.



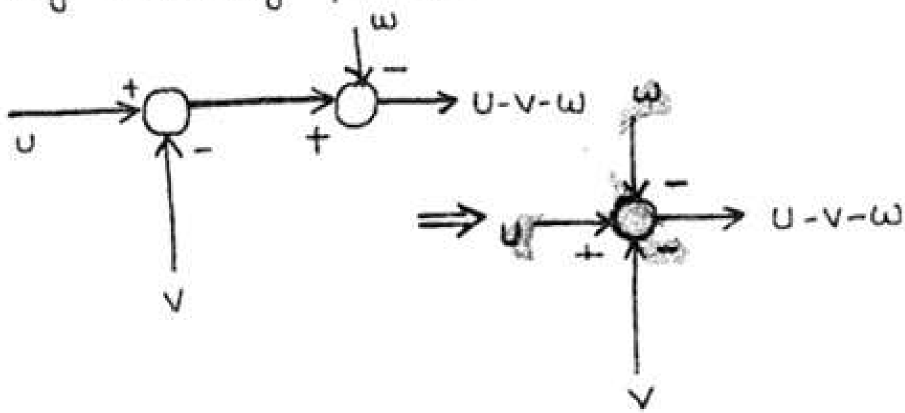
7) Moving a Summing point before the block.



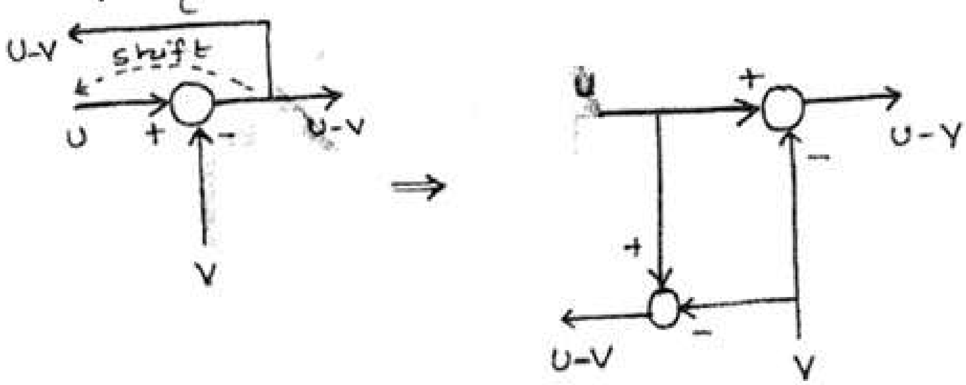
8) Rearranging Summing Points.



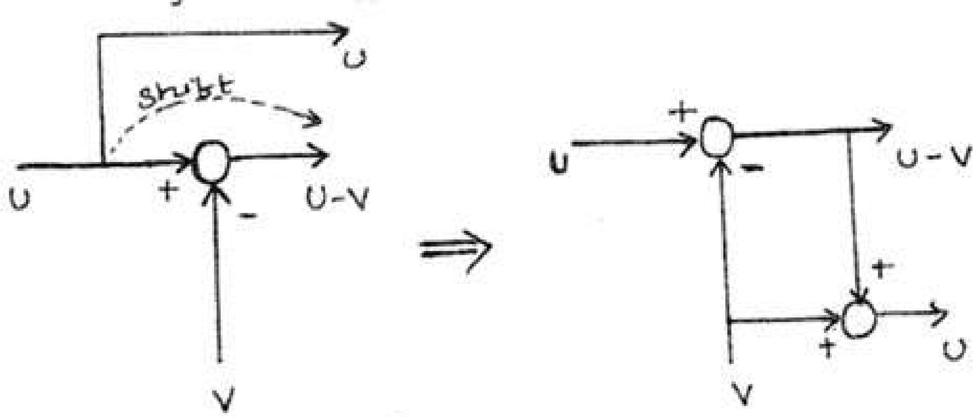
9) Adding Summing points.



10) Shifting a take-off point before a summing Point



11) Shifting take off point after the summing point.



* Steps for reduction of Complicated Block diagram:

step1:- Combine all Cascade blocks.

step2:- Combine all parallel blocks.

step3:- Eliminate all minor feed back loops.

step4:- Shift Summing points to the left and take off Points to the right of the major loop.

step5:- Repeat steps 1 to 4 until the Canonical form has been achieved for a particular input.

step6:- Repeat steps 1 to 5 for each input as required

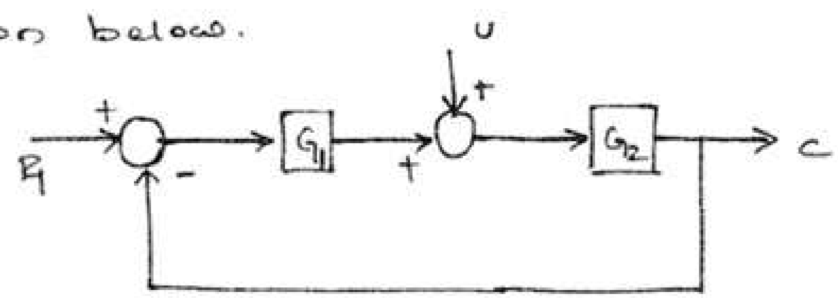
* Superposition of Multiple Inputs:-

→ Some time it is necessary to Evaluate System Performance when several inputs are simultaneously applied at different points of the System.

→ When multiple inputs are present in a linear System, Each input is treated Independently of the others.

For Example:-

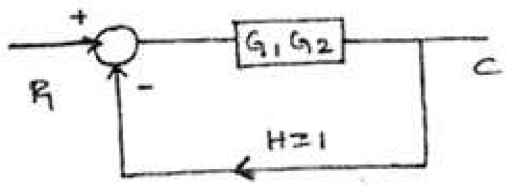
Determine the transfer function for the block diagram shown below.



Step1:- Consider any one input at a time, (U)

Put $U = 0$.

Step 2:- Block diagram is redrawn as shown below.



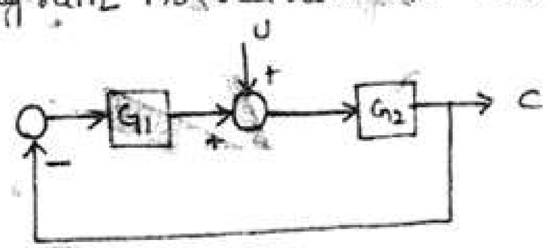
Step3:- The Output Equation $\frac{C}{R}$ is given by

$$\boxed{\frac{C}{R} = \frac{G_1 G_2}{1 + G_1 G_2}}$$

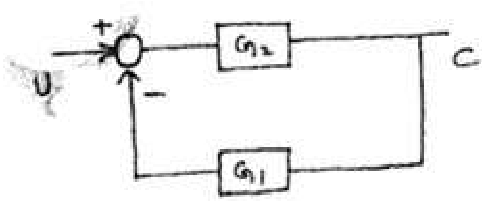
Step4:- By considering second input (R)

Put $R = 0$

Step5:- Block diagram is redrawn as shown below.



⇓ Similar.



The output transfer function is given by

$$\frac{C}{U} = \frac{G_2}{1 + G_2 G_1}$$

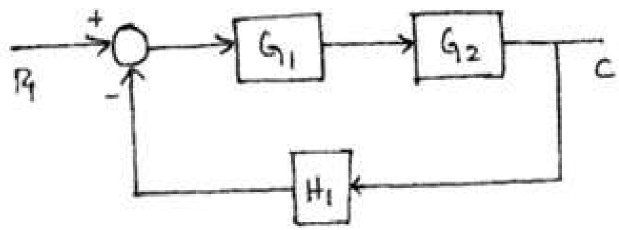
The total output is given by

$$C = \left[\frac{G_1 G_2}{1 + G_1 G_2}\right] R + \left[\frac{G_2}{1 + G_2 G_1}\right] U$$

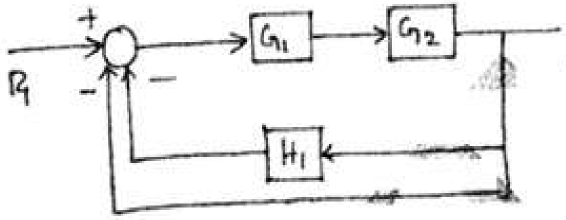
Problems to be solved in Class

⇒ For the block diagram shown in the figure find the Overall transfer function.

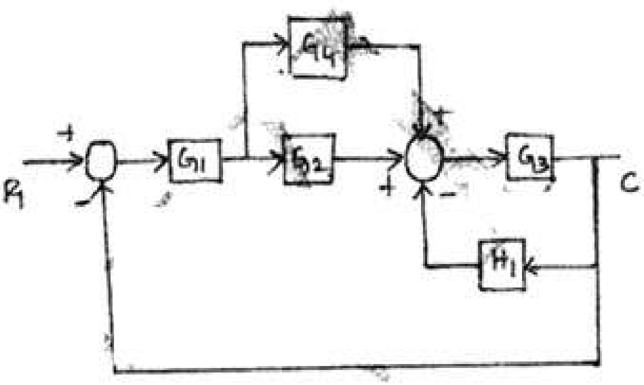
1)



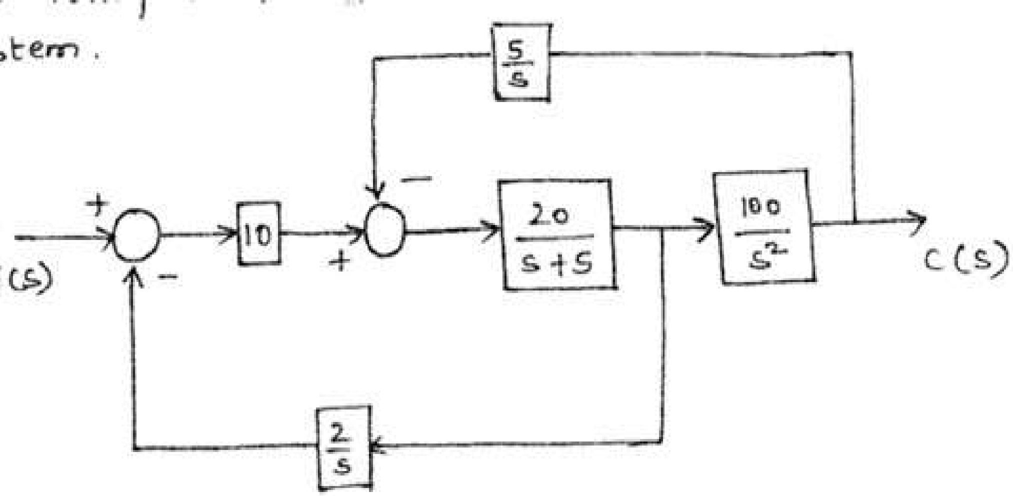
2)



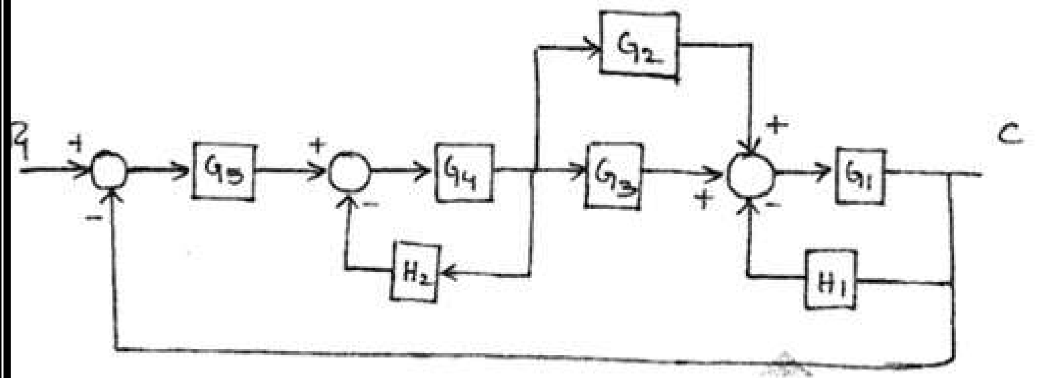
3)



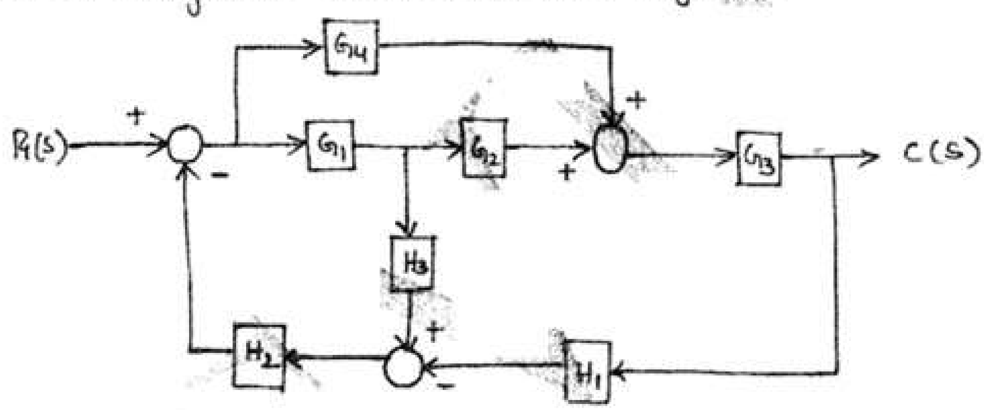
⇒ Reduce the block diagram shown in the figure to its simplest form and Determine the order of the system.



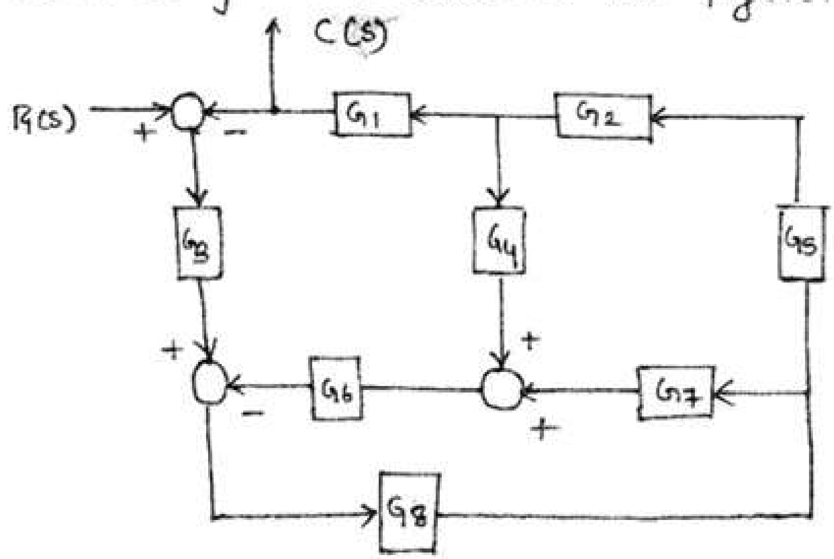
⇒ Find the overall transfer function for the block diagram shown below. Using block diagram reduction technique.



⇒ Determine the overall transfer function for the block diagram shown in the figure.



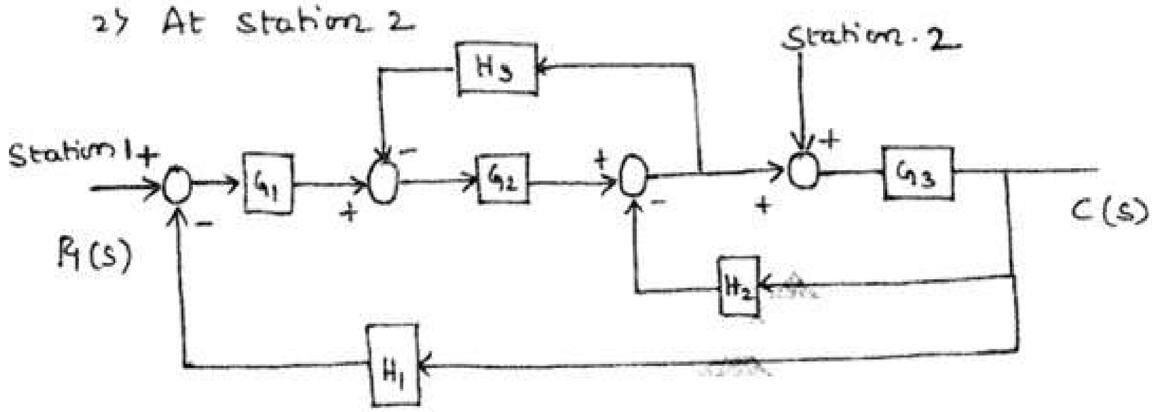
⇒ Find the overall transfer function $\frac{C(S)}{R(S)}$ for the block diagram shown in the figure.



=> For the system represented by the block diagram shown in the figure. Evaluate the closed loop transfer function when the input are is

i) At station 1

2) At station 2



=> The performance equations of a controlled system are given by the following set of linear algebraic equation draw the block diagram and determine the overall transfer function $\frac{C(s)}{R(s)}$ by reducing the block diagram in steps.

$$E_1(s) = R(s) - H_3(s)\, C(s)$$

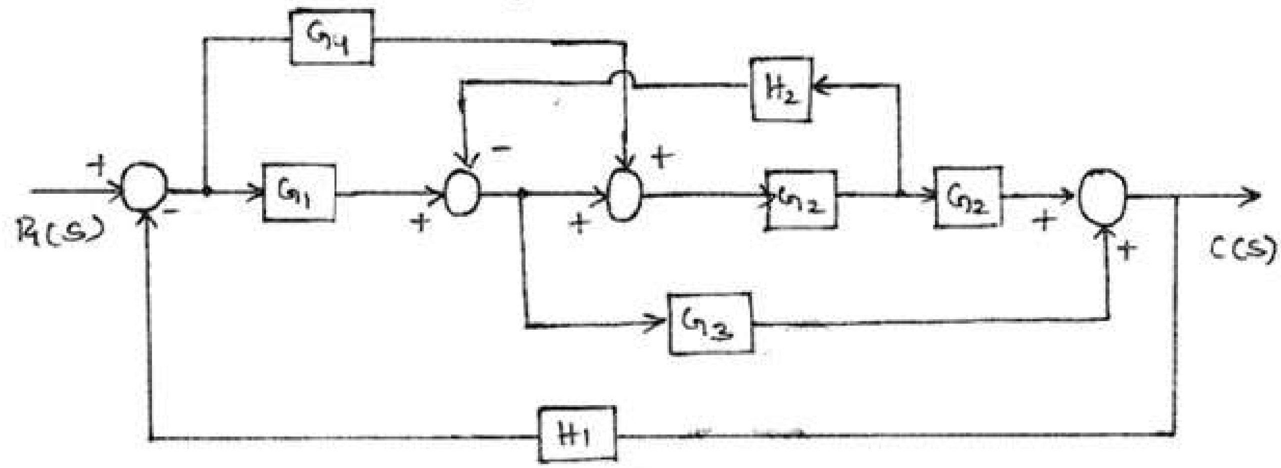
$$E_2(s) = E_1(s) - H_1(s)\, E_4(s)$$

$$E_3(s) = G_1(s)\, E_2(s) - H_2(s)\, C(s)$$

$$E_4(s) = E_3(s)\, G_2(s)$$

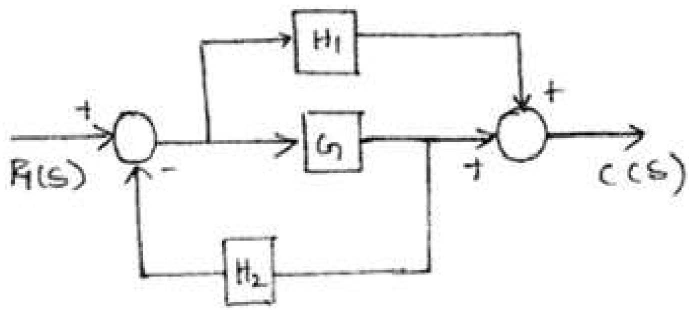
$$C(s) = G_3(s)\, E_4(s)$$

=> Determine the Overall transfer function for the system represented by the block diagram shown in the figure using block diagram reduction technique.



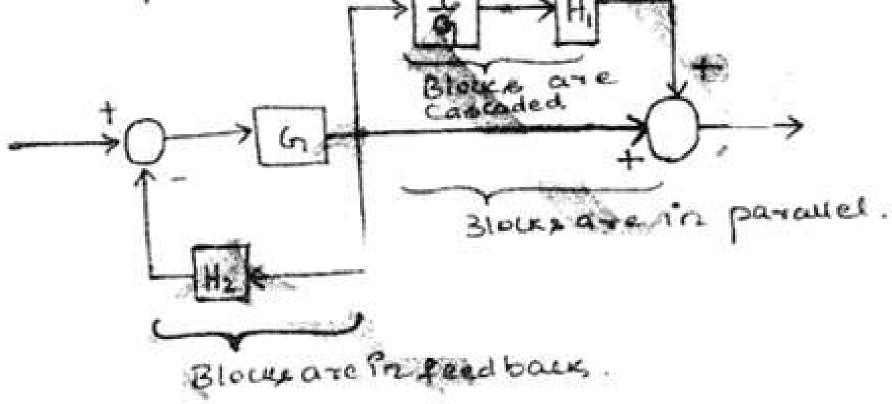
Problems on Block diagram Algebra:-

1) Simplify the block diagram shown in the figure and obtain the overall transfer function $\frac{C(s)}{R(s)}$

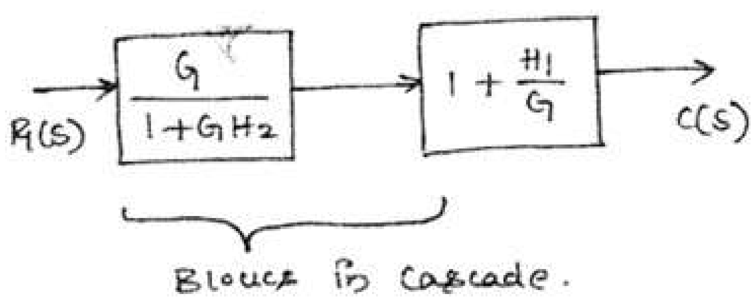


Solution:-

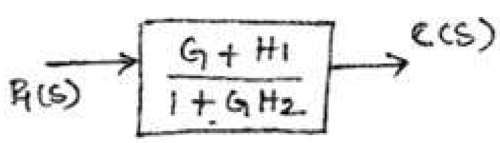
Step 1: By shifting the take off point after the block G.



Step 2:- Eliminating the feedback loop and combining the blocks in parallel and cascaded.



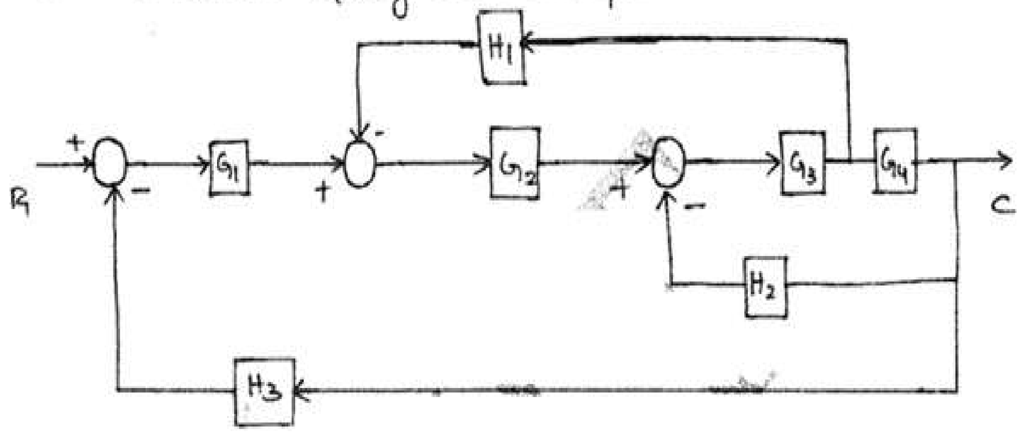
Step 3:- Combining blocks in cascade.



Hence the overall transfer function is given by

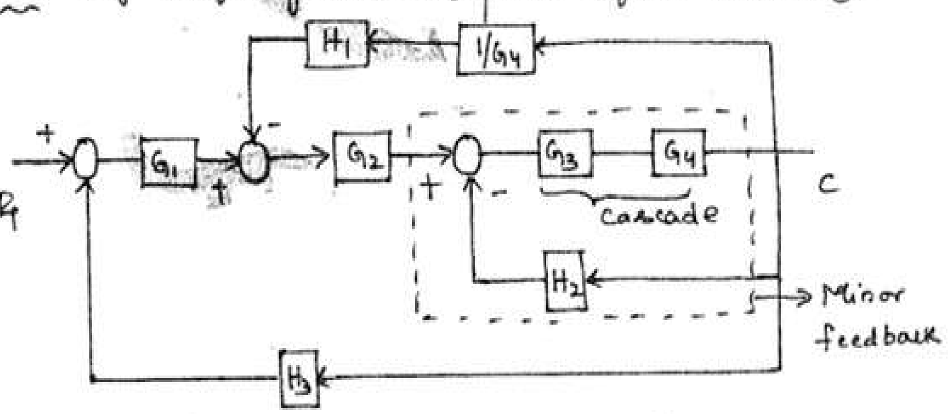
$$\boxed{\frac{C(S)}{R(S)} = \frac{G_1 + H_1}{1 + G_1 H_2}}$$

2) Find the ratio $\frac{C(S)}{R(S)}$ for the block diagram shown below using block diagram reduction technique.

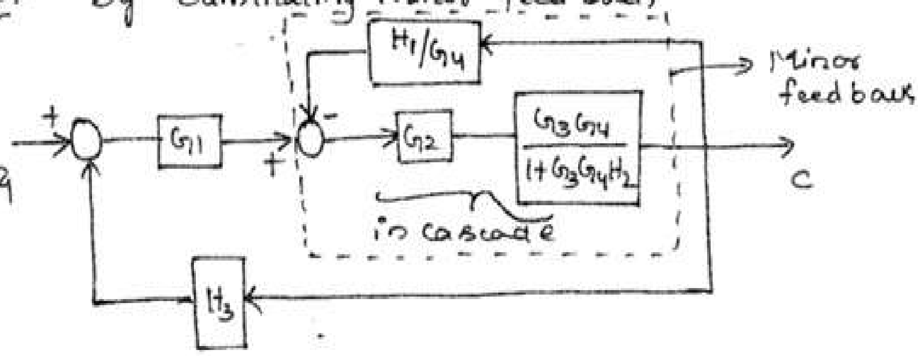


Solution:-

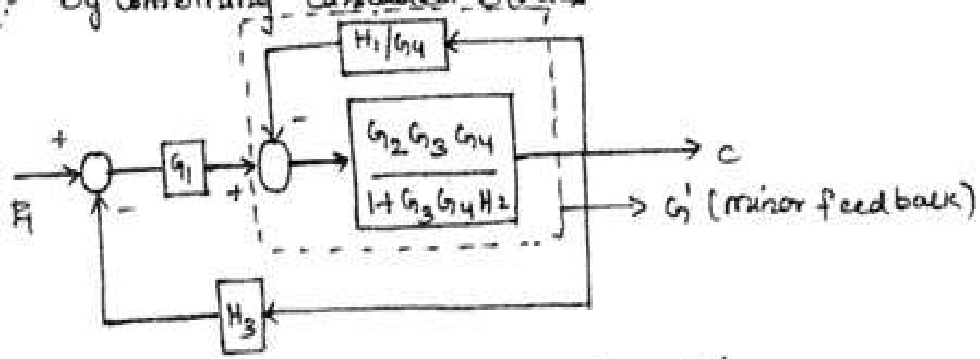
Step1: By shifting take off point after block G_3.



Step 2: By Eliminating minor feed back



Step 3:- By Combining Cascaded Blocks

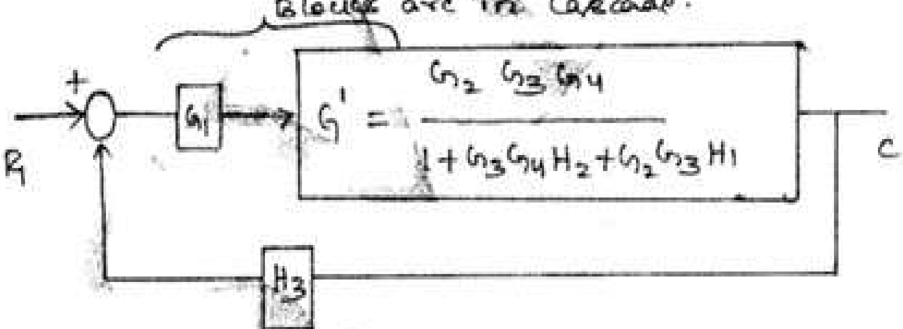


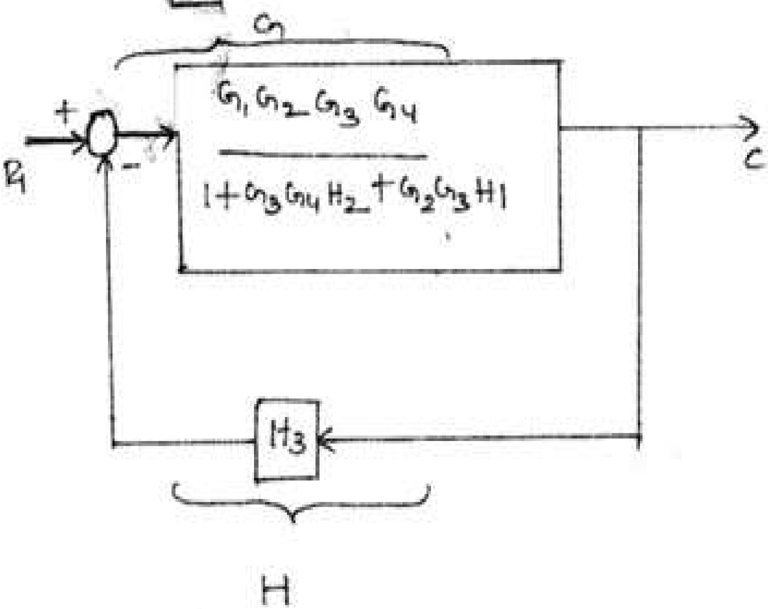
Step 4:- By Eliminating minor feed back G'

$$G' = \frac{\dfrac{G_2 G_3 G_4}{1+G_3 G_4 H_2}}{1 + \dfrac{G_2 G_3 G_4}{1+G_3 G_4 H_2} \cdot \dfrac{H_1}{G_4}}$$

$$\boxed{G' = \frac{G_2 G_3 G_4}{1+G_3 G_4 H_2 + G_2 G_3 H_1}}$$

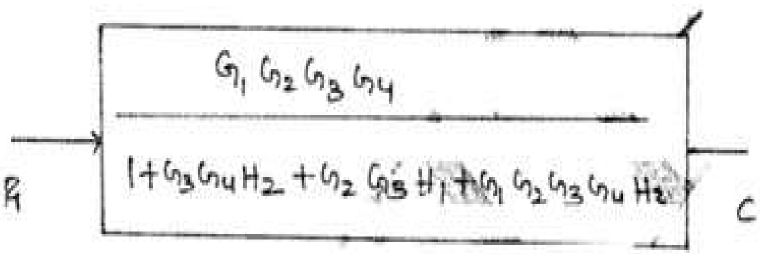
Blocks are in cascade.



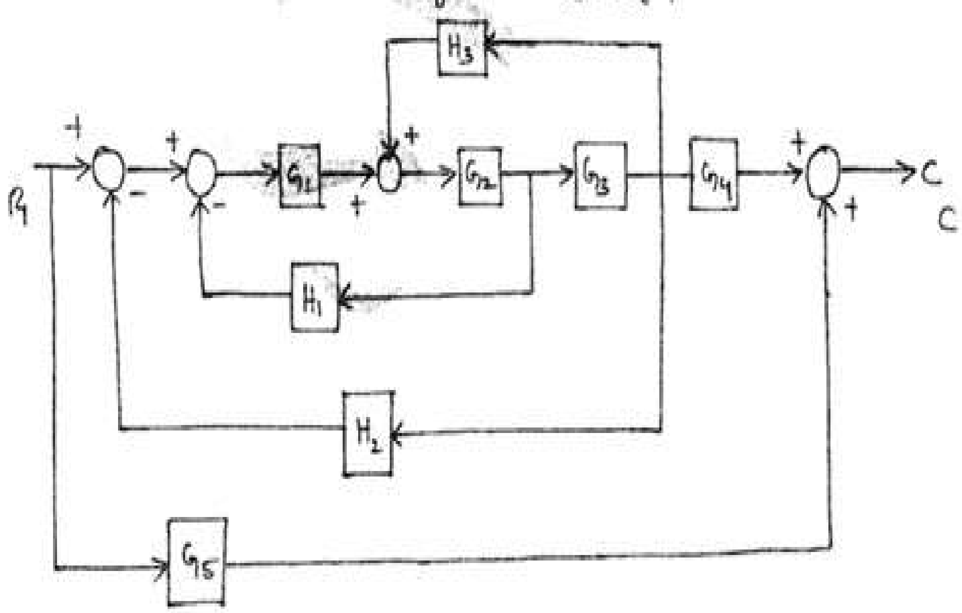


$$\frac{C}{R} = \frac{G}{1+GH} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_2+G_2G_3H_1}}{1+\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_2+G_3G_2H_1}\cdot H_3}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_2+G_2G_3H_1+G_1G_2G_3G_4H_3}$$

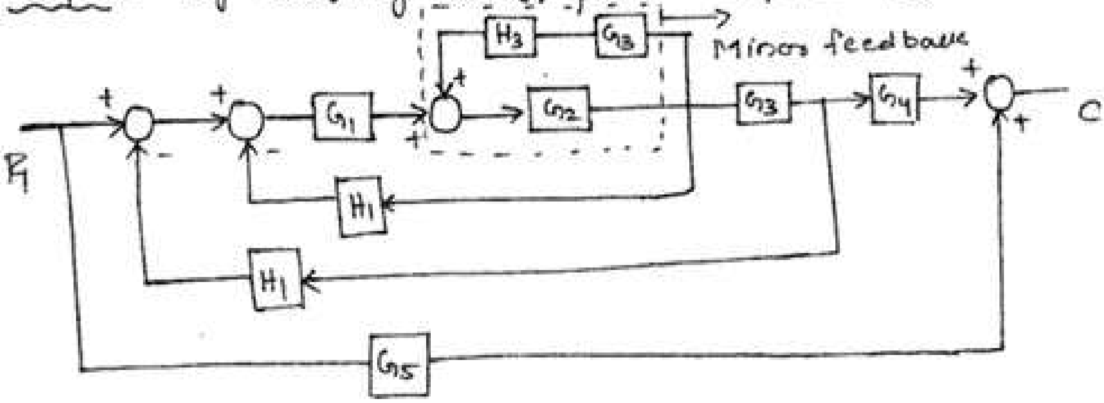


3) For the block diagram shown in the figure determine the overall transfer function.

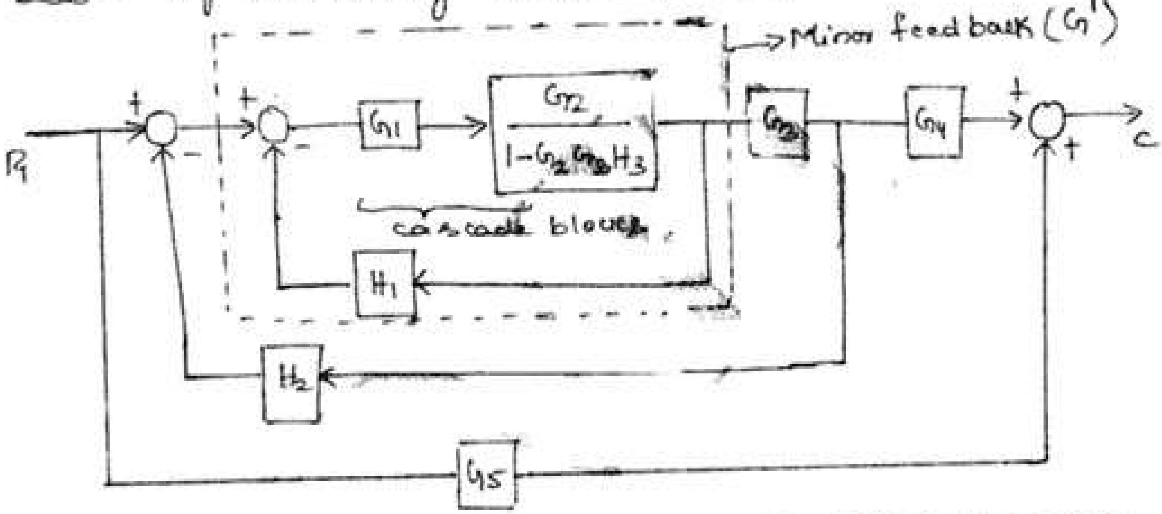


Solution:-

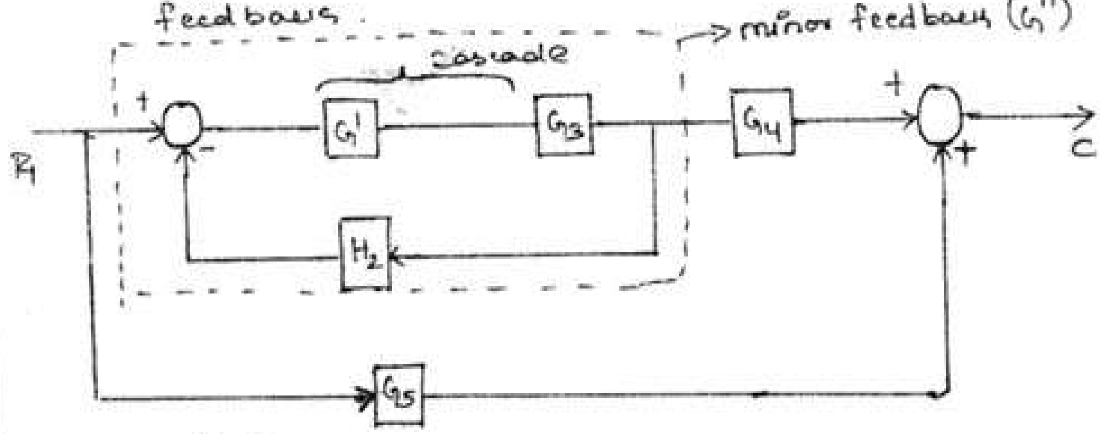
Step 1:- By shifting takeoff point before G_3



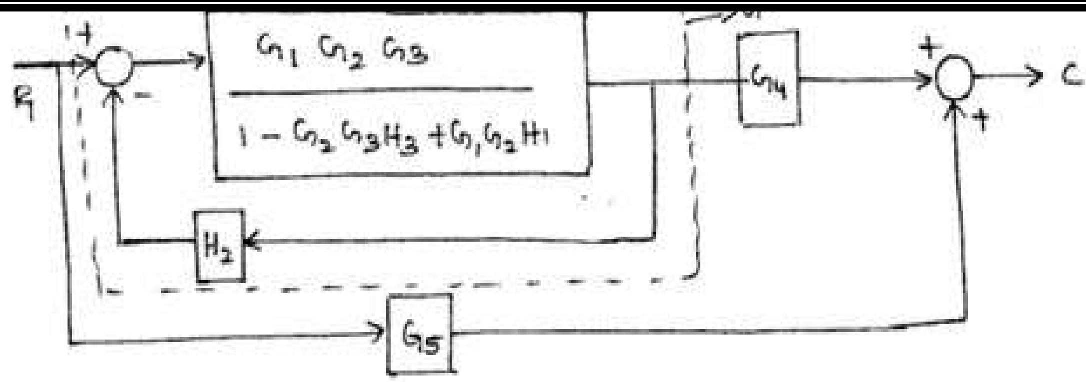
Step 2:- By Eliminating minor feedback.



Step 3:- By combining cascaded blocks and Eliminating minor feedback.



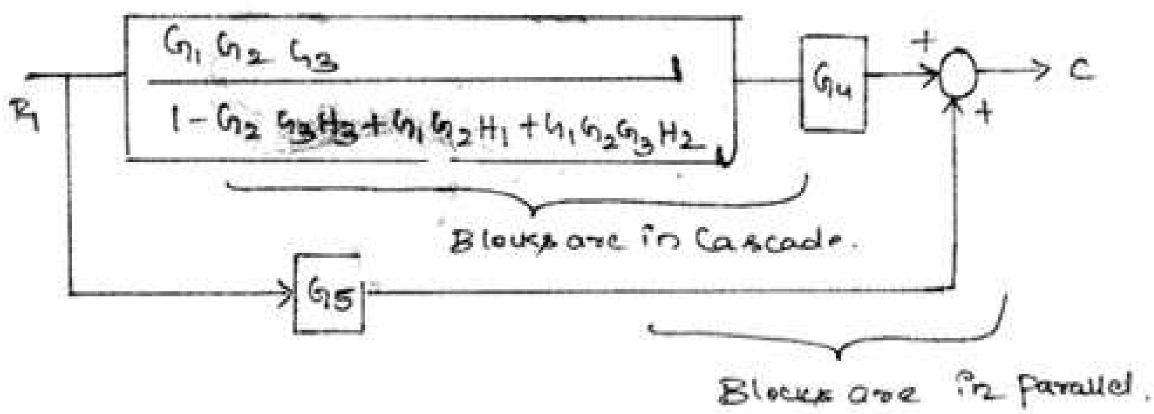
$$G' = \frac{\dfrac{G_1 G_2}{1 - G_2 G_3 H_3}}{1 + \dfrac{G_1 G_2}{1 - G_2 G_3 H_3} \cdot H_1} = \frac{G_1 G_2}{1 - G_2 G_3 H_3 + G_1 G_2 H_1}$$



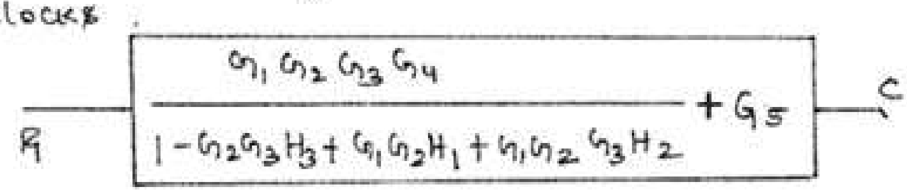
step4: By Eliminating minor feedbacks G''

$$G'' = \frac{\dfrac{G_1 G_2 G_3}{1 - G_2 G_3 H_3 + G_1 G_2 H_1}}{1 - \dfrac{G_1 G_2 G_3}{1 - G_2 G_3 H_3 + G_1 G_2 H_1} \cdot H_2}$$

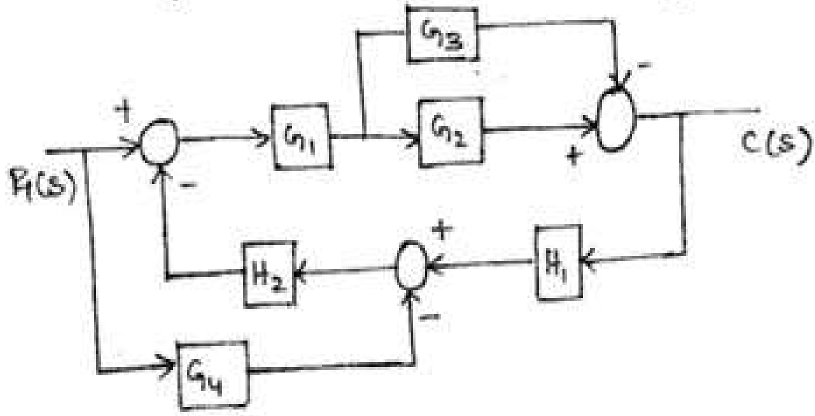
$$\boxed{G'' = \frac{G_1 G_2 G_3}{1 - G_2 G_3 H_3 + G_1 G_2 H_1 + G_1 G_2 G_3 H_2}}$$



Step5: By Combining cascade blocks and parallel.
Blocks.



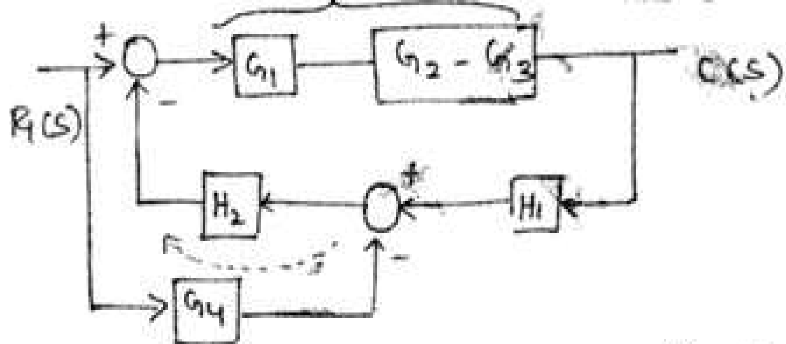
4) Determine the Overall transfer function for the block diagram shown in the figure.



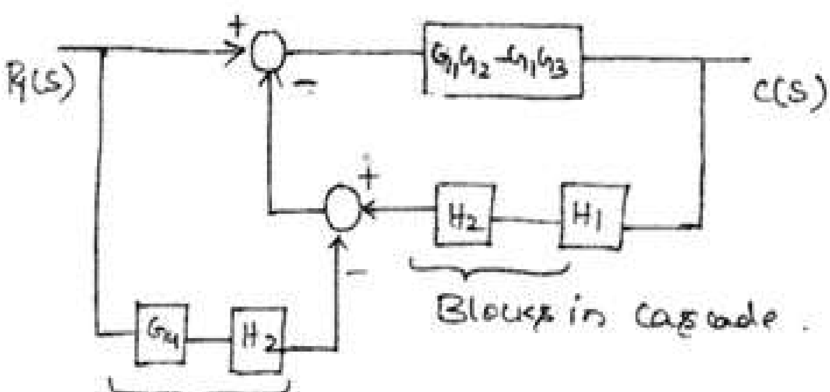
Solution:

Step1: By combining parallel blocks $(G_2 + G_3)$

Blocks are in cascade.

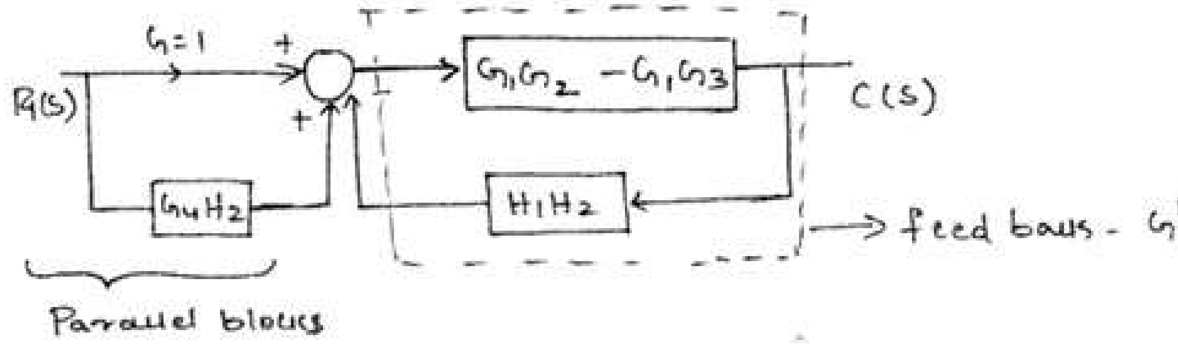


Step2: By combining Cascaded blocks $(G_1, (G_2 - G_3))$ and By shifting summing point after Block (H_2)



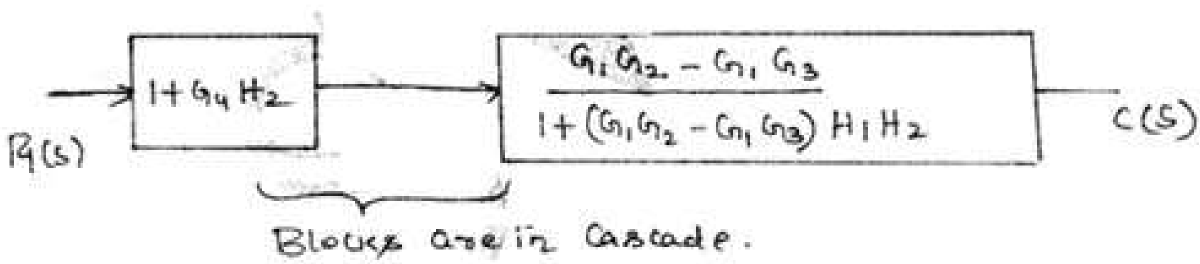
Blocks in cascade.

Step 3:- By Combining cascaded blocks and By Combining summing blocks (adding)



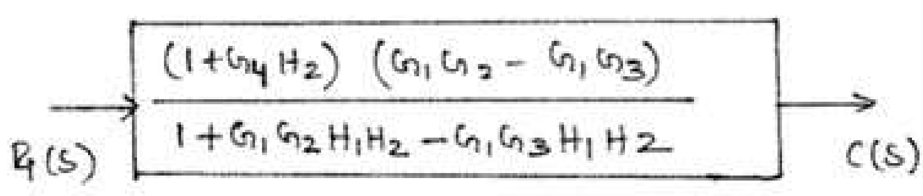
Step 4:- By Combining parallel blocks and by Eliminating feed back G'

$$G' = \frac{G}{1+GH} = \frac{G_1G_2 - G_1G_3}{1+(G_1G_2 - G_1G_3)H_1H_2}$$

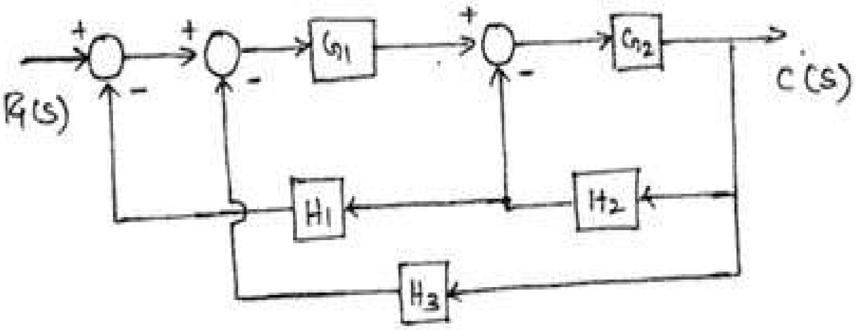


Step 5:- By Combining cascaded block.

$$\frac{C(S)}{R(S)} = \frac{(1+G_4H_2)(G_1G_2 - G_1G_3)}{1+(G_1G_2 - G_1G_3)H_1H_2}$$

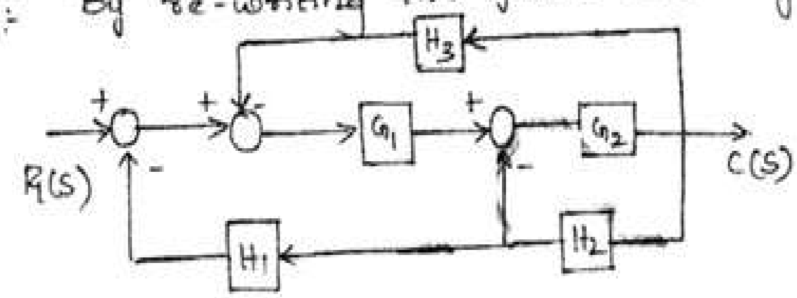


5) Determine the transfer function for the block diagram shown in the figure.

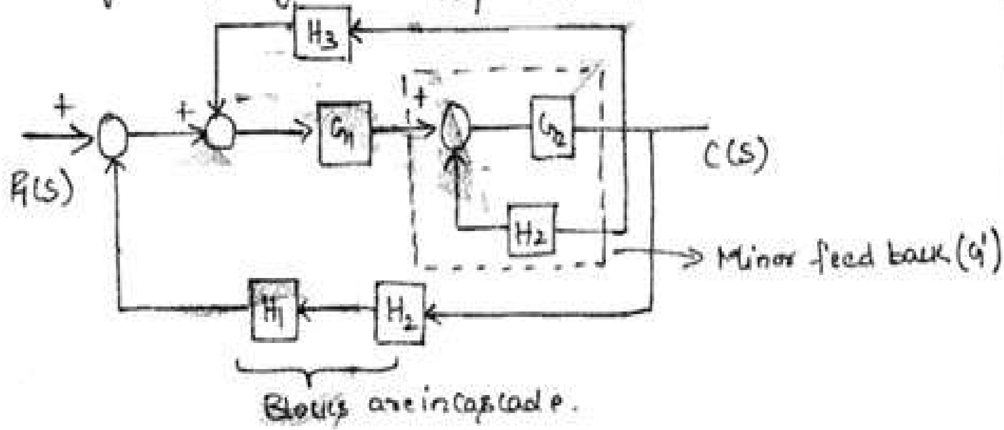


Solution:-

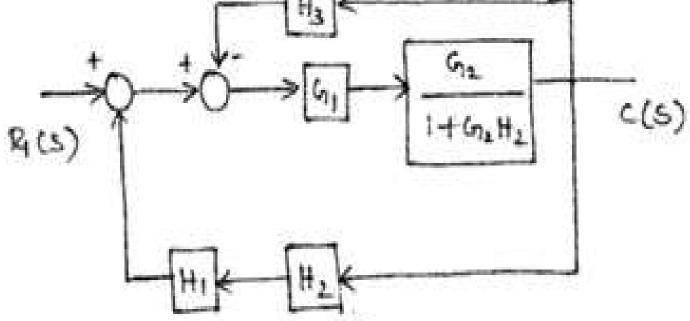
Step1:- By re-writing the given block diagram.



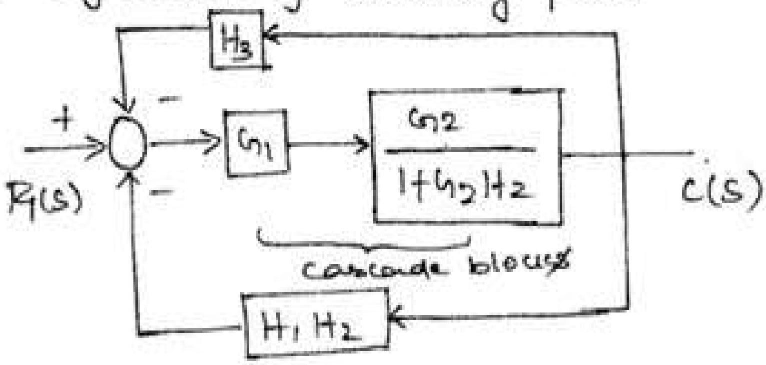
Step2:- By shifting take-off point befor H_2



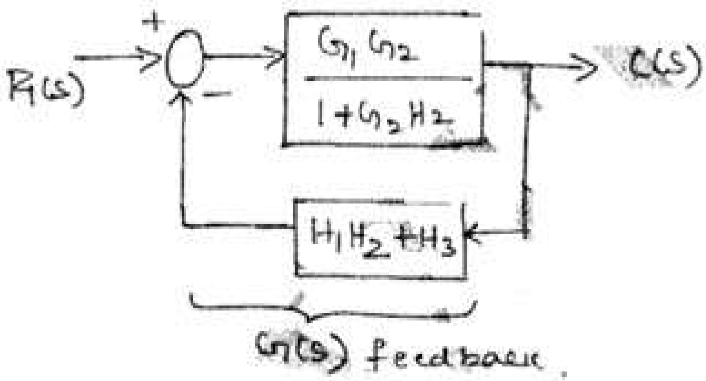
$$G' = \frac{G}{1+GH} = \frac{G_2}{1+G_2H_2}$$



Step 3:- By Combining Summing point



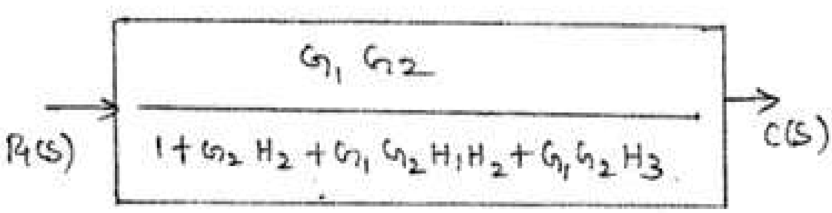
Step 4:- By Combining Cascaded blocks and parallel blocks (H_3, H_1H_2)



Step 5:- By eliminating the feed back.

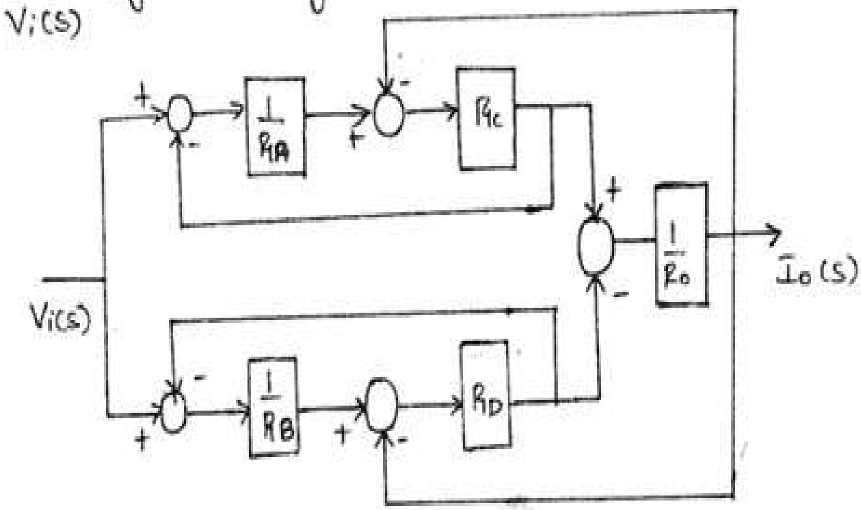
$$G(S) = \frac{C(S)}{R(S)} = \frac{G}{1+GH} = \frac{\frac{G_1G_2}{1+G_2H_2}}{1+\frac{G_1G_2}{1+G_2H_2}\cdot(H_1H_2+H_3)}$$

$$\frac{C(S)}{R(S)} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_1H_2+G_1G_2H_3}$$



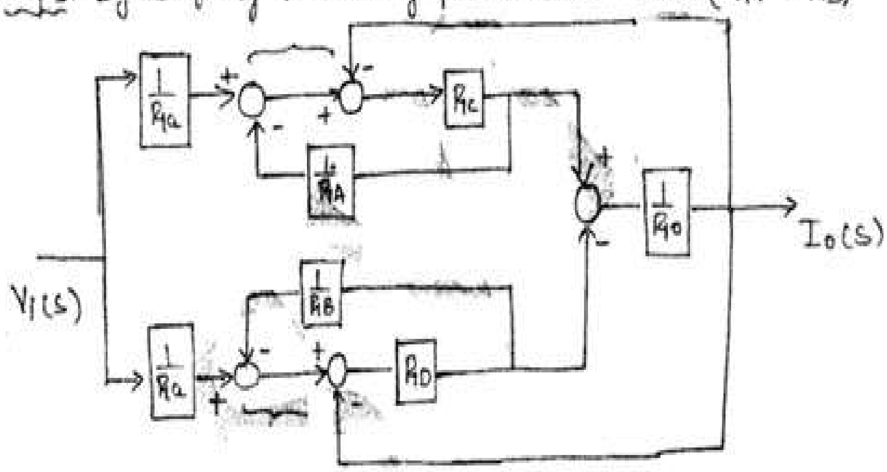
6) For the system shown in the figure determine

$\frac{I_o(S)}{V_i(S)}$ by block diagram reduction technique.

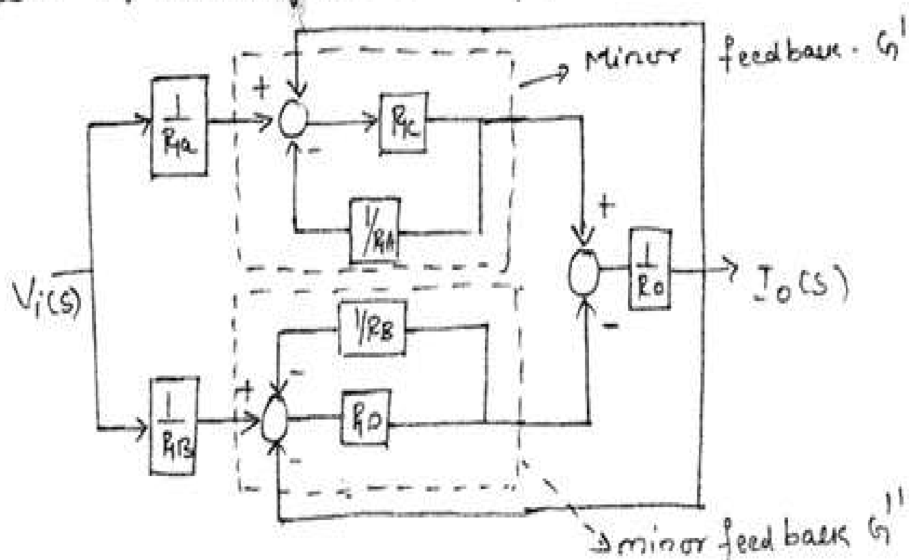


Solution :-

Step 1. By shifting summing points after block $\left(\frac{1}{R_A} + \frac{1}{R_B}\right)$



Step 2: By combining the summing point



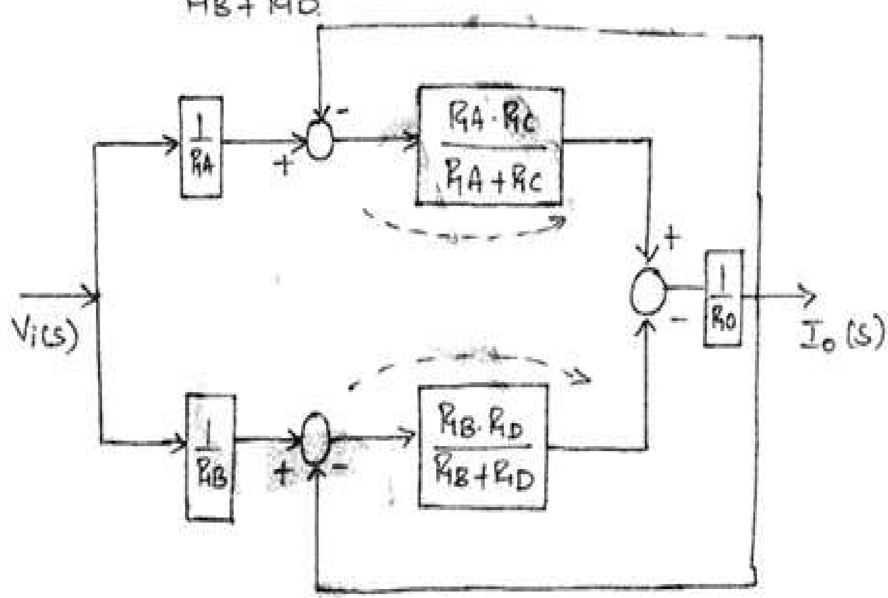
Step 3: By Eliminating the feedback G' and G''

$$G' = \frac{G}{1+GH} = \frac{R_C}{1+R_C\cdot\frac{1}{R_A}} = \frac{R_C}{\frac{R_A+R_C}{R_A}} = \frac{R_A R_C}{R_A+R_C}$$

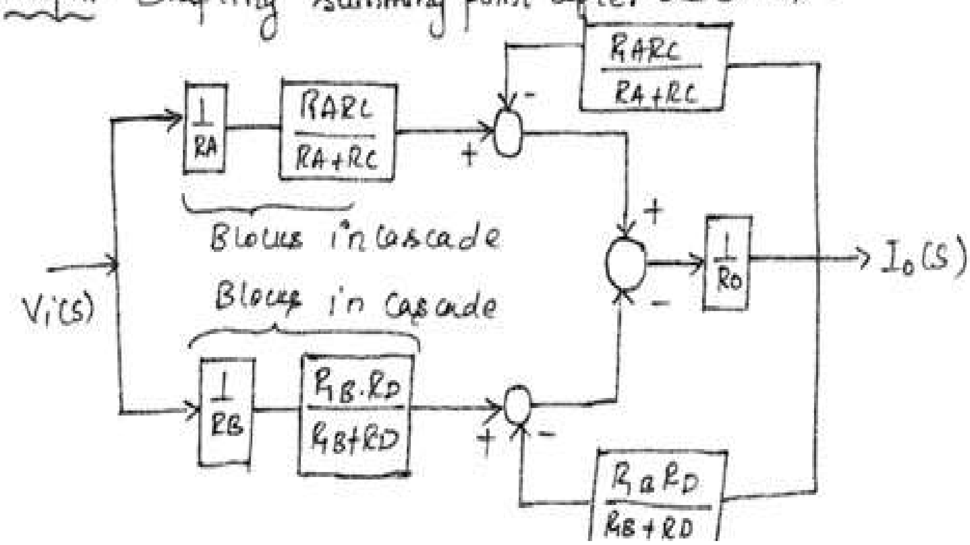
$$G' = \frac{R_A R_C}{R_A+R_C}$$

$$G'' = \frac{G}{1+GH} = \frac{R_D}{1+R_D\cdot\frac{1}{R_B}} = \frac{R_D}{\frac{R_B+R_D}{R_B}} = \frac{R_B\cdot R_D}{R_B+R_D}$$

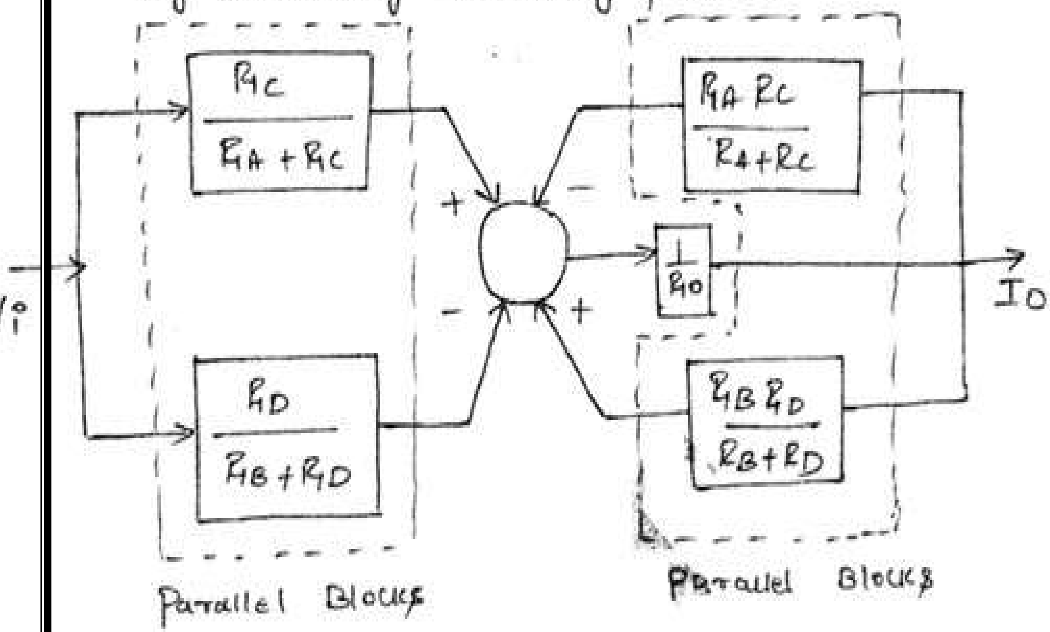
$$G'' = \frac{R_B\cdot R_D}{R_B+R_D}$$



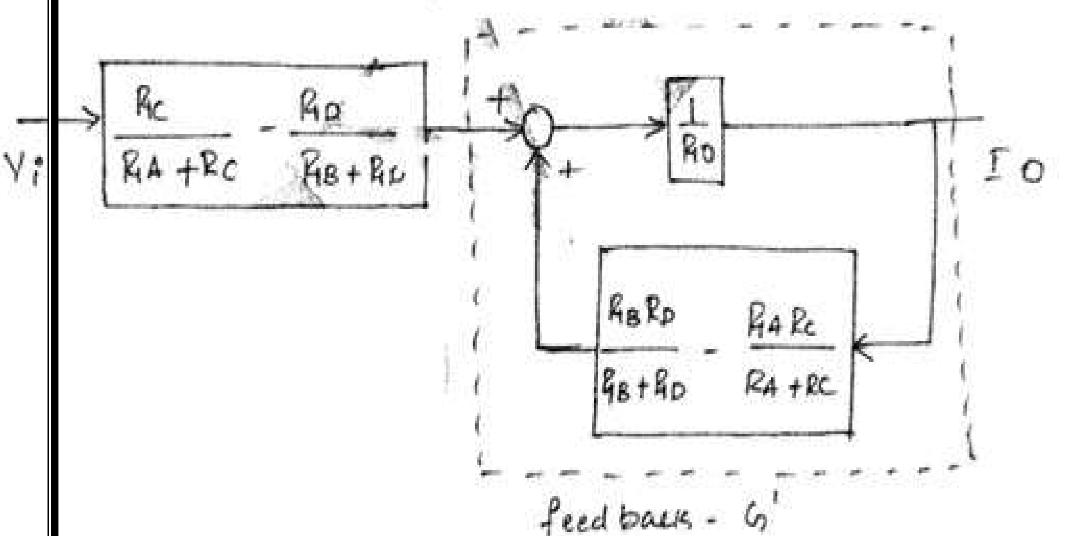
Step 4: Shifting summing point after the blocks.



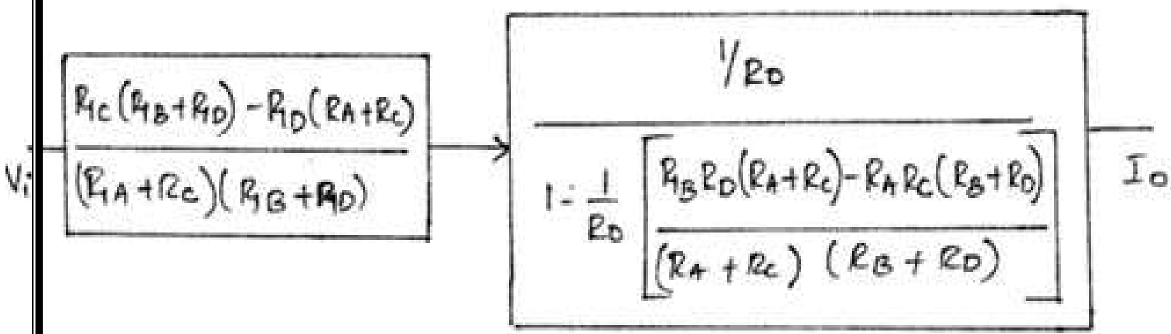
Step 5:- By Combining Cascaded blocks and By Combining Summing points.



Step 6:- By Combining parallel Blocks



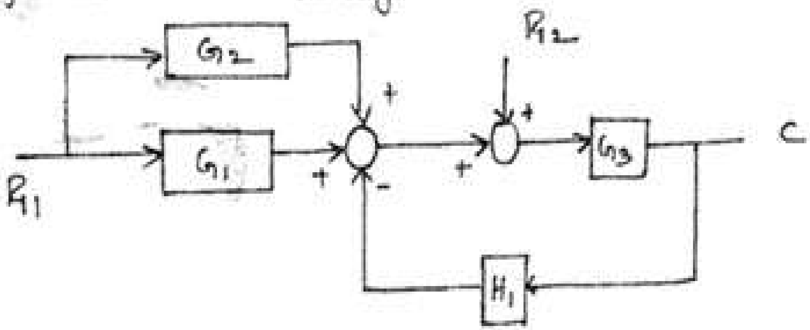
$$G' = \frac{G}{1+GH} = \frac{G}{1-GH} = \frac{1/R_O}{1 - \frac{1}{R_O}\left[\frac{R_B R_D}{R_B+R_D} - \frac{R_A R_C}{R_A+R_C}\right]}$$



$$\frac{I_o(s)}{V_i(s)} = \frac{\left[R_C(R_B+R_D)-R_D(R_A+R_C)\right]}{(R_A+R_C)(R_B+R_D)} * \frac{(R_A+R_C)\,(R_B+R_D)}{R_D\,(R_A+R_C)(R_B+R_D) - R_BR_D(R_A+R_C) + R_AR_C(R_B+R_D)}$$

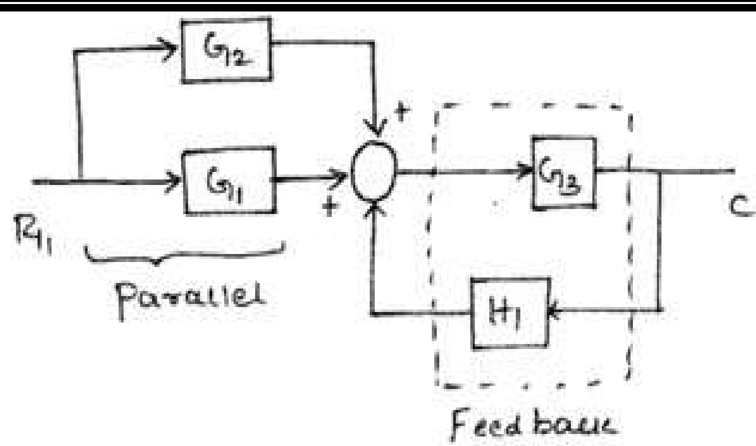
$$\boxed{\frac{I_o(s)}{V_i(s)} = \frac{R_C\,(R_B+R_D) - R_D\,(R_A+R_C)}{R_D(R_A+R_C)(R_B+R_D) - R_BR_D(R_A+R_C) + R_AR_C(R_B+R_D)}}$$

7) Determine the transfer function $\frac{C}{R_1}$ and $\frac{C}{R_2}$ for the Block diagram shown in figure.

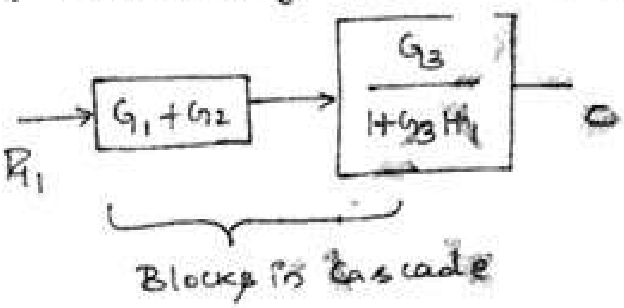


Solution:

Case 1: By applying Superposition principle i.e Considering input at a time, Consider R_1 with R_2 to be 0. Block diagram as shown below.



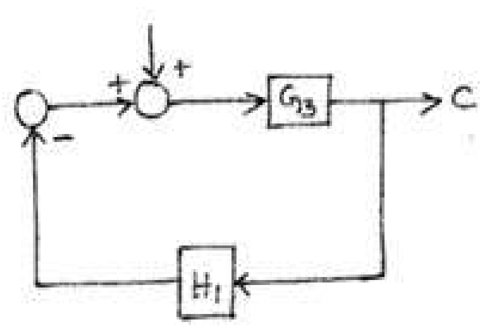
Step 1:- By combining parallel blocks and By eliminating feedback blocks.



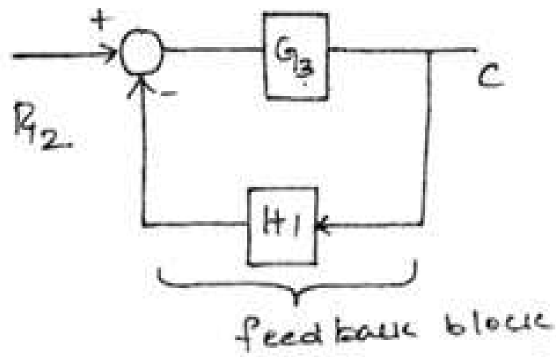
Step 2:- By eliminating cascaded blocks the Transfer function $\frac{C}{R_1}$ is given by

$$\boxed{\frac{C}{R_1} = \frac{(G_1+G_2)G_3}{1+G_3H_1}}$$

Case ii:- Considering the input R_2 with $R_1=0$ the block diagram is as shown below



Step 1:- By Combining summing points.

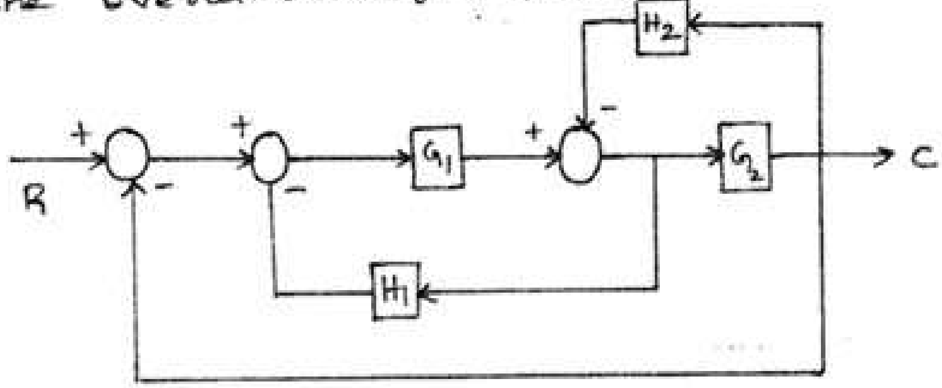


The Overall transfer function $\frac{C}{R_2}$ is given by

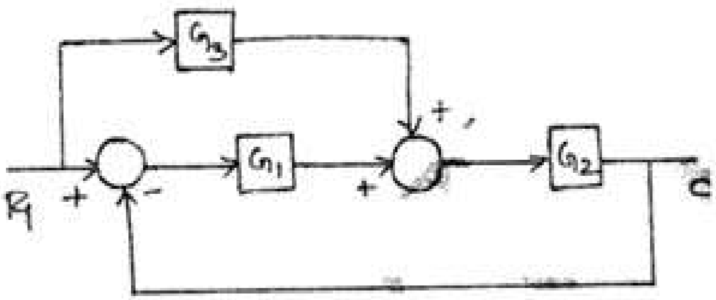
$$\boxed{\frac{C}{R_2} = \frac{G_3}{1+G_3H_1}}$$

Practice Problems:-

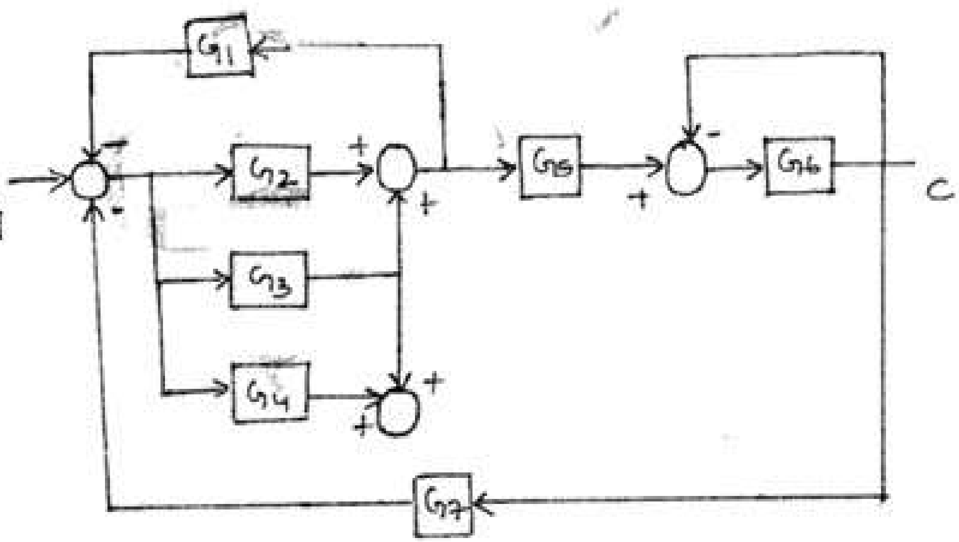
1) using block diagram reduction techniques, obtain the overall transfer function



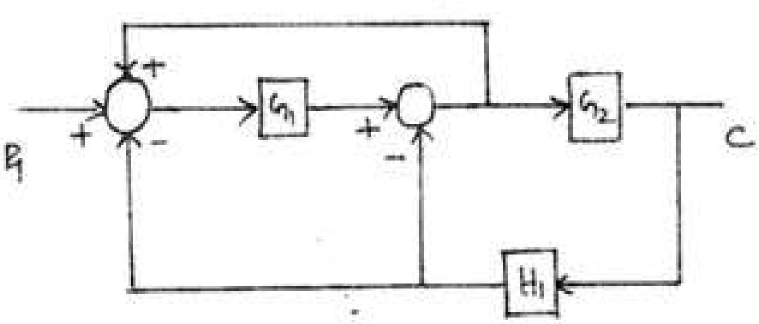
2)



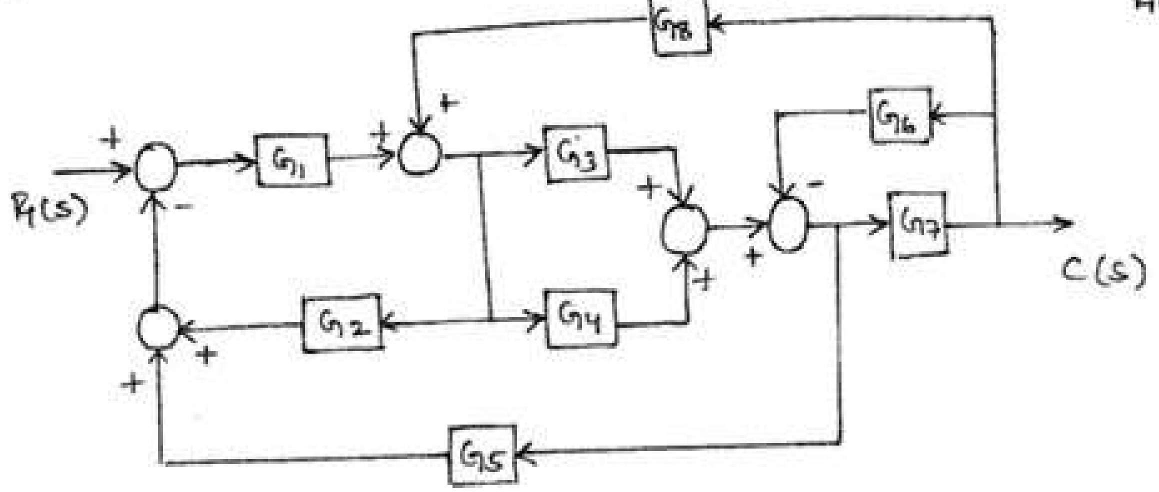
3)



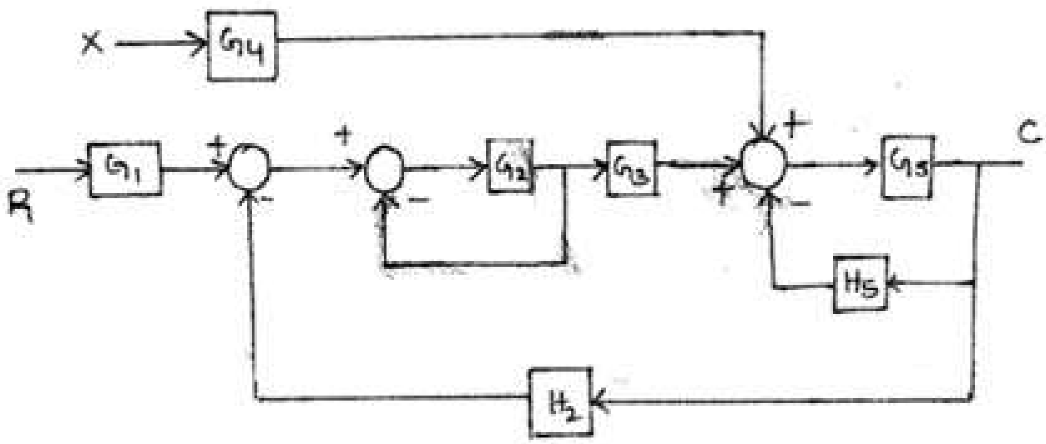
4)



5) Determine the Overall transfer function $\frac{C(S)}{R(S)}$

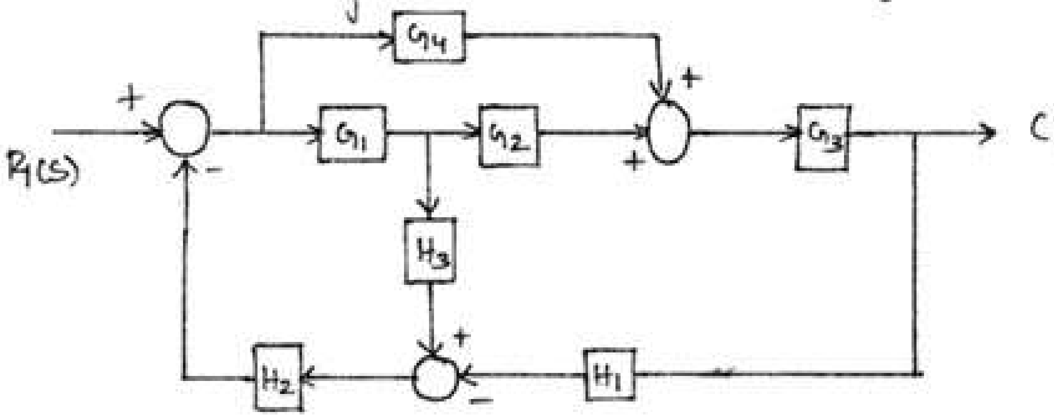


6) using the block diagram reduction technique, find the transfer function for Each input

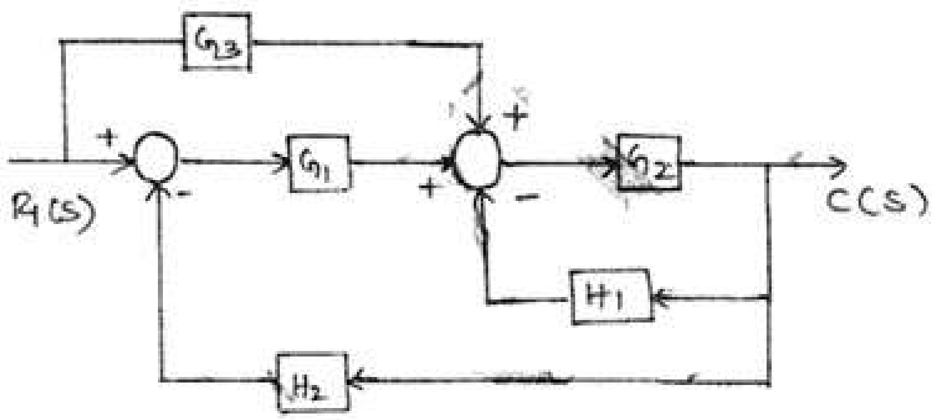


Assignment Problems On Block Diagram Algebra

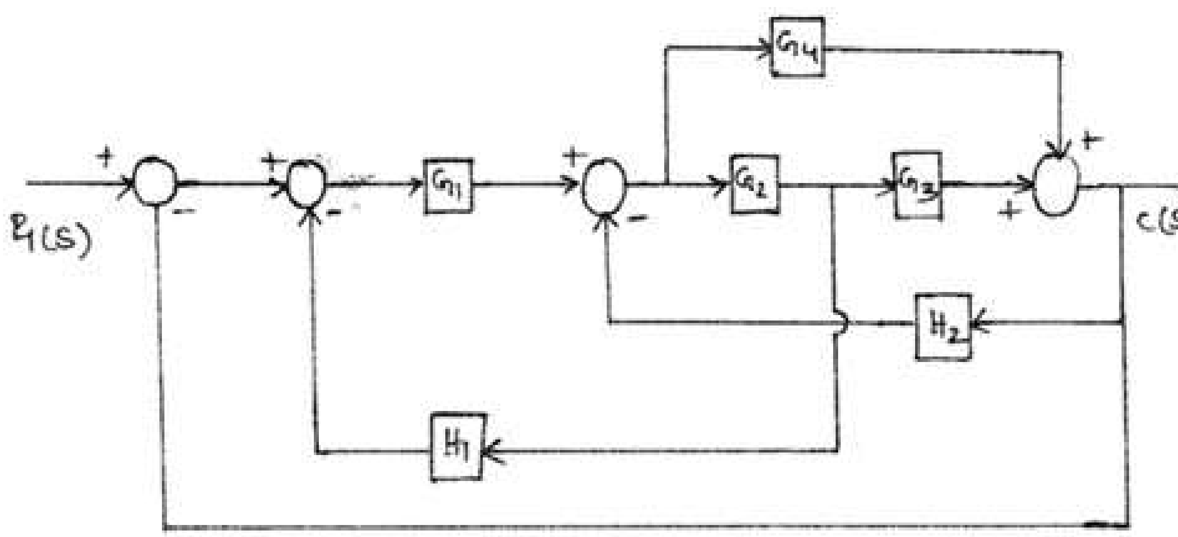
1) Determine the Overall transfer function for the block diagram shown in the figure.



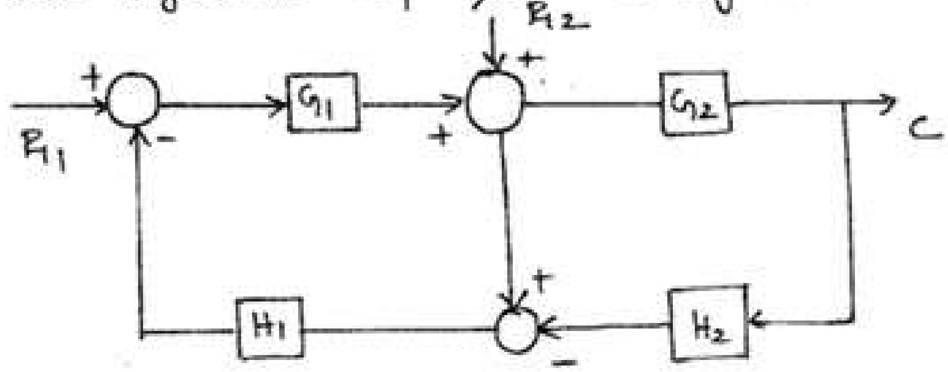
2)



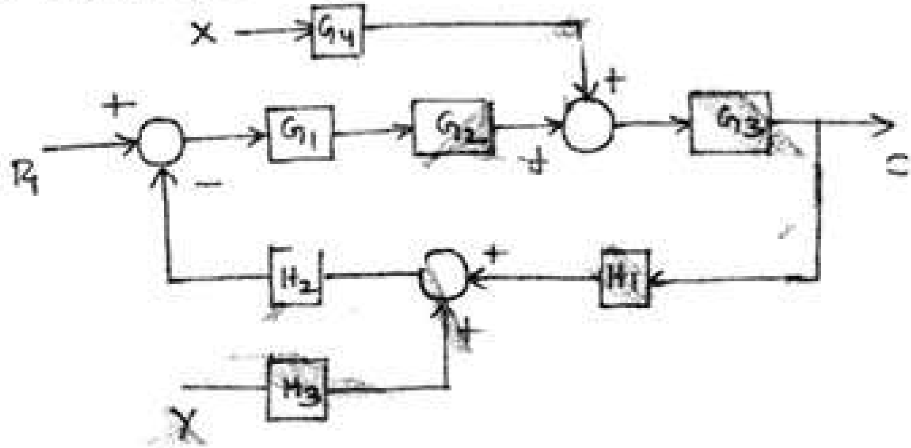
3)



4) Derive an Expression for the total output for the system represented by the block diagram.



5) Using Block diagram reduction Technique find the over transfer function.



Signal Flow Graph

* It is defined as the graphical representation of a set of simultaneous Equations or transfer function.

* The Overall transfer function for a signal flow graph Can be determined by using Mason's gain formula.

It is given by.

$$\boxed{M(s) = \sum_{K=1}^{N} \frac{P_K \Delta_K}{\Delta}}$$

Where. $\Delta = 1 - \Sigma P_{m1} + \Sigma P_{m2} - \Sigma P_{m3} + \cdots\cdots\cdots$

P_K is the forward path gain of the K^{th} forward path.

Δ_K is the Co-factor.

* Forward path: It is the Journey from input to Output node in the direction of arrows without touching in between more than once.

$N \rightarrow$ no of forward path present in signal flow graph.

* Loop: A path is said to be a loop if we start from a node and come back to the same node in the direction of arrows without touching any nodes in between more than once.

* Two-non-touching Loops: Two loops are said to be non-touching, if they don't have a Common node or Variable between them.

Δ is the determinant of the signal flow graph.

ΣP_{m1} is the sum of loop gains of all possible Combinations of single loops

$\Sigma P m_2$ is sum of product of loop gain of all possible combinations of 2 non-touching loops.

$\Sigma P m_3$ is the sum of product of the loop gain of all combinations of 3 non-touching loops.

Δ_K is the co-factor of the graph. The expression for Δ_K is similar to Δ, but it must be applied to that part of the graph not touching the K^{th} forward path.

If a forward path touches all single loop present in the graph then the corresponding $\Delta K = 1$

⇒ Procedure to Construct Signal flow graph from linear Equations:

* let us consider a system described by a set of linear equations.

$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4$$
$$x_3 = a_{23}x_2$$
$$x_4 = a_{24}x_2 + a_{34}x_3 + a_{44}x_4$$
$$x_5 = a_{25}x_2 + a_{45}x_4$$

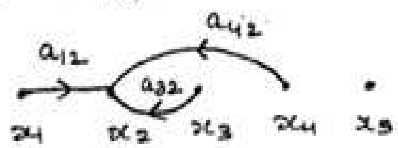
* Where x_1 is the input variable and x_5 is the output variable.

* When constructing signal flowgraph, the nodes are used to represent variables.

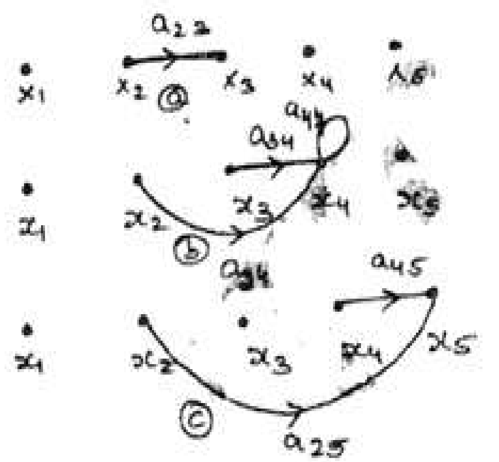
* Therefore locate the nodes x_1, x_2, x_3, x_4, x_5 as shown in the figure below.

x_1 x_2 x_3 x_4 x_5

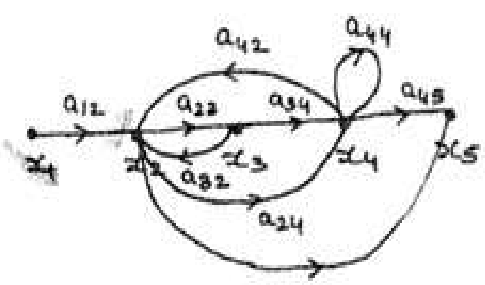
* The first Equation states that x_2 is equal to the sum of three incoming signals and it's signal flow graph is shown in the figure.



* Similarly the signal flow graph for the remaining three Equations are shown in the figure.



* The Complete signal flowgraph is a Combination of all the four parts as shown in the figure.



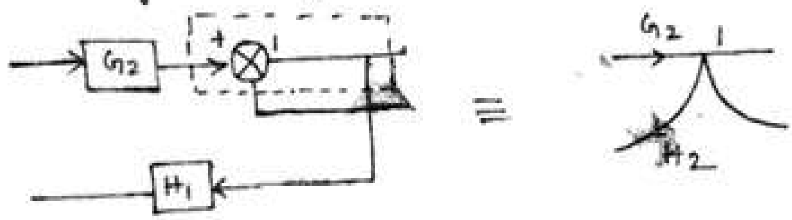
⇒ **Procedure to draw signal flow graph from Block diagram.**

Step1:- Replace the input signal and output signal by nodes.

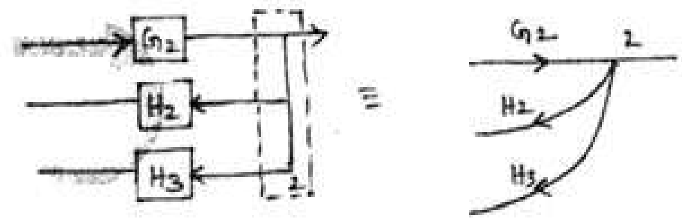
Step2:- Replace all the summing points by nodes.

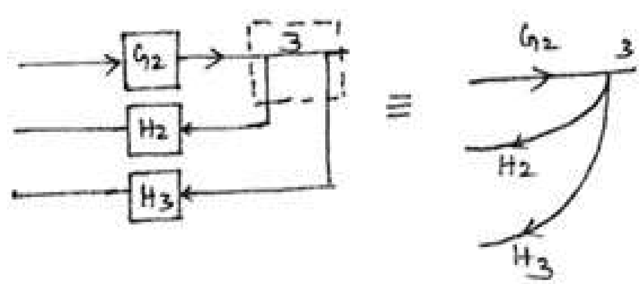
Step3:- Replace all the take off points by nodes.

Step4:- If the branch connecting a summing point and take off point has unity gain, then the summing point and take off point can be combined and represented by a single node.

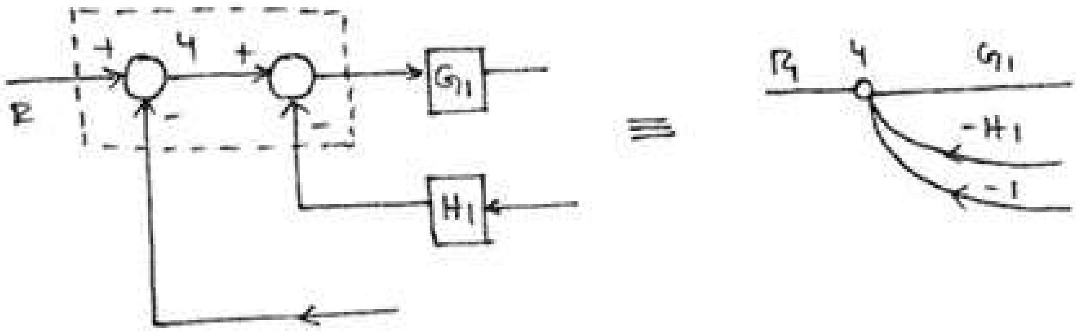


Step5:- If there are more take off points from the same signal then all the take off points can be combined and represented by a single node.





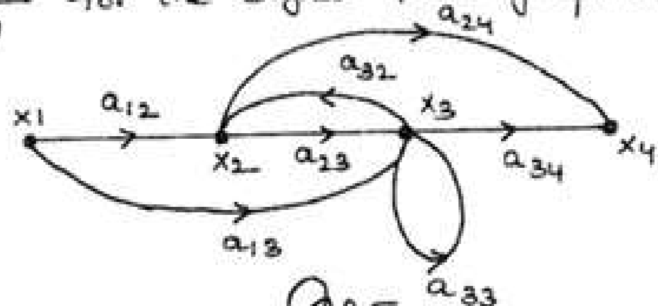
Step 6:- If the gain of the link connecting two summing points is one, then the two summing points can be combined and can be replaced by a single node as shown in the figure.



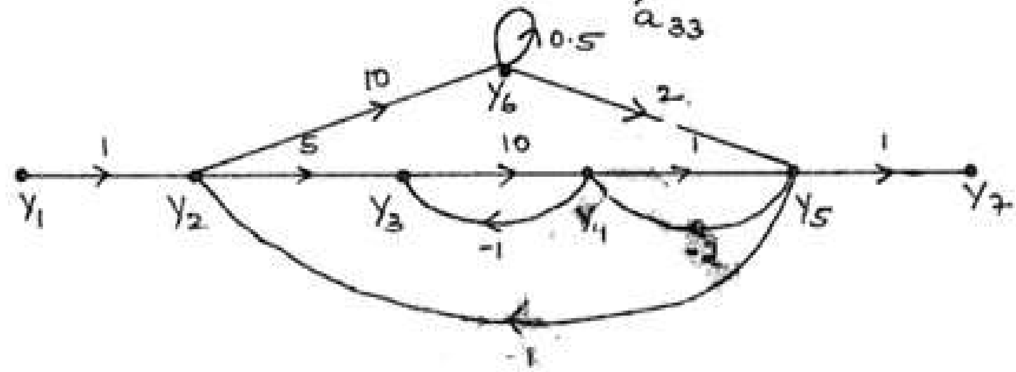
Step 7:- In summing point, subtract a signal instead of adding and then multiply the transmittance by -1 while representing in signal flow graph as shown in the figure above.

Problems to be solved in the class.

1) Using mason's gain formula find the Overall transfer function. $\frac{X_4}{X_1}$ for the Signal flow graph shown below.

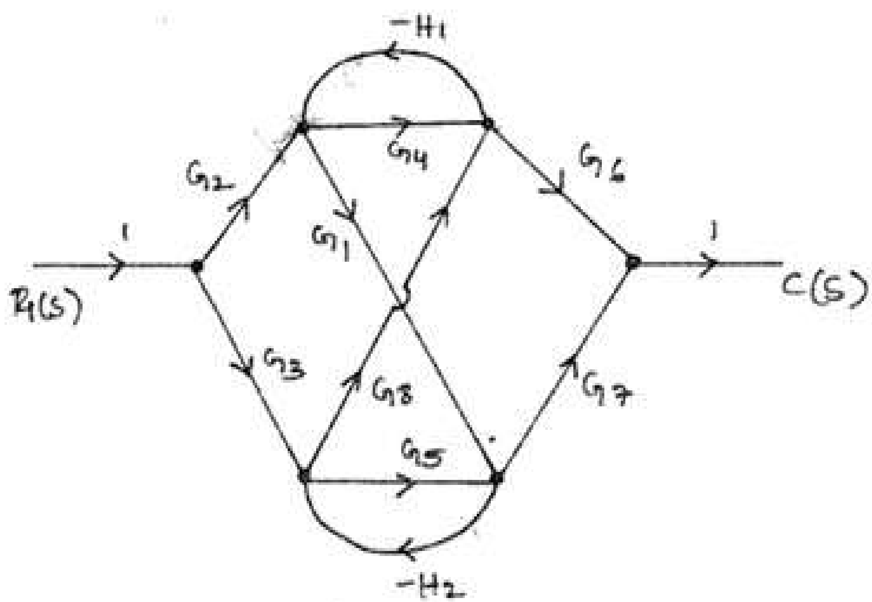


2)



using mason's gain formula determine $\frac{Y_7}{Y_1}$ and $\frac{Y_3}{Y_1}$ for the signal flow graph shown in the figure.

3) Determine the Overall transfer function of system whose signal flow graph as shown below.



4) Draw the signal flow graph for the system of Equations given below and Obtain the transfer function using mason's gains formula.

$$X_2 = G_1X_1 - H_1X_2 - H_2X_3 - H_6X_6$$

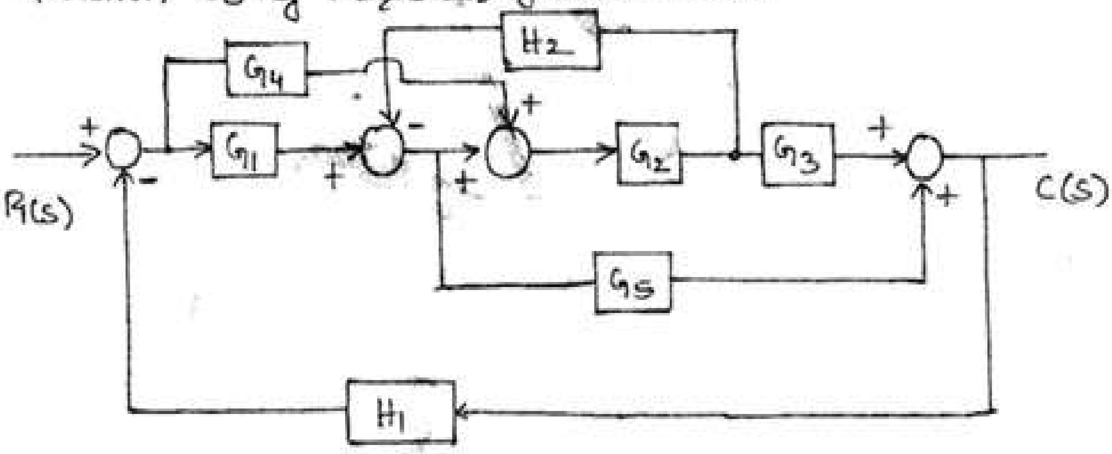
$$X_3 = G_1X_1 + G_2X_2 - H_3X_3$$

$$X_4 = G_2X_2 + G_3X_3 - H_4X_5$$

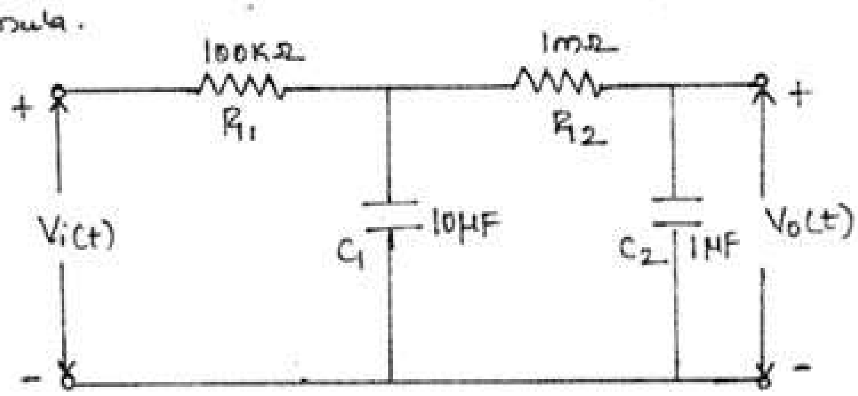
$$X_5 = G_5X_4 - H_5X_6$$

$$X_6 = G_5X_5$$

5) Draw the Signal flow graph for ~~the~~ block diagram shown in the figure. Determine the overall transfer function using mason's gain formula.

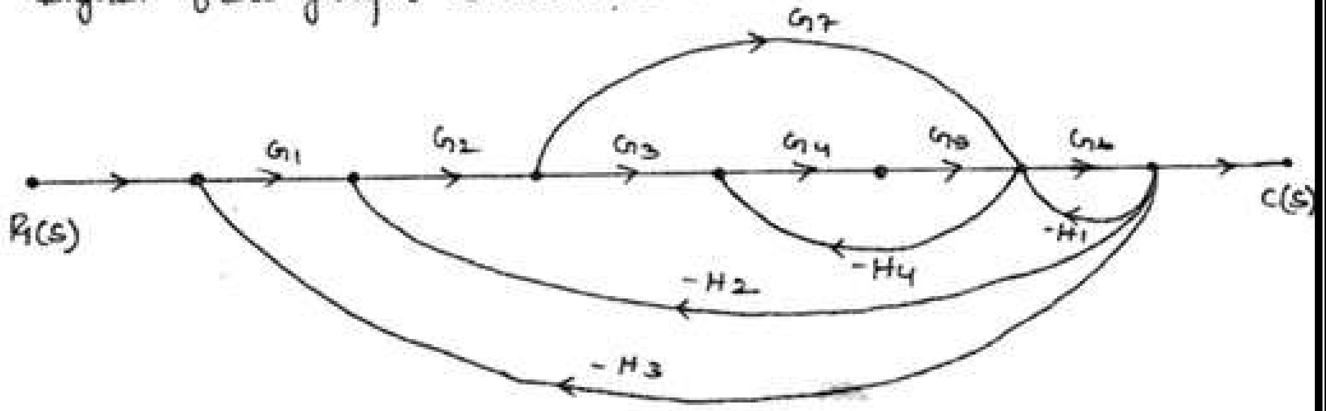


6) For the Electrical Circuit shown in the figure, find the overall transfer function using Mason's gain formula.



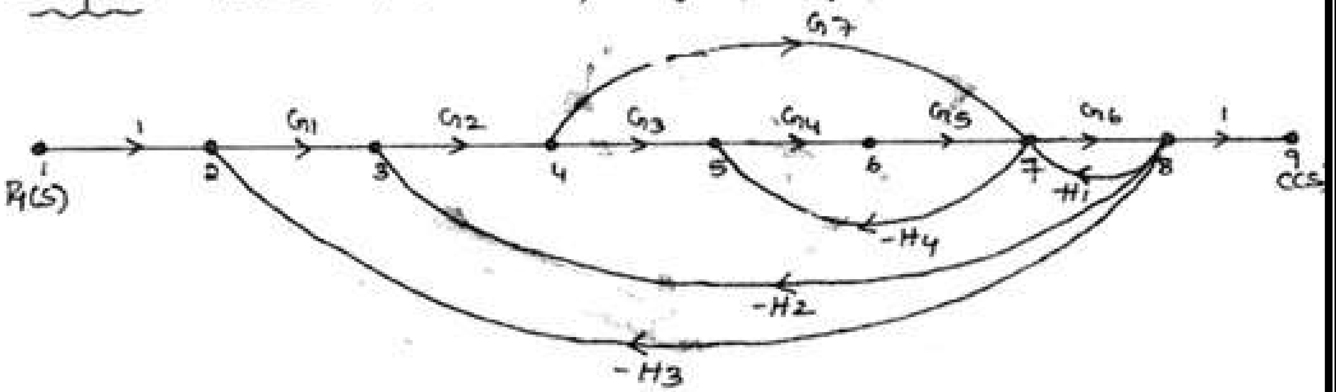
Problems on Signal flow graph

1) Obtain the transfer function of the system for the Signal flow graph shown below



Solution:-

Step 1:- Name the nodes by using numbers.



Step 2:- Forward path gains.

$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$ (1, 2, 3, 4, 5, 6, 7, 8, 9)

$P_2 = G_1 G_2 G_7 G_6$ (1, 2, 3, 4, 7, 8, 9)

Number of forward path $n = 2$,

Step 3:- Single loop gains

$P_{11} = -G_4 G_5 H_4$ (5, 6, 7)

$P_{21} = -G_6 H_1$ (7, 8)

$P_{31} = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$ (2, 3, 4, 5, 6, 7, 8)

$P_{41} = -G_2 G_3 G_4 G_5 G_6 H_2$ (3, 4, 5, 6, 7, 8)

$P_{51} = -G_2 G_7 G_6 H_2 \quad (3,4,7,8)$

$P_{61} = -G_1 G_2 G_7 G_6 H_3 \quad (2,3,4,7,8)$

Step 4:- Two non-touching loop gains.

* Two non touching loops and higher order is absent

$\therefore \Sigma P_{m_2}$ + onwards is zero.

Step 5:- To find ΔK.

Note:- * Number of forward path is equal to number of co-factor (ΔK)

* By comparing forward path gains with single loop gains.

for Ex:- $\Delta_1 = 1 - \underbrace{P_{11} + P_{31} - P_{51}}_{\text{Sequence which is not common}}$

$\Delta_1 = 1 - (0) = 1$

$\Delta_2 = 1 - (0) = 1$

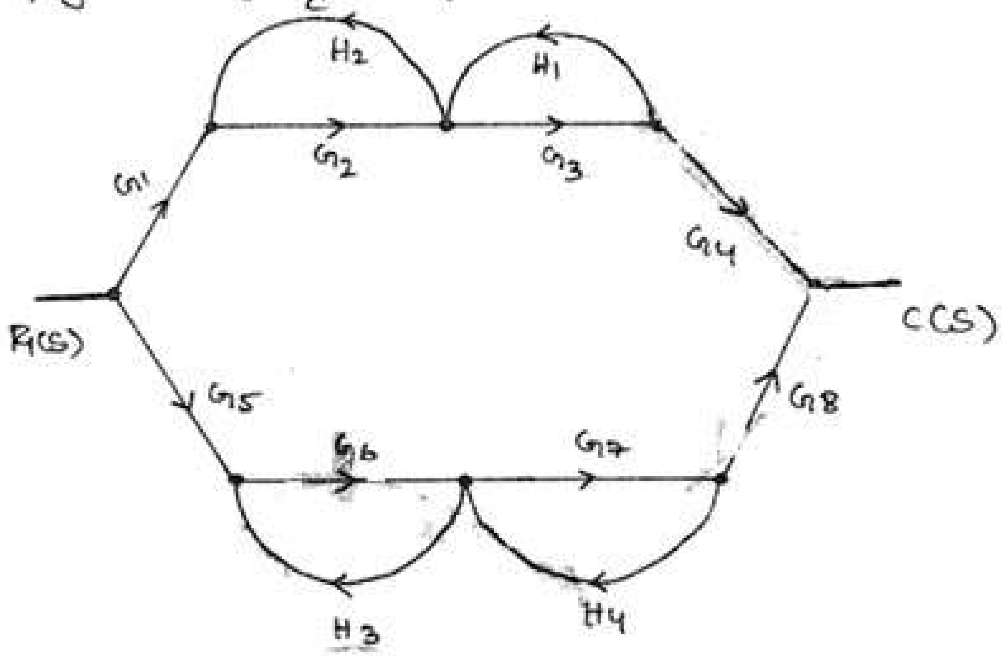
Step 6:- Find the overall transfer function using mason's gain formula.

$$\frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{n=2} P_K \Delta_K}{1 - \sum_{m=1}^{6} P_{m_1} + 0}$$

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51} + P_{61})}$$

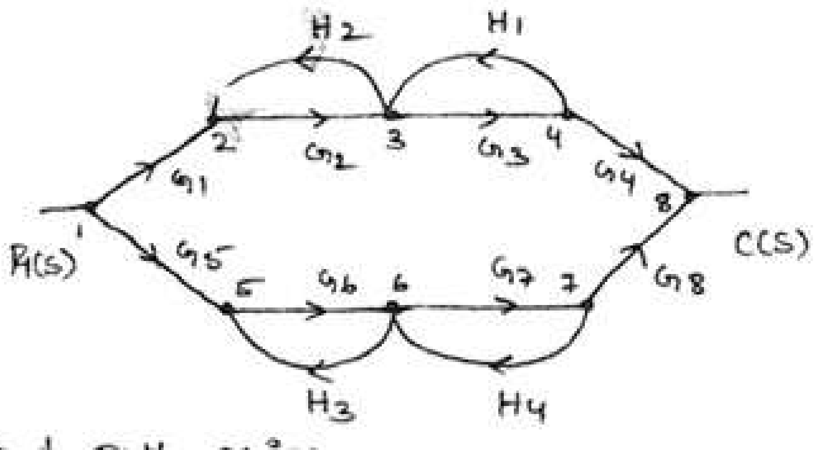
$$\frac{C(S)}{R(S)} = \frac{(G_1G_2G_3G_4G_5G_6)(1) + (G_1G_2G_7G_6)(1)}{1-\big((-G_4G_5H_4)+(-G_6H_1)+(-G_1G_2G_3G_4G_5G_6H_3) + (-G_2G_3G_4G_5G_6H_2)+(-G_2G_7G_6H_2) + (-G_1G_2G_7G_6H_3)\big)}$$

2) Obtain the overall transfer for the system shown in figure using mason's rule.



Solution:

⇒



⇒ Forward path gains.

$P_1 = G_1\ G_2\ G_3\ G_4 \quad (1, 2, 3, 4, 8)$

$P_2 = G_5\ G_6\ G_7\ G_8 \quad (1, 5, 6, 7, 8)$

Number of forward path $n = 2$.

$\Rightarrow$ Single loop gains:-

$P_{11} = G_2 H_2 \ (2,3)$

$P_{21} = G_3 \ H_1 \ (3,4)$

$P_{31} = G_6 \ H_3 \ (5,6)$

$P_{41} = G_7 \ H_4 \ (6,7)$

$\Rightarrow$ Two non-touching loop gains.

$P_{12} = P_{11} \ P_{31} = G_2 H_2 G_6 H_3 \ (2,3,5,6)$

$P_{22} = P_{11} \ P_{41} = G_2 H_2 G_7 H_4 \ (2,3,6,7)$

$P_{32} = P_{21} \ P_{31} = G_3 H_1 G_6 H_3 \ (3,4,5,6)$

$P_{42} = P_{21} \ P_{41} = G_3 H_1 G_7 H_4 \ (3,4,6,7)$

ΣP_{m3} + onwards is zero.

$\Rightarrow$ Co-factor of graph

$\Delta_1 = 1 - (P_{31} + P_{41}) + 0 = 1 - (G_6 H_3 + G_7 H_4)$

$\Delta_2 = 1 - (P_{11} + P_{21}) + 0 = 1 - (G_2 H_2 + G_3 H_1)$

$\Rightarrow$ Overall transfer function

$$M(S) = \sum_{K=1}^{N} \frac{P_K \Delta_K}{\Delta}$$

where Δ is $1 - \Sigma P_{m1} + \Sigma P_{m2} - \Sigma P_{m3} + - + \cdots$

$$M(S) = \frac{\sum_{K=1}^{2} P_K \Delta_K}{1 - \sum_{m=1}^{4} P_{m1} + \sum_{m=1}^{4} P_{m2} - 0} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{1 - (P_{11} + P_{21} + P_{31} + P_{41}) + (P_{12} + P_{22} + P_{32} + P_{42})}$$

$$\frac{C(S)}{R(S)} = \frac{(G_1G_2G_3G_4)(1-G_6H_3-G_7H_4) + (G_5G_6G_7G_8)(1-G_2H_2+G_3H_1)}{1-[G_2H_2+G_3H_1+G_6H_3+G_7H_4]+[G_2H_2G_6H_3+G_2H_2G_7H_4+G_3H_1G_6H_3+G_3H_1G_7H_4]}$$

3) Draw the signal flow graph for the system of equations given below and obtain the overall transfer function using mason's rule.

$$x_2 = x_1 - H_3x_8$$

$$x_3 = G_1x_2 - H_2x_8$$

$$x_4 = G_2x_3$$

$$x_5 = G_3x_4 - H_4x_6$$

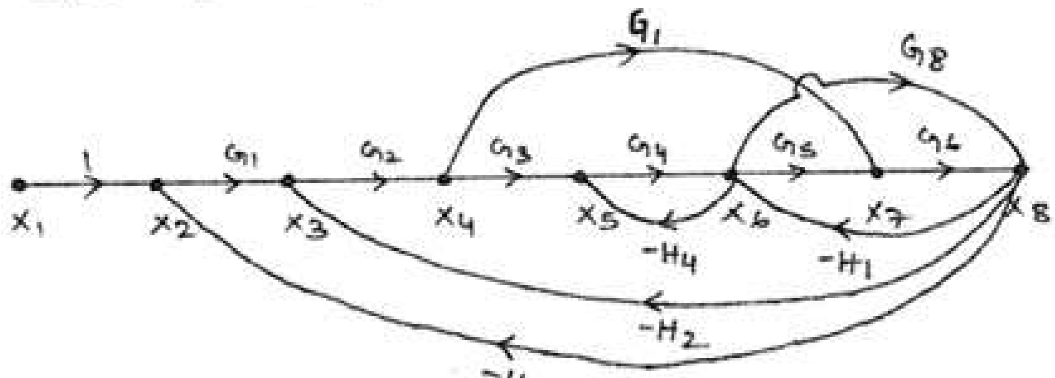
$$x_6 = G_4x_5 - H_1x_8$$

$$x_7 = G_7x_4 + G_5x_6$$

$$x_8 = G_8x_6 + G_6x_7$$

Solution:-

* The signal flow graph satisfying the above equation is as shown below.



⇒ Forward path gains.

$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$ (1, 2, 3, 4, 5, 6, 7, 8)

$P_2 = G_1 G_2 G_7 G_6$ (1, 2, 3, 4, 7, 8)

$P_3 = G_1 G_2 G_3 G_4 G_8$ (1, 2, 3, 4, 5, 6, 7, 8)

Number of forward path $n = 3$.

⇒ Single loop gains.

$P_{11} = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$ (2, 3, 4, 5, 6, 7, 8)

$P_{21} = -G_1 G_2 G_7 G_6 H_3$ (2, 3, 4, 7, 8)

$P_{31} = -G_1 G_2 G_3 G_4 G_8 H_3$ (2, 3, 4, 5, 6, 8)

$P_{41} = -G_2 G_3 G_4 G_5 G_6 H_2$ (3, 4, 5, 6, 7, 8)

$P_{51} = -G_2 G_7 G_6 H_2$ (3, 4, 7, 8)

$P_{61} = -G_2 G_3 G_4 G_8 H_2$ (3, 4, 5, 6, 8)

$P_{71} = -G_4 H_4$ (5, 6)

$P_{81} = -G_5 G_6 H_1$ (6, 7, 8)

$P_{91} = -G_8 H_1$ (6, 8)

⇒ Two-non touching loop gains

$P_{12} = P_{21} P_{71} = G_1 G_2 G_7 G_6 H_3 G_4 H_4$ (2, 3, 4, 5, 6, 7, 8)

$P_{22} = P_{51} P_{71} = G_2 H_7 G_6 H_2 G_4 H_4$ (3, 4, 5, 6, 7, 8)

ΣP_{m3} and Onwards is zero.

⇒ Co-factor of Graph.

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - P_{71} + 0 = 1 + G_4 H_4$

$\Delta_3 = 1$

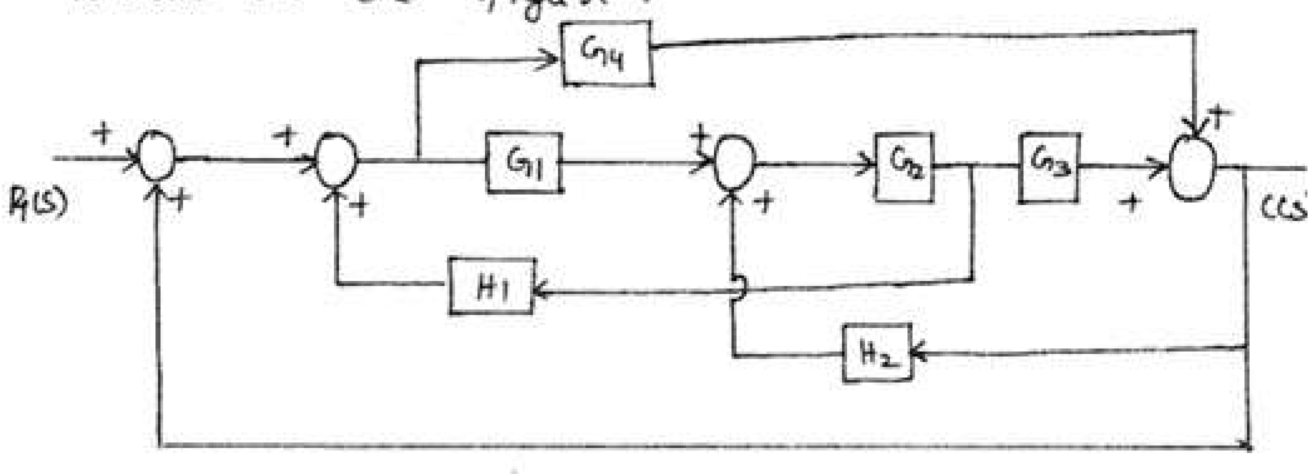
⇒ Overall transfer function.

$$M(s) = \frac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}$$

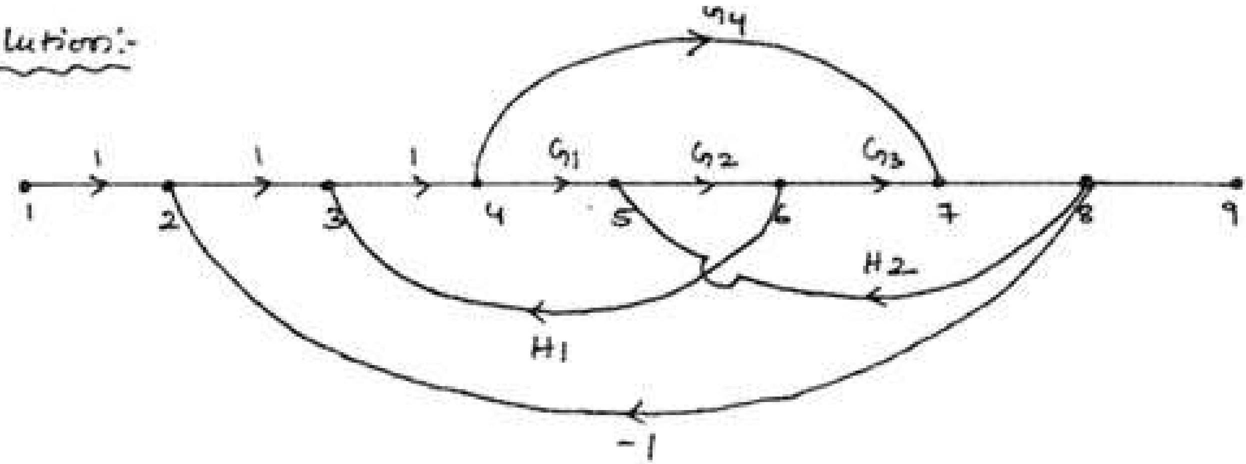
$$\frac{X_8}{X_1} = \frac{\sum_{K=1}^{3} P_K \Delta_K}{1 - \sum_{m=1}^{9} P_{m1} + \sum_{m=1}^{2} P_{m2} - 0} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51} + P_{61} + P_{71} + P_{81} + P_{91}] + [P_{12} + P_{22}]}$$

$$\frac{X_8}{X_1} = \frac{G_1G_2G_3G_4G_8 + (G_1G_2G_7G_6)(1 + G_4H_4) + G_1G_3G_3G_4G_5G_6}{1 - [-G_1G_2G_3G_4G_5G_6H_3 - G_1G_2G_7G_6H_3 - G_1G_2G_3G_4G_8H_3 - G_2G_3G_4G_5G_6H_2 - G_2G_7G_6H_2 - G_2G_3G_4G_8H_2 - G_4H_4 - G_8H_1 + [G_1G_2G_7G_6H_3G_4H_4 + G_2G_7G_6H_2G_4H_4]]}$$

4) Draw the signal Flow graph and determine the overall transfer function for the block diagram shown in the figure.



Solution:-



The Signal flow graph for the given block diagram is as shown above.

$\Rightarrow$ Forward path gains.

$P_1 = G_1 G_2 G_3 \; (1, 2, 3, 4, 5, 6, 7, 8, 9)$

$P_2 : \; G_4 \quad (1, 2, 3, 4, 7, 8, 9)$

$\Rightarrow$ Single loop gains

$P_{11} = G_1 G_2 G_3 \; (2, 3, 4, 5, 6, 7, 8)$

$P_{21} = G_1 G_2 H_1 \; (3, 4, 5, 6)$

$P_{31} = G_2 G_3 H_2 \; (5, 6, 7, 8)$

$P_{41} = G_4 \; (2, 3, 4, 7, 8)$

$P_{51} = G_4 H_2 G_2 H_1 \; (3, 4, 7, 8, 5, 6)$

ΣP_{m2} 4 onwards is zero.

$\Rightarrow$ To find Δ_K

$\Delta_1 = 1 - (0) = 1$

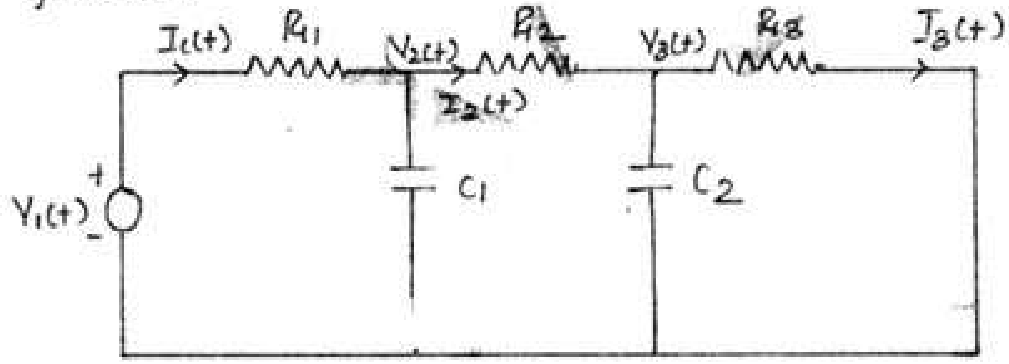
$\Delta_2 = 1 - (0) = 1$

Overall transfer Function

$$\frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{n=2} P_K \Delta_K}{1-\sum_{m=1}^{5} P_{m1} + 0} = \frac{P_1\Delta_1 + P_2\Delta_2}{1-\left[P_{11}+P_{21}+P_{31}+P_{41}+P_{51}\right]+0}$$

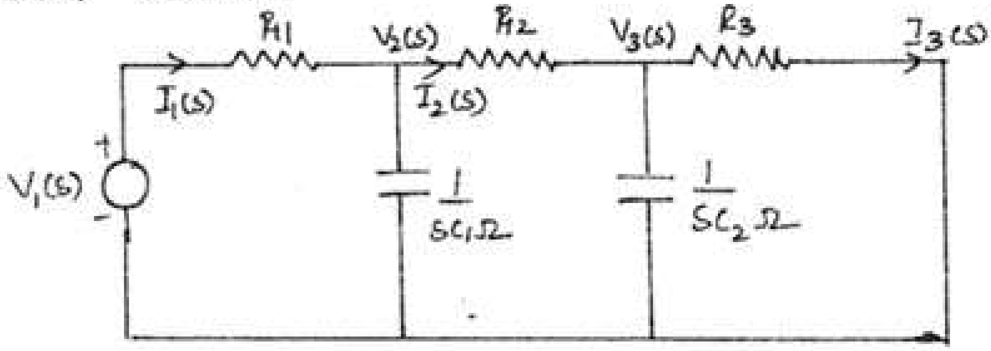
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_4}{1-\left[G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_4 + G_4H_2G_2H_1\right]}$$

5) For the Circuit shown in the Figure write the Performance Equation considering the Voltage & Current Variable as indicated. draw the Corresponding signal flow graph and determine $\frac{I_3(s)}{V_1(s)}$ using mason's gain formula.



Solution:-

By taking Laplace transform, the network is as shown below.



$$I_1(s) = \frac{V_1(s) - V_2(s)}{R_1}$$

$$I_1(s) = \frac{1}{R_1} V_1(s) - \frac{1}{R_1} V_2(s) \longrightarrow (1)$$

$$I_2(s) = \frac{V_2(s) - V_3(s)}{R_2}$$

$$= \frac{1}{R_2} V_2(s) - \frac{1}{R_2} V_3(s) \longrightarrow (2)$$

$$I_3(s) = \frac{V_3(s)}{R_3} \longrightarrow (3)$$

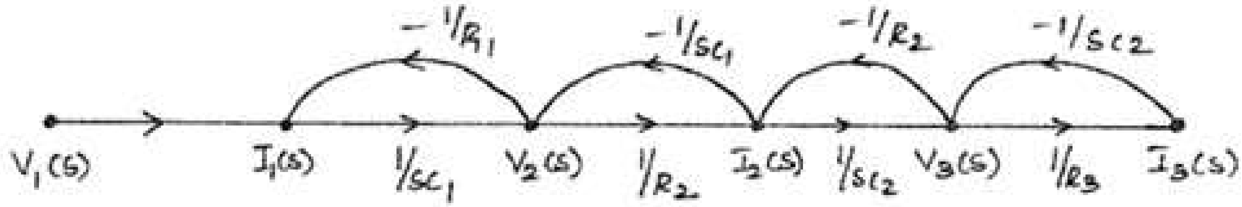
$$V_2(s) = \frac{1}{sC_1}\left(I_1(s) - I_2(s)\right)$$

$$V_2(s) = \frac{1}{sC_1} I_1(s) - \frac{1}{sC_1} I_2(s) \longrightarrow (4)$$

$$V_3(s) = \frac{1}{sC_2}\left(I_2(s) - I_3(s)\right)$$

$$V_3(s) = \frac{1}{sC_2} I_2(s) - \frac{1}{sC_2} I_3(s) \text{ — } (5)$$

=> Signal flow graph is drawn is as shown below.



$\Rightarrow$ Forward path gains

$$P_1 = \frac{1}{R_1} \cdot \frac{1}{sC_1} \cdot \frac{1}{R_2} \cdot \frac{1}{sC_2} \cdot \frac{1}{R_3} = \frac{1}{s^2 C_1 C_2 R_1 R_2 R_3}$$

Number of forward path gain, $n = 1$

$\Rightarrow$ Single Loop gains

$$P_{11} = -\frac{1}{sC_1 R_1}$$

$$P_{21} = -\frac{1}{sC_1 R_2}$$

$$P_{31} = -\frac{1}{sC_2 R_2}$$

$$P_{41} = -\frac{1}{sC_2 R_3}$$

$\Rightarrow$ Two-non-touching Loop gains

$$P_{12} = P_{11} P_{31} = \frac{1}{s^2 C_1 C_2 R_1 R_2}$$

$$P_{22} = P_{11} P_{41} = \frac{1}{s^2 C_1 C_2 R_1 R_3}$$

$$P_{32} = P_{21} P_{41} = \frac{1}{s^2 C_1 C_2 R_2 R_3}$$

ΣP_{m3} and onwards is zero.

$\Rightarrow$ To find Δ_K

$$\Delta_1 = 1 - 0 = 1$$

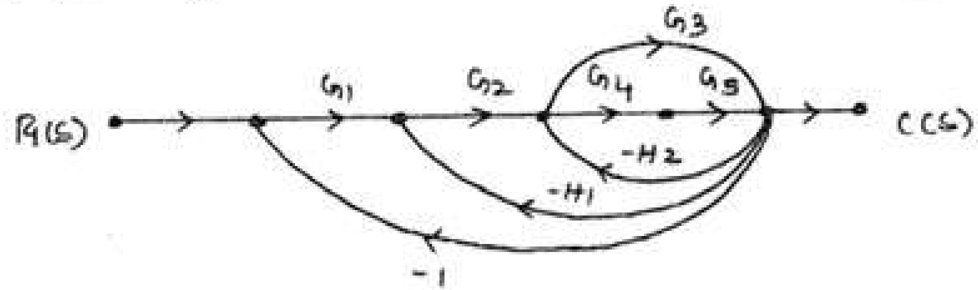
=> The Overall transfer function is given by

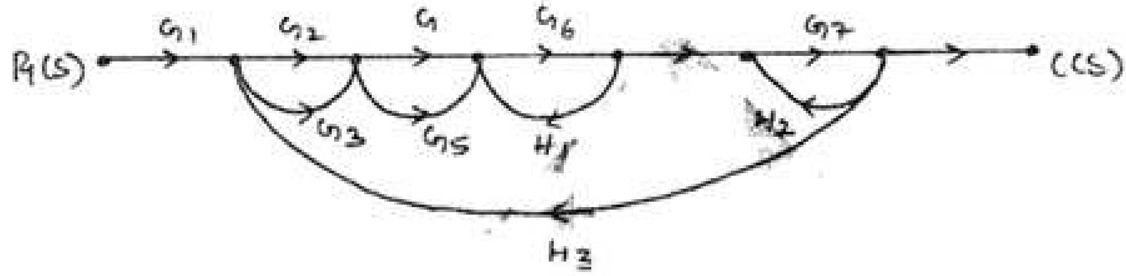
$$T.F = \frac{I_3(s)}{V_1(s)} = \frac{\sum_{K=1}^{n=1} P_K \Delta_K}{1 - \sum_{m=1}^{4} P_{m1} + \sum_{m=1}^{3} P_{m2} - 0.}$$

$$\frac{I_3(s)}{V_1(s)} = \frac{P_1 \Delta_1}{1 - (P_{11} + P_{21} + P_{31} + P_{41}) + (P_{12} + P_{22} + P_{32})}$$

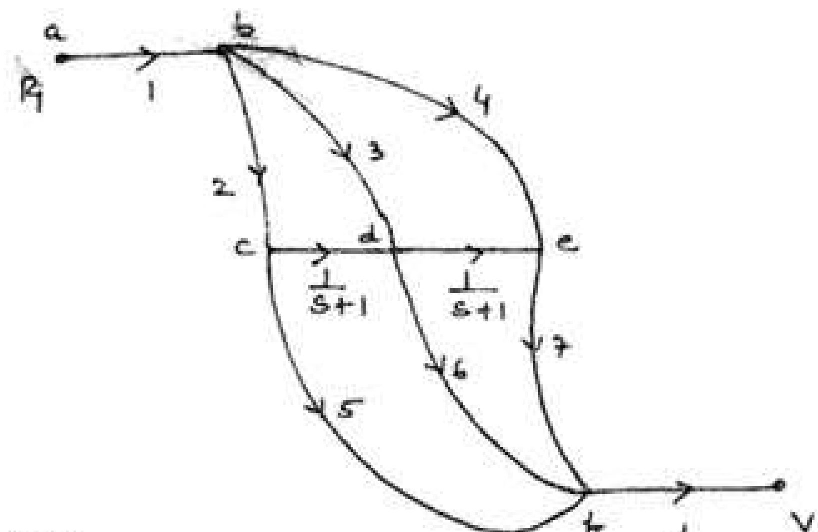
Practice Problems on Signal flow graph

1) Determine the overall transfer function for the system shown in the figure.

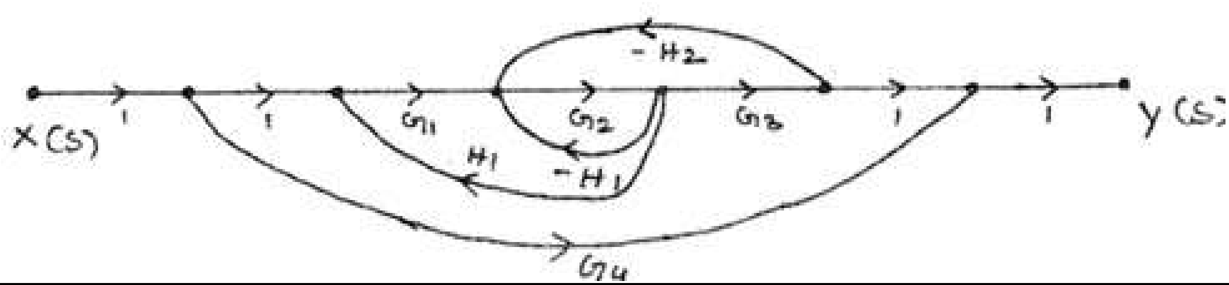




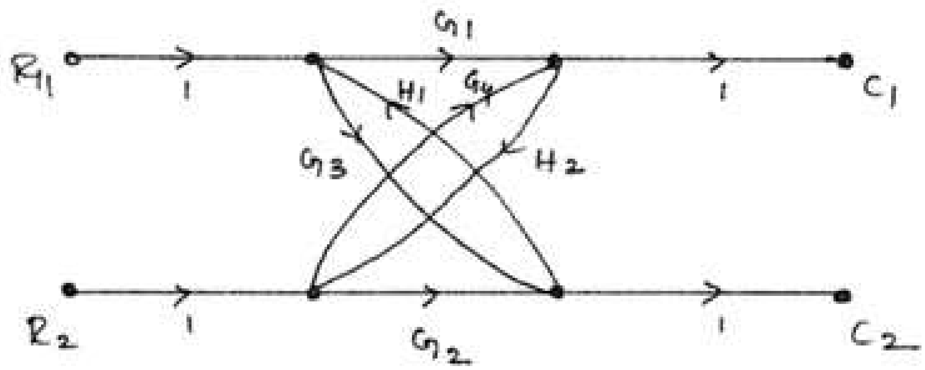
3) Determine the transmittance of the signal flow graph



4) Determine $\frac{Y(S)}{X(S)}$ using mason's rule.

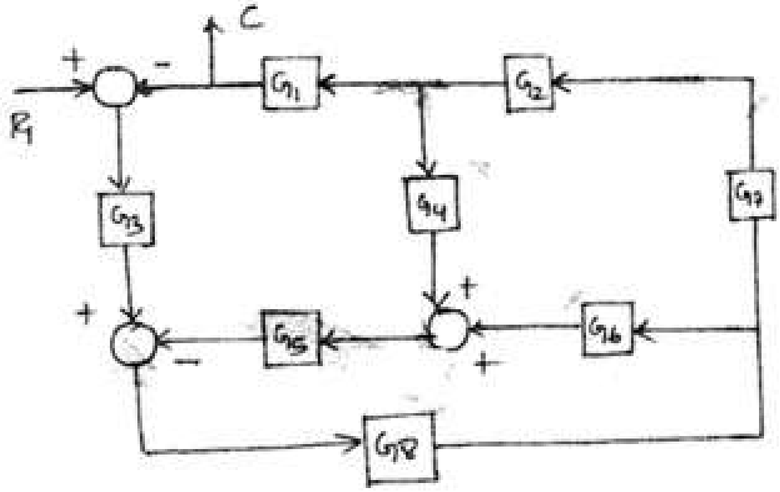


5)



Determine the overall transfer function using mason's rule.

6) Draw a signal flow graph ~~for~~ the block diagram shown in the figure.



Assignment Problems On Signal flow graph

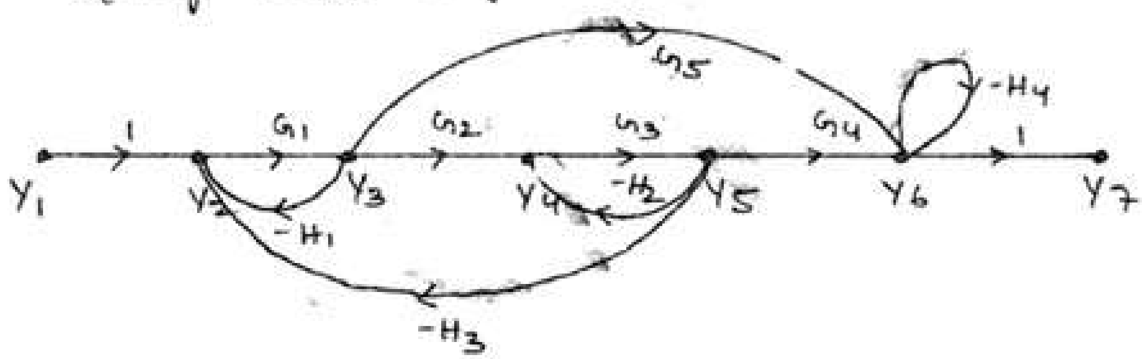
1) Draw Signal flow graph from the following Equations and obtain the Overall transfer function.

$$x_2 = a_{21} x_1 + a_{23} x_3$$

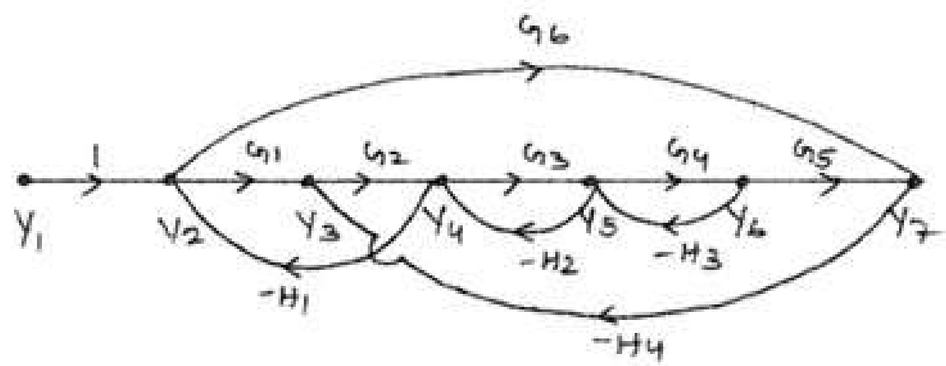
$$x_3 = a_{31} x_1 + a_{32} x_2 + a_{33} x_3$$

$$x_4 = a_{42} x_2 + a_{43} x_3$$

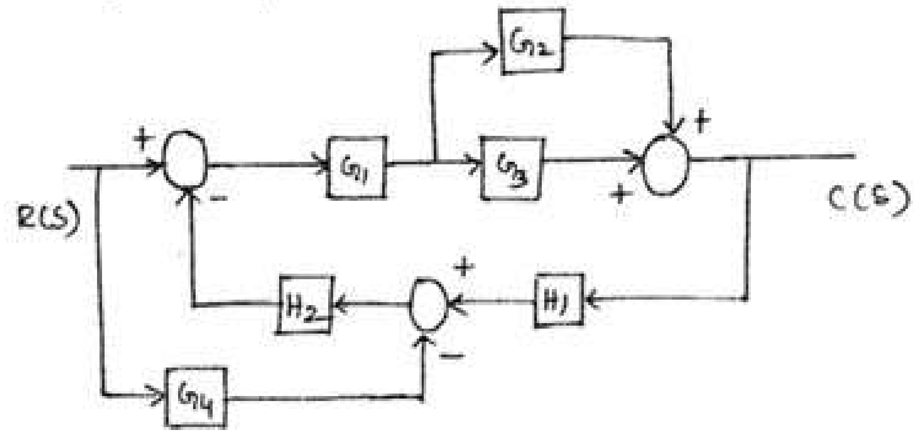
2) Determine the Overall transfer function $\frac{Y_7}{Y_4}$ of the signal flow graph shown in the figure. using mason's gain formula



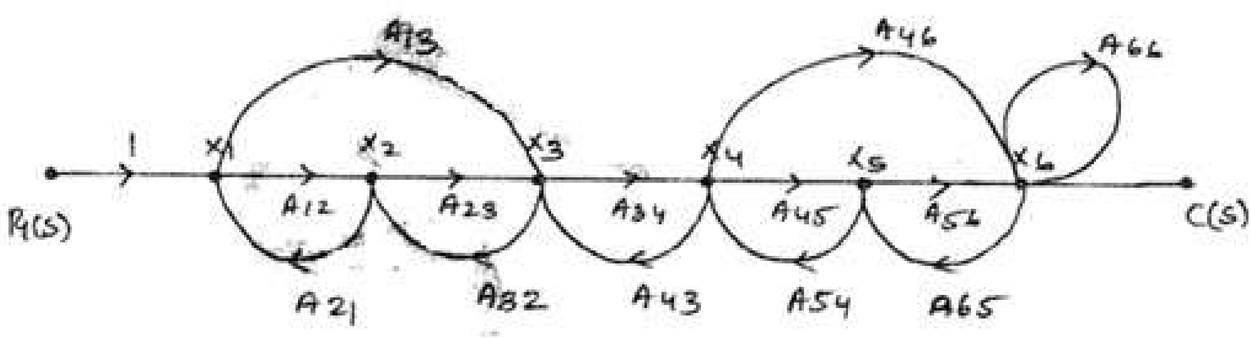
3) Find the ~~transfer~~ function $\frac{Y_7}{Y_1}$ using mason's gain formula for the Signal flow graph shown in figure.



4) Draw the signal flow graph for the block diagram shown in the figure and evaluate $\frac{C(S)}{R(S)}$ using mason's formula.



5) Find the overall transfer function $\frac{C(S)}{R(S)}$ using mason's rule for the system shown in the figure.



Solution for Practice Problems:-

Translational System:-

Problem -1

*** → F-V Analogy:-**

Equation 1 → $V(t) = L_1 \frac{d}{dt} i_1;$

Equation 2 → $0 = L_2 \frac{d}{dt} i_2 + \frac{1}{C_1}\int (i_2 - i_3)\,dt + R_1(i_2 - i_3);$

Equation 3 → $\frac{1}{C_1}\int (i_2 - i_3)\,dt + R(i_2 - i_3) = L_2 \frac{d}{dt} i_3 + \frac{1}{C_2}\int i_3\,dt;$

*** → F-I Analogy:-**

Equation 1 → $I(t) = C_1 \frac{d}{dt} V_1.$

Equation 2 → $0 = C_2 \frac{d}{dt} V_2 + \frac{1}{L_1}\int (V_2 - V_3)\,dt + G_1(V_2 - V_3);$

Equation 3 → $\frac{1}{L_1}\int (V_2 - V_3)\,dt + G_1(V_2 - V_3) = C_2 \frac{d}{dt} V_3 + \frac{1}{L_2}\int V_3\,dt;$

Problem 2:-

*** → F-V Analogy:**

Equation 1: $V(t) = L_1 \frac{d}{dt} i_1 + R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt;$

Equation 2: $R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt = L_2 \frac{d}{dt} i_2 + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt;$

*** → F-I Analogy:-**

Equation 1: $I(t) = C_1 \frac{d}{dt} V_1 + G_1(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt;$

Equation 2: $G_1(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_2)\,dt = C_2 \frac{d}{dt} V_2 + G_2 V_2 + \frac{1}{L_2}\int V_2\,dt;$

Problem 3:-

* → F-V Analogy:-

Equation 1:- $V(t) = L_1 \frac{d}{dt} i_1 + \frac{1}{C_1}\int (i_1 - i_4)\,dt + R_1 (i_1 - i_2)$;

Equation 2:- $R_1 (i_1 - i_2) = L_2 \frac{d}{dt} i_2 + \frac{1}{C_2}\int (i_2 - i_3)\,dt$;

Equation 3:- $\frac{1}{C_1}\int (i_1 - i_4)\,dt = R_2 (i_4 - i_3)$;

Equation 4:- $R_2 (i_4 - i_3) + \frac{1}{C_2}\int (i_2 - i_3)\,dt = L_3 \frac{d}{dt} i_3 + R_3 i_3 + \frac{1}{C_3}\int i_3\,dt$;

* → F-I Analogy:-

Equation 1: $I(t) = C_1 \frac{d}{dt} V_1 + \frac{1}{L_1}\int (V_1 - V_4)\,dt + G_1 (V_1 - V_2)$;

Equation 2:- $G_1 (V_1 - V_2) = C_2 \frac{d}{dt} V_2 + \frac{1}{L_2}\int (V_2 - V_3)\,dt$;

Equation 3:- $\frac{1}{L_1}\int (V_1 - V_4)\,dt = G_2 (V_4 - V_3)$;

Equation 4:- $G_2 (V_4 - V_3) + \frac{1}{L_2}\int (V_2 - V_3)\,dt = C_3 \frac{d}{dt} V_3 + G_3 V_3 + \frac{1}{L_3}\int V_3\,dt$;

Problem 4:-

* → F-V Analogy:-

Equation 1:- $V(t) = L_1 \frac{d}{dt} i_1 + R_1 i_1 + \frac{1}{C_1}\int i_1\,dt + \frac{1}{C_2}\int (i_1 - i_2)\,dt$;

Equation 2:- $\frac{1}{C_2}\int (i_1 - i_2)\,dt = L_1 \frac{d}{dt} i_2 + \frac{1}{C_3}\int (i_2 - i_3)\,dt + R_2 (i_2 - i_3)$;

Equation 3:- $\frac{1}{C_3}\int (i_2 - i_3)\,dt + R_2 (i_2 - i_3) = L_2 \frac{d}{dt} i_3$;

* → F-I Analogy:-

Equation 1:- $I(t) = C_1 \frac{d}{dt}V_1 + G_1 V_1 + \frac{1}{L_1}\int V_1 dt + \frac{1}{L_2}\int (V_1 - V_2) dt$;

Equation 2: $\frac{1}{L_2}\int (V_2 - V_3) dt = C_1 \frac{dV_2}{dt} + \frac{1}{L_3}\int (V_2 - V_3) dt + G_2 (V_2 - V_3)$;

Equation 3:- $\frac{1}{L_3}\int (V_2 - V_3) dt + G_2 (V_2 - V_3) = C_2 \frac{d V_3}{dt}$;

Problem 5:

* → F-V Analogy:

Equation 1: $V(t) = L_2 \frac{d}{dt} i_2 + \frac{1}{C_3}\int i_2 dt + \frac{1}{C_1}\int (i_2 - i_1) dt + R_2 (i_2 - i_3)$

Equation 2: $\frac{1}{C_1}\int (i_2 - i_1) dt = L_1 \frac{d}{dt} i_1 + R_1 i_1$;

Equation 3: $R_2 (i_2 - i_3) = L_3 \frac{d i_3}{dt} + \frac{1}{C_2}\int i_3 dt$;

* → F-I Analogy:

Equation 1: $I(t) = C_2 \frac{d V_2}{dt} + \frac{1}{L_3}\int V_2 dt + \frac{1}{L_1}\int (V_2 - V_1) dt +$

$G_2 (V_2 - V_3)$;

Equation 2: $\frac{1}{L_1}\int (V_2 - V_1) dt = C_1 \frac{d}{dt} V_1 + G_1 V_1$;

Equation 3: $G_2 (V_2 - V_3) = C_3 \frac{d}{dt} V_3 + \frac{1}{L_2}\int V_3 dt$;

Problem 6:-

* → F-V Analogy:

Equation 1: $V(t) = L_2 \frac{d}{dt} i_2 + R_2 i_2 + \frac{1}{C_2}\int i_2 dt + \frac{1}{C_1}\int (_2 - i_1) dt$

$+ R_1 (i_2 - i_1)$;

Equation 2: $\frac{1}{C_1}\int (i_2 - i_1) dt + R_1 (i_2 - i_1) = L_1 \frac{d}{dt} i_1$;

* → F-I Analogy:-

Equation 1: $I(t) = C_2 \frac{dV_2}{dt} + G_2 V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_1}\int (V_2 - V_1)\,dt$

$+ G_1 (V_2 - V_1);$

Equation 2: $\frac{1}{L_1}\int (V_2 - V_1)\,dt + G_1 (V_2 - V_1) = C_1 \frac{d}{dt} V_1;$

Transfer Function:

Problem 1:-

$$\frac{V_o(S)}{V_i(S)} = \frac{R_2}{R_1 + R_2}\left[\frac{1 + SCR_1}{1 + \frac{SCR_1R_2}{R_1 + R_2}}\right]$$

Problem 2:-

$$\frac{V_o(S)}{V_i(S)} = \frac{R}{S^2 LCR + SL + R}$$

Problem 3:-

$$\frac{E_o(S)}{E_i(S)} = \frac{R_1 + SL}{R + R_1 + S(L + RR_1C) + S^2 RLC}$$

Problem 4:-

$$\frac{E_o(S)}{E_i(S)} = \frac{1 + S^2 R_1R_2C_1C_2 + SR_2(C_1 + C_2)}{1 + S^2 R_1R_2C_1C_2 + S(R_1C_2 + R_2C_1 + R_2C_2)}$$

Problems 5:-

$$\frac{\theta(S)}{T(S)} = \frac{K}{(J_1J_2S^4 + J_1BS^3 + (KJ_1 + KJ_2)S^2 + KBS)}$$

Problem 6 :-

$$\frac{\theta_1(s)}{T(s)} = \frac{(J_2 s^2 + K_2) K_1}{J_1 J_2 J_m s^6 + (K_2 J_m J_1 + J_2 J_1 K_2 + J_2 J_1 K_1 + J_2 J_m K_1) s^4 + (J_2 K_2 K_1 + J_1 K_1 K_2 + J_m K_1 K_2) s^2}$$

Problem 7 :-

$$\frac{X(s)}{F(s)} = \frac{B_{12} s + K_1}{s(M_1 M_1 s^3 + (M_2 B_{12} + M_1 B + M_1 B_{12}) s^2 + (m_1 K_1 + M_2 K_1 + B B_{12}) s + K_1 B)}$$

Block Diagram Algebra

Problem 1 :-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 + G_1 H_1}$$

Problem 2 :-

$$\frac{C(s)}{R(s)} = \frac{G_2 (G_1 + G_3)}{1 + G_1 G_2}$$

Problem 3 :-

$$\frac{C(s)}{R(s)} = \frac{G_3 G_6 + G_4 G_6 + G_2 G_5 G_6 + G_3 G_5 G_6}{1 + G_1 G_2 + G_1 G_3 + G_6 + G_1 G_2 G_6 + G_1 G_3 G_6 + G_3 G_6 G_7 + G_4 G_6 G_7 + G_2 G_5 G_6 G_7 + G_3 G_5 G_6}$$

Problem 4 :-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H_1 + G_2 H_1 - G_1}$$

Problem 5:-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_7 (G_3 + G_4)}{1 + G_6 G_7 - G_3 G_7 G_8 - G_4 G_7 G_8 + G_1 G_3 G_5 + G_1 G_4 G_5 + G_1 G_2 + G_1 G_2 G_6 G_7}$$

Problem 6:-

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_5}{1 + G_2 + G_5 H_5 + G_2 G_5 H_5 + G_2 G_3 G_5 H_2}$$

Signal Flow Graph

Problem 1:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_4 G_5 + G_1 G_2 G_3}{1 + H_2 G_4 G_5 + G_2 G_4 G_5 H_1 + G_1 G_2 G_4 G_5 + G_1 G_2 G_3 + G_2 G_3 H_1 + G_3 H_2}$$

Problem 2:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{G_1 G_6 G_7 (G_2 + G_3)(G_4 + G_5)}{1 - G_6 G_7 H_3 (G_2 + G_3)(G_4 + G_5) - G_6 H_1 - G_7 H_2 + G_6 G_7 H_1 H_2}$$

Problem 3:-

$$M(s) = \frac{C(s)}{R(s)} = \frac{56(s+1)^2 + 33(s+1) + 14}{(s+1)^2}$$

Problem 4:-

$$M(s) = \frac{Y(s)}{X(s)} = \frac{G_1 G_2 G_3 + G_4 (1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2)}{1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}$$

Problem 5:-

$$M(S) = \frac{C_2(S)}{R_1(S)} = \frac{G_3(1-G_4H_2) + G_1G_2H_2}{(1-G_4H_2) + H_1\left(G_3(G_4H_2-1) - G_1G_2H_2\right)}$$

Problem 6:-

$$M(S) = \frac{C(S)}{R(S)} = \frac{(1 + G_5G_6G_8 + G_2G_4G_5G_7G_8)}{1 + G_5G_6G_8 + G_2G_4G_5G_7G_8 + G_1G_2G_3G_7G_8}$$

Module no 2 Questions

Q2-1: Define transfer function and what are its properties. **Jun. 2013, 5 Marks**

Q2-2: Illustrate how to perform the following, in connection with block diagram reduction rules:

(i) Shifting a take-off point after a summing point.

(ii) Shifting a take-off point before a summing point. **Dec. 2012, 4 Marks**

Q2-3: The performance equations of a controlled system are given by the following set of linear algebraic equations: **Dec. 2012, 8 Marks**

$$E_1(s) = R(s) - H_3(s)C(s);$$
$$E_2(s) = E_1(s) - H_1(s)E_4(s);$$
$$E_3(s) = G_1(s)E_2(s) - H_2(s)C(s);$$
$$E_4(s) = G_2(s)E_3(s);$$
$$C(s) = G_3(s)E_4(s);$$

(i) Draw the block diagram.

(ii) Find the overall transfer function $\frac{C(s)}{R(s)}$ using block diagram reduction technique

Q2-4: For the system represented by the following equations, find the transfer function $\frac{X(s)}{U(s)}$ by signal flow graph, technique **Dec. 2014, 10 Marks**

$$x = x_1 + \alpha_3 u$$
$$\dot{x}_1 = -\beta_1 x_1 + x_2 + \alpha_2 u \; \dot{x}_2 = -\beta_2 x_1 + \alpha_1 u.$$

Q2-5: Obtain the transfer function of field controlled servo motors. **Jun. 2013, 8 Marks**

Q2-6: Obtain the transfer function of an armature controlled dc servo motor. **Jun. 2009, 6 Marks**

Q2-7: Define the transfer function. Explain Mason's gain formula for determining the transfer function from signal flow graphs. **Dec. 2010, 6 Marks**

Q2-8: Explain briefly the following terms: **Jul. 2009, 8 Marks**
(i) Forward path. (ii) Path gain. (iii) Loop gain. (iv) Canonical form.

Q2-9: Draw the signal flow graph for the system of equation given below and obtain the overall transfer function $\frac{X_6(s)}{X_1(s)}$ using MGF **Jul. 2007, 12 Marks**

$$X_2 = G_1X_1 - H_1X_2 - H_2X_3 - H_6X_6$$
$$X_3 = G_1X_1 + G_2X_2 - H_3X_3$$
$$X_4 = G_2X_2 + G_3X_3 - H_4X_5$$
$$X_5 = G_4X_4 - H_5X_6$$
$$X_6 = G_5X_5$$

Q2-10: For a negative feedback control system, starting from fundamentals, show that the closed loop transfer function M(s) is given by **Jan. 2007, 8 Marks**

$$M(s) = \frac{N_g D_h}{D_g D_h + N_g N} ,$$

where $G(s) = N/D_g$ forward path gain

$$H(s) = \frac{N_h}{D_h}$$ feedback gain

Q2-11: Define the term "transfer function". The unit step response of single loop, unity feedback control system is given by,

$$c(t) = 1 - 1.25e^{-2t} + 0.25e^{-10t}$$

Determine its closed loop and open loop transfer functions. **Jul. 2006, 08 Marks**

Q2-12: Define the term "transfer function". The unit step response of single loop, unity feedback control system is given by,

$$c(t) = 1 - 3e^{-2t} + 2e^{-3t}$$

Determine its closed loop and open loop transfer functions. **Jan. 2006, 06 Marks**

Q2-13: Obtain a block diagram representation and evaluate the transfer function of an armature controlled dc motor. **Jan. 2006, 08 Marks**

Q2-14: Illustrate how to perform the following in connection with block diagram reduction techniques: **Jul. 2005, 10 Marks**

(i) moving a summing point ahead of a block and behind a block.

(ii) moving a take off point ahead of a block and behind a block.

(iii) Transforming a non unity feedback to a unity feedback.

Q2-15: Consider the system shown in 1. An armature-controlled dc servomotor drives a load consisting of the moment of inertia J_L and load torque T_L. Field current i_f is constant. The torque developed by the motor is T_m. The moment of inertia of the motor rotor is J_m. The viscous friction coefficient of motor is B_m. The angular displacements of the motor rotor and the load element are θ_m and θ_L, respectively. The gear ratio is $n = \frac{N_1}{N_2} = \frac{\theta_L}{\theta_m}$. Draw a block diagram and obtain the transfer functions $\frac{\Theta_L(s)}{E_a(s)}$ and $\frac{\Theta_L(s)}{T_L(s)}$.

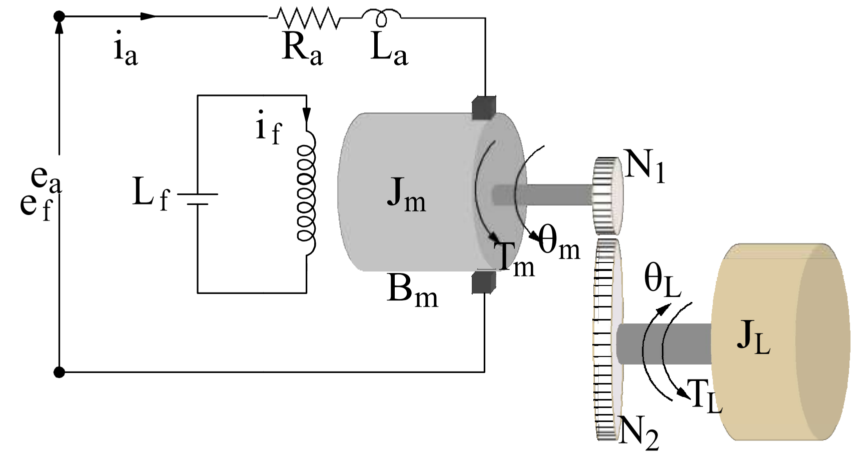


Figure 1: Armature-controlled dc servomotor system.

Q2-16: Consider the system shown in 2. An field-controlled dc servomotor drives a load consisting of the moment of inertia J_L and load torque T_L. Armature current i_a is constant. The torque developed by the motor is T_m. The moment of inertia of the motor rotor is J_m. The viscous friction coefficient of motor is B_m. The angular displacements of the motor rotor and the load element are θ_m and θ_L, respectively. The gear ratio is $n = \frac{N_1}{N_2} = \frac{\theta_L}{\theta_m}$. Draw a block diagram and obtain the transfer functions $\frac{\Theta_L(s)}{E_f(s)}$ and $\frac{\Theta_L(s)}{T_L(s)}$.

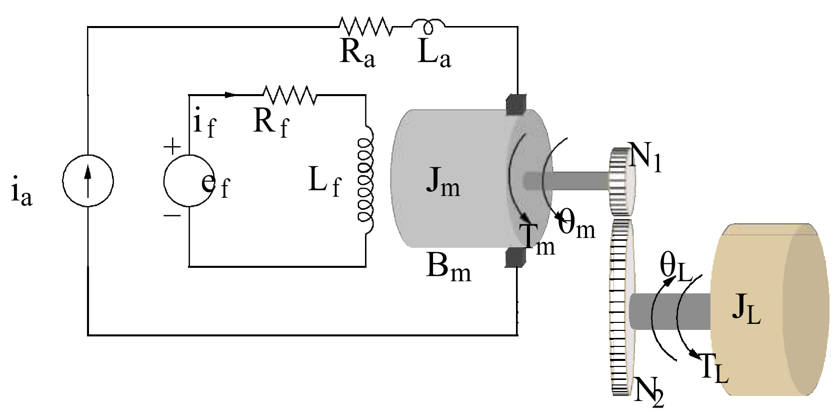


Figure 2: Field-controlled dc servomotor system.

Q2-17: Consider the system shown in 3. An armature-controlled dc servomotor drives a load with torque T_L. The torque developed by the motor is T_m. The moment of inertia of the motor rotor is J_m. The viscous friction coefficient of motor is B_m. The angular speed of the motor rotor is ω. Obtain the transfer functions $\frac{\omega(s)}{V(s)}$ and $\frac{\omega(s)}{T_L(s)}$.

Q2-18: Consider the system shown in 4. An armature-controlled dc servomotor drives a load consisting of the moment of inertia J_L and load torque T_L. Field current i_f is constant. The torque developed by the motor is T_m. The moment of inertia of the motor rotor is J_m. The viscous friction coefficient of motor is B_m. The angular displacements of the motor

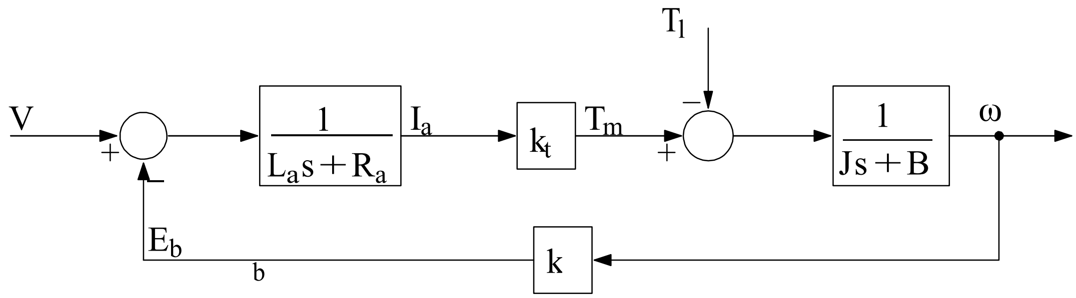


Figure 3: Block diagram of an armature-controlled dc servomotor system.

rotor and the load element are θ_m and θ_L, respectively. Draw a block diagram and obtain the transfer functions $\frac{\Theta_L(s)}{E_a(s)}$ and $\frac{\Theta_L(s)}{T_L(s)}$.

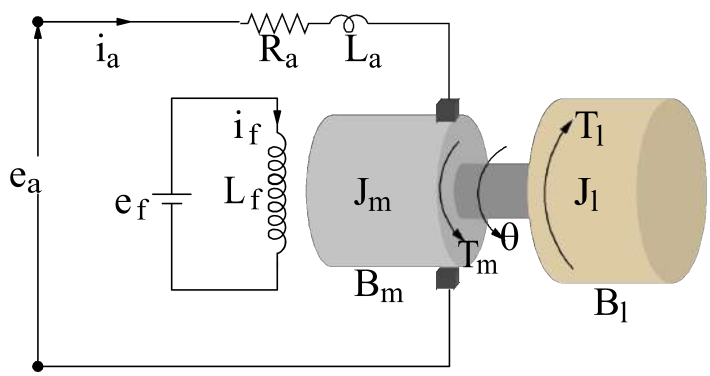


Figure 4: Armature-controlled dc servomotor system.

Q2-19: Consider the system shown in 5. An field-controlled dc servomotor drives a load consisting of the moment of inertia J_L and load torque T_L. Armature current i_a is constant. The torque developed by the motor is T_m. The moment of inertia of the motor rotor is J_m. The viscous friction coefficient of motor is B_m. The angular displacements of the motor rotor and the load element are θ_m and θ_L, respectively. The gear ratio is $n = \frac{N_1}{N_2} = \frac{\theta_L}{\theta_m}$. Draw a block diagram and obtain the transfer functions $\frac{\Theta_L(s)}{E_f(s)}$ and $\frac{\Theta_L(s)}{T_L(s)}$.

$i_f \quad R_f$

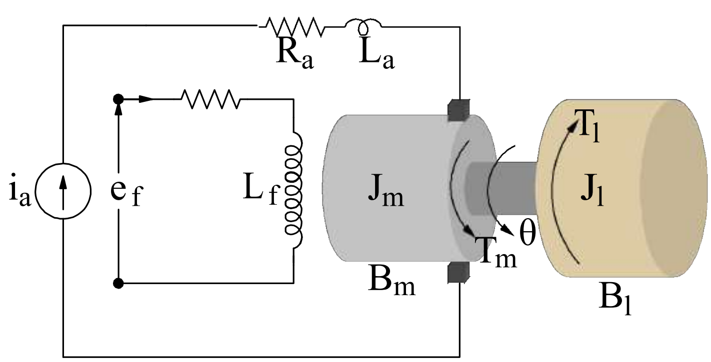


Figure 5: Field-controlled dc servomotor system.

Q2-20: Find the transfer function $\frac{X(s)}{E(s)}$ for the electro-mechanical system shown in Figure.6.

For a simplified analysis, assume that the coil has a back emf $e_b = K_b\frac{dx}{dt}$ and the coil current i produces a force $F_c = K_f i$ on the mass M.

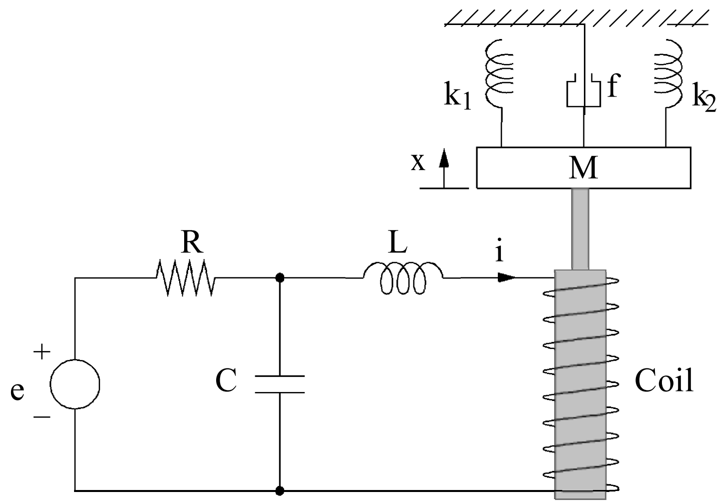


Figure 6: Electro-mechanical system.

Q2-21: Write the dynamic equations and draw a block diagram for the circuit shown in Figure. 7, also determine $\frac{V_1(s)}{E_1(s)}, \frac{V_2(s)}{E_1(s)}, \frac{V_1(s)}{E_2(s)}$ and $\frac{V_2(s)}{E_2(s)}$ by block diagram reduction technique.

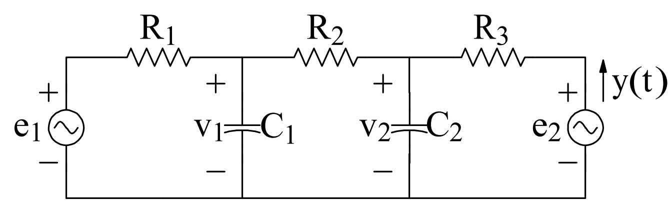


Figure 7:

Q2-22: Write the dynamic equations and draw a block diagram for the circuit shown in Figure. 8, also determine $\frac{V_1(s)}{V_s(s)}, \frac{V_2(s)}{V_s(s)}, \frac{V_1(s)}{I_s(s)}$ and $\frac{V_2(s)}{I_s(s)}$ by block diagram reduction technique.

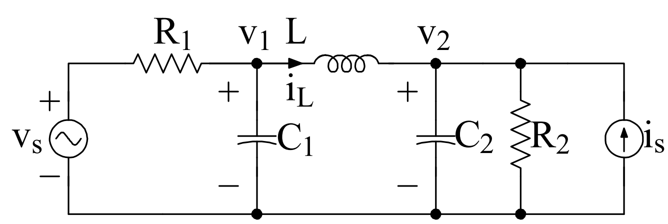


Figure 8:

Q2-23: Write the dynamic equations for the notch circuit shown in Figure. 9 and determine $\frac{E_o(s)}{E_i(s)}$. Also draw a block diagram and determine $\frac{E_o(s)}{E_i(s)}$ by block diagram reduction technique.

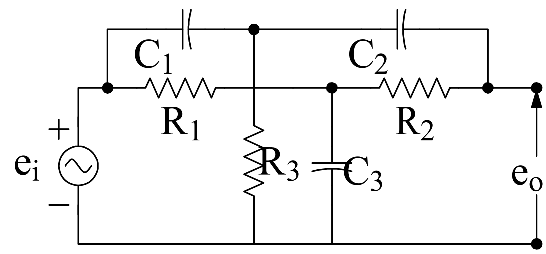


Figure 9:

Q2-24: An equivalent circuit of an electronic amplifier is shown in Figure.10. Determine its transfer function $\frac{V_{out}(s)}{V_{in}(s)}$. Take $v_{Gk} = v_G - v_K$

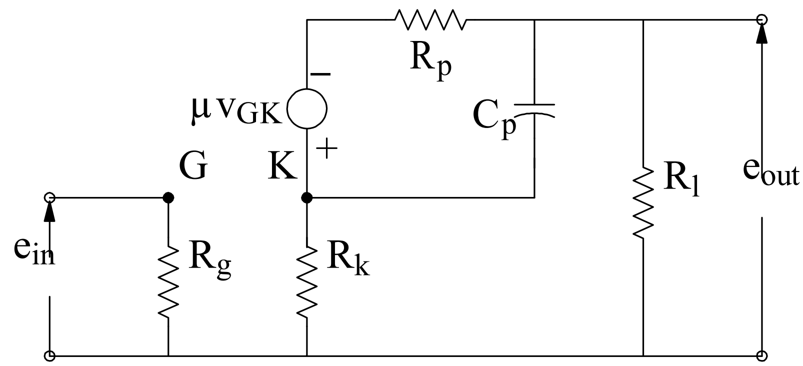


Figure 10:

Q2-25: Using block diagram reduction techniques, determine transfer functions $\frac{C}{R}$ of the system shown in Figure.11.

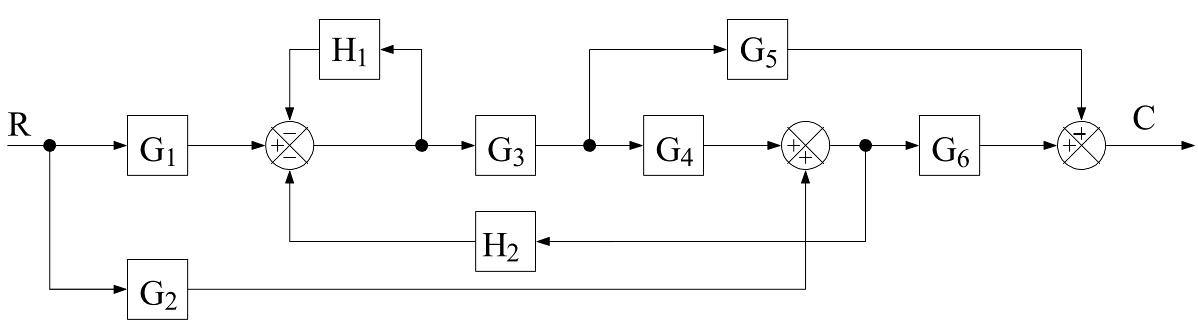


Figure 11:

Q2-26: Using block diagram reduction techniques, determine transfer functions $\frac{C}{R}$ of the system shown in Figure.12.

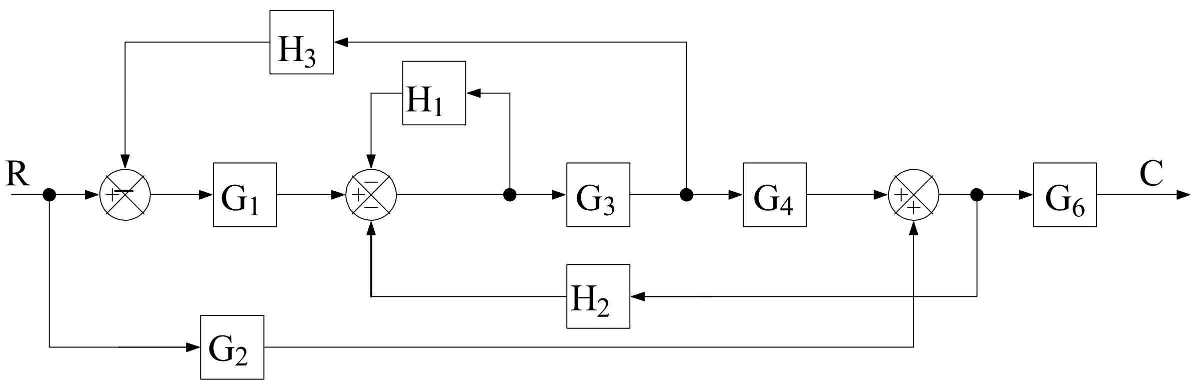


Figure 12:

Q2-27: Using block diagram reduction techniques, determine transfer functions $\frac{C}{R}$ of the system shown in Figure.13.

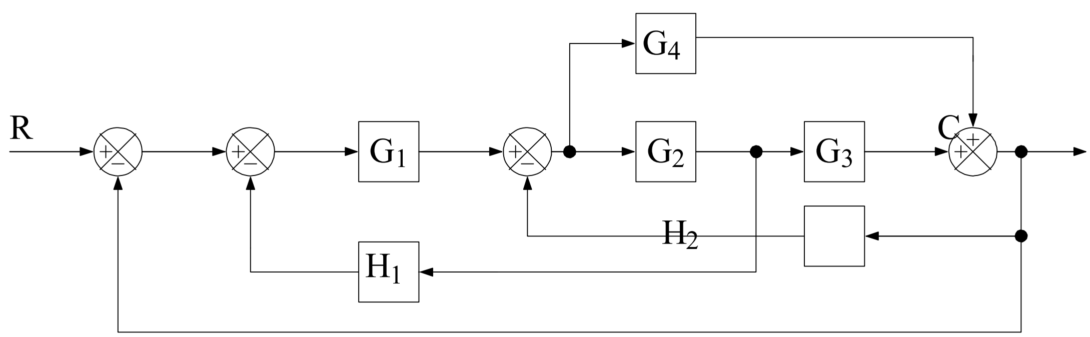


Figure 13:

Q2-28: Using block diagram reduction techniques, determine transfer functions $\frac{C_1}{R_1}$, $\frac{C_2}{R_1}$, $\frac{C_1}{R_2}$, and $\frac{C_2}{R_2}$ of the system shown in Figure.14.

Q2-29: Using block diagram reduction techniques, determine transfer functions $\frac{C}{R_1}$, and $\frac{C}{R_2}$ of the system shown in Figure.15.

Q2-30: For the signal flow graph shown in Figure 16 obtain transfer function $\frac{C(s)}{R(s)}$.

Q2-31: For the signal flow graph shown in Figure 17 obtain transfer function $\frac{C(s)}{R(s)}$.

Q2-32: For the signal flow graph shown in Figure 18 obtain transfer functions $\frac{C_1(s)}{R_1(s)}$, $\frac{C_2(s)}{R_1(s)}$, $\frac{C_1(s)}{R_2(s)}$ and $\frac{C_2(s)}{R_2(s)}$.

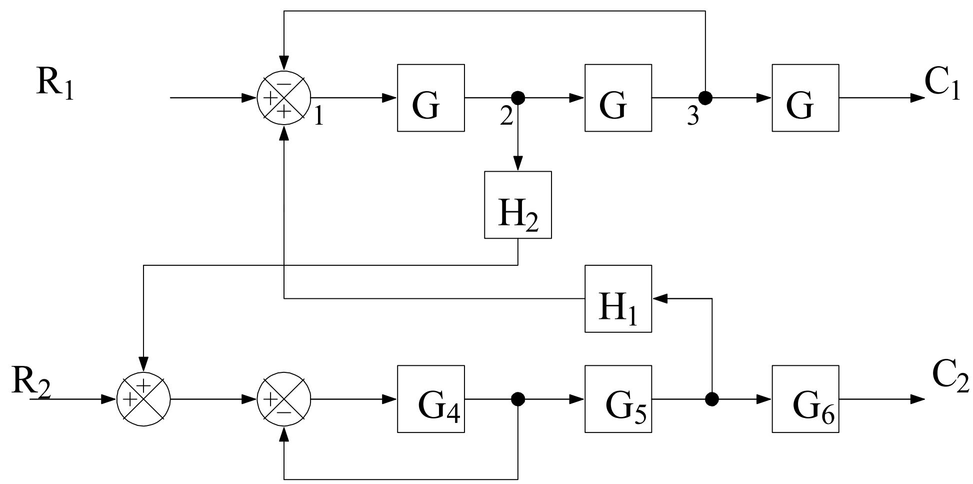


Figure 14:

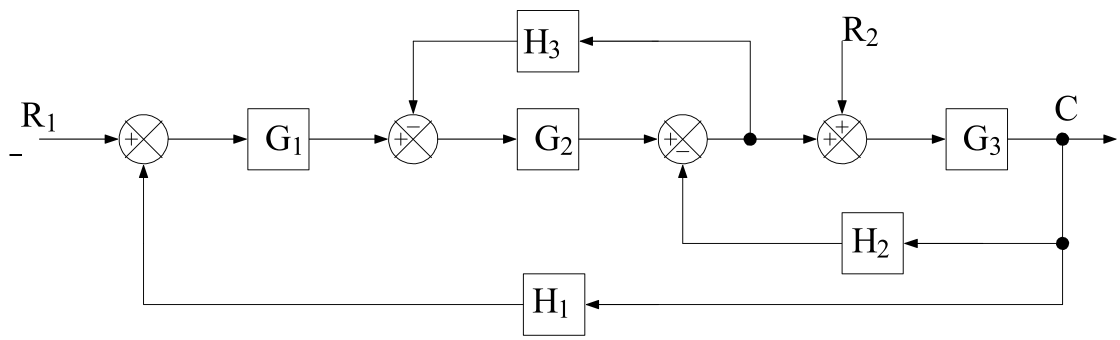


Figure 15:

Q2-33: Block diagram of a controlled system is shown in Figure 12

(i) Draw a equivalent signal flow graph.

(ii) Find the overall transfer function $\frac{C(s)}{R(s)}$ using Mason's gain formula.

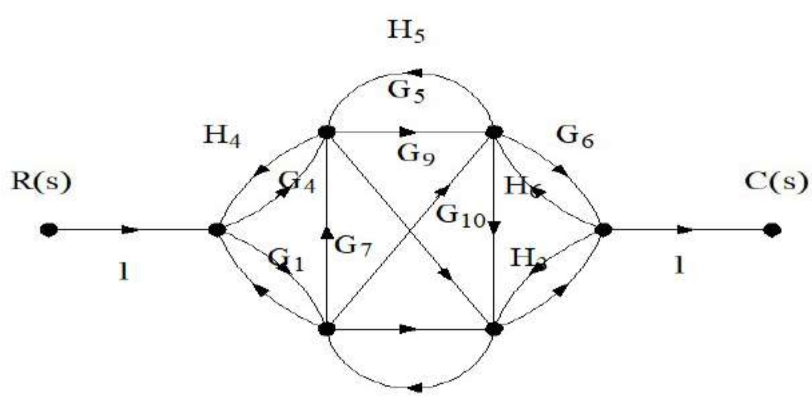


Figure 16:

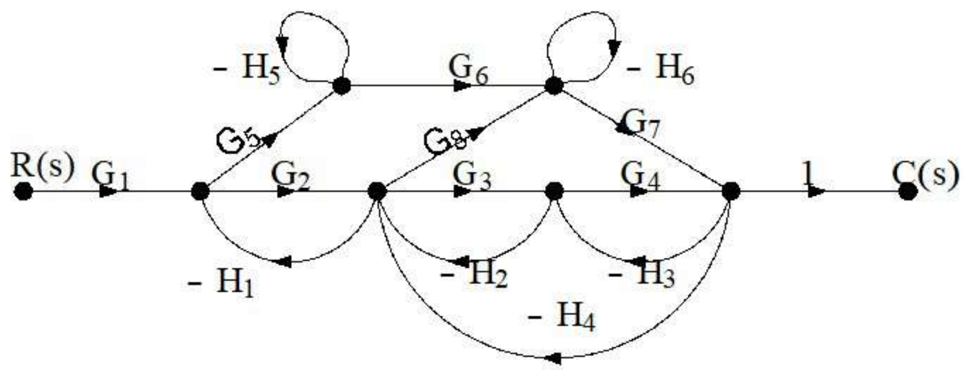


Figure 17:

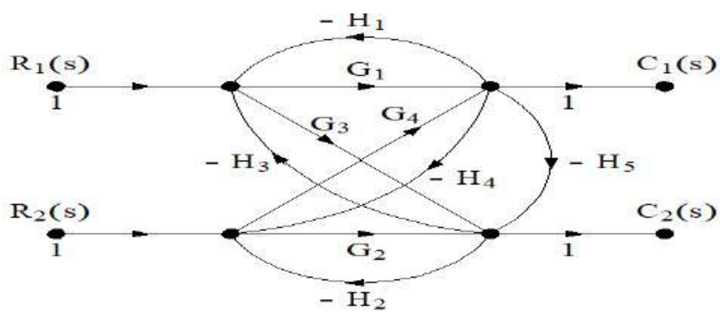


Figure 18:

MODULE-3

Syllabus: Time Response of feedback control systems: Standard test signals, Unit step response of First and Second order Systems. Time response specifications, Time response specifications of second order systems, steady state errors and error constants. Introduction to PI, PDand PID Controllers (excluding design).

Study Material Referred:

- Modern Control engineering-K Ogata.
- Control Systems Engineering- J.Nagarath and M.Gopal.
- Automatic Control Systems-Benjamin C. Kuo.
- Linear Control Systems –B S Manke.
- Control Systems- Anand Kumar.
- **VTU Previous year Question papers (2010-2016)**.
- Problems and solution of controls systems -AK Jairath.

CONTENTS

Note: 1) Assignment Carries 05 Marks (To be submitted before 2ND test).

Module 2: Time Response Of Feedback Control Systems

* → Time response is the variation of the controlled output of the system with respect to time, when the system is subjected to test signal.

or

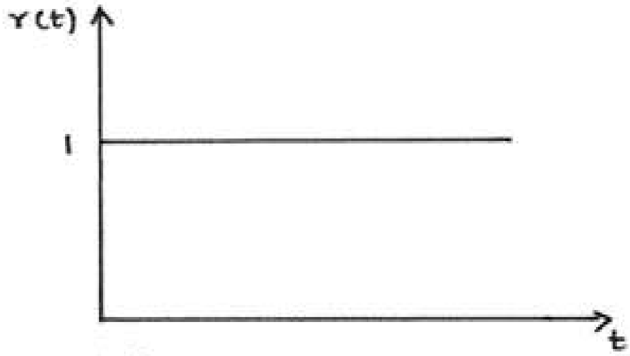
* → Time response of control system is defined as, To how a system behaves in accordance with time when a specified input test signal is applied.

* → Thus the time response of a control system is divided into two parts they are

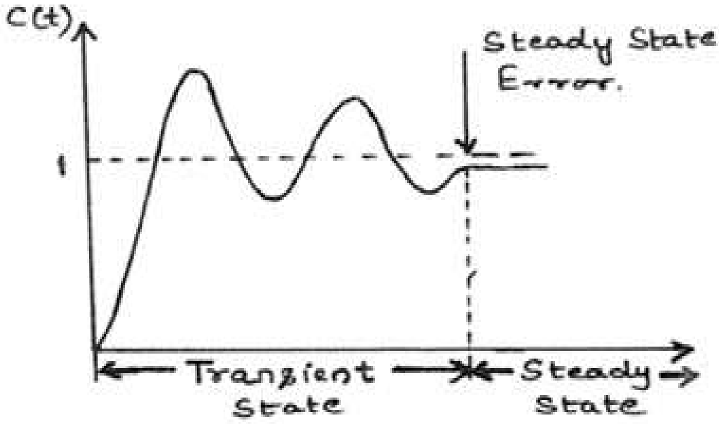
  a) Transient Response

  b) Steady State Response

* → The typi[illegible] response of a controlled system for a specified input test signal is as shown below.



(a) Input test Signal.



(b) Output response.

* → Transient response of the system is defined as the part of time response that goes to zero as time becomes very large. Thus $C_t(t)$ has property that

i.e, $\lim_{t \to \infty} C_{tr}(t) = 0$

* → The transient part of the time response reveals the nature of the response and gives an indication of speed

* → The steady state response is the part of total response that remains after the transient has died out.

* → The steady state part reveals the accuracy of control system.

* → Steady state error is observed if the actual o/p doesnot match with in the output.

* → From the fig, the o/p c(t) is given by.

$$\boxed{c(t) = C_{tr}(t) + C_{ss}(t)}$$

* → The different input signals that are used to test a control system.

1) Step input.

2) Ramp input.

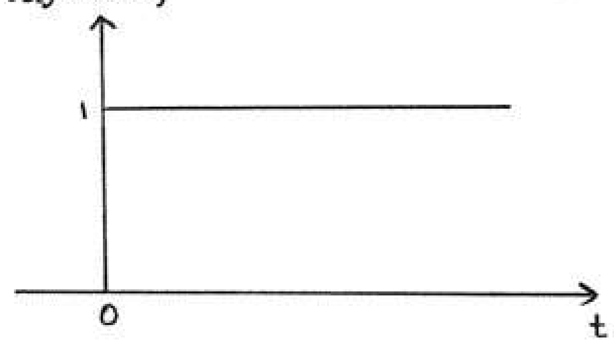
3) Parabolic input.

4) Impulse input.

5) Sinusoidal input.

1) Step Input:-

* → It is defined as $r(t) = u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$



By taking Laplace transform of step function.

$$L\{u(t)\} = \frac{1}{s}$$

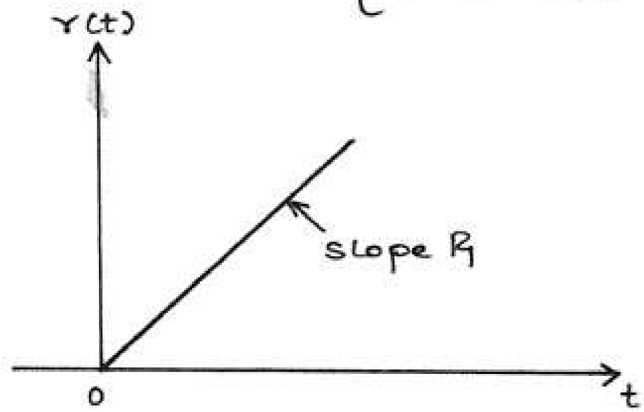
if $r(t) = A.u(t)$

Where A is the Amplitude of the step signal.

$$L[r(t)] = \frac{A}{s}$$

2) Ramp input:-

* → It is defined as $r(t) = \begin{cases} At, & t \geq 0 \\ 0, & t < 0 \end{cases}$



* → For A=1, then $r(t) = t$ and the ramp function is called unit ramp function.

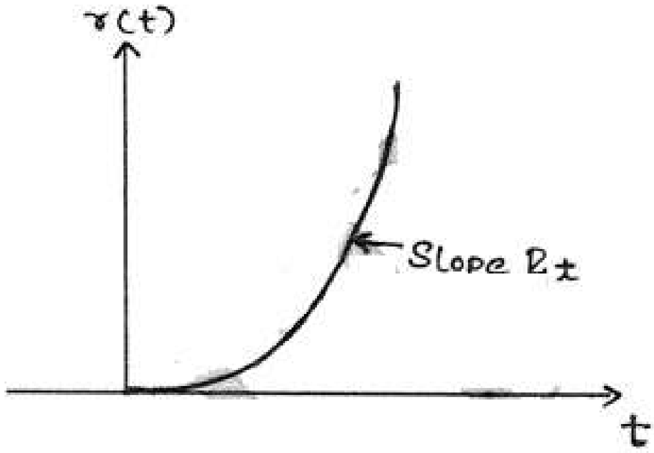
* → By taking Laplace transform

$$L[r(t)] = L[t \cdot u(t)] = \frac{1}{s^2}$$

for $r(t) = A \cdot t\, u(t)$ then $L[r(t)] = \frac{A}{s^2}$

3) Parabolic Input:-

It is defined as $r(t) = \begin{cases} At^2/2, & \text{for } t \geq 0 \\ 0, & \text{for } t < 0 \end{cases}$



*→ For $A = 1$, then $r(t) = \frac{t^2}{2}$ and the parabolic function is called unit parabolic function.

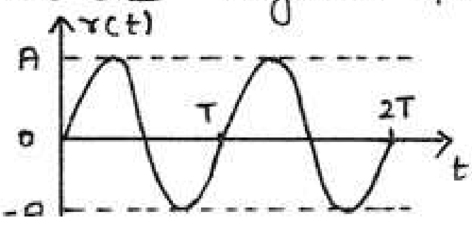
*→ By taking laplace transform.

$$L[r(t)] = L\left[\frac{t^2}{2}\right] = \frac{1}{s^3}$$

4) Sinusoidal Input:-

It is defined as $r(t) = \begin{cases} A \sin\omega t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

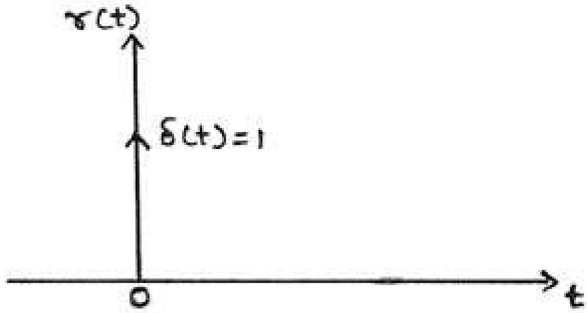
*→ Where A is the Amplitude of the sinusoidal signal and ω is the Angular frequency.



$$L\left[A \sin(\omega t)\right] = \frac{A\cdot\omega}{s^2+\omega^2}$$

5) Impulse Input:

*→ It is defined as $r(t) = \begin{cases} \delta(t) = 1, & t = 0; \\ 0, & t \neq 0; \end{cases}$



*→ By taking laplace transforms

$$L\left[r(t)\right] = L\left[\delta(t)\right] = 1$$

Note :-

1) $L\left[A\cdot u(t)\right] = \frac{A}{s}$

2) $L\left[t^n\cdot u(t)\right] = \frac{n!}{s^{n+1}}$

3) $L\left[e^{\pm at}\right] = \frac{1}{s\mp a}$

4) $L\left[\sin\omega t\right] = \frac{\omega}{s^2+\omega^2}$

5) $L\left[\cos\omega t\right] = \frac{s}{s^2+\omega^2}$

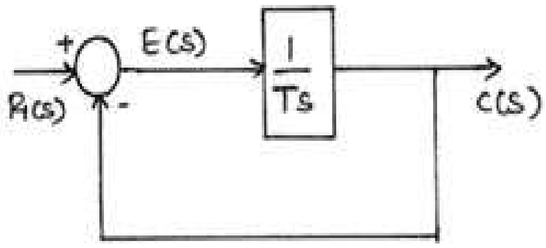
6) $L\left[e^{\pm at}\sin\omega t\right] = \frac{\omega}{(s\mp a)^2+\omega^2}$

7) $L\left[e^{\pm at}\cos\omega t\right] = \frac{(s\mp a)}{(s\mp a)^2+\omega^2}$

8) $L\left[\delta(t)\right] = 1$

Time Response Of First Order Control System.

*→ Consider the block diagram of a general first order control system is as shown below.



*→ The overall transfer function is given by.

$$\frac{C(s)}{R(s)} = \frac{1/Ts}{1+\frac{1}{Ts}\cdot 1} = \frac{1}{Ts+1}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{1}{sT+1}}$$

*→ 'T' is the time constant of the system.

*→ The highest power of 's' in the denominator of the overall transfer function represents the order of the control system.

*→ The output of the system is expressed as.

$$C(s) = R(s)\cdot\frac{1}{sT+1}$$

W.K.T that $R(s) = \frac{1}{s}$. $\because R(t) = 1,\ L[R(t)] = \frac{1}{s}$

$$\therefore\ C(s) = \frac{1}{s}\cdot\frac{1}{sT+1}$$

Breaking R.H.S into partial fractions

$$C(S) = \frac{1}{S} \cdot \frac{1}{ST+1} = \frac{A}{S} + \frac{B}{ST+1} \longrightarrow (1)$$

$$A = \lim_{S \to 0} \left[S \cdot C(S) \right] = \lim_{S \to 0} \left\{ \frac{1}{ST+1} \right\} = 1$$

$$B = \lim_{S \to -1/T} \left[(ST+1)\ C(S) \right] = \lim_{S \to -1/T} \left\{ \frac{1}{S} \right\} = -T$$

By substituting the value A and B in Equation (1)

$$C(S) = \frac{1}{S} - \frac{T}{ST+1}$$

$$C(S) = \frac{1}{S} - \frac{1}{S + \frac{1}{T}}$$

By taking inverse laplace transform

$$C(t) = 1 - e^{-t/T} \quad \text{for } t \geq 0. \longrightarrow (2)$$

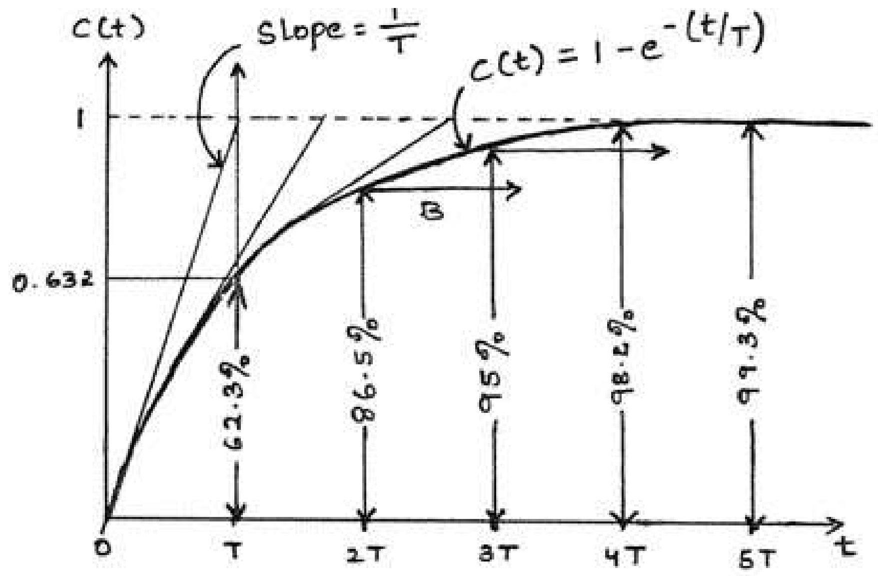
* → Equation 2 states that initially the output $C(t)$ is zero and finally it becomes unity.

* → At 't' = 'T', the value $C(t) = 0.632$, or the response $C(t)$ has reached 63.2% of its total change.

i.e. $C(T) = 1 - e^{-T/T} = 1 - e^{-1} = 0.632$

$$\boxed{C(T) = 0.632}$$

* → Note that the smaller the time constan T, the faster the system response. The response is as shown below.



*→ The slope of the tangent line at $t=0$ is $1/T$ Since

$$\left.\frac{dc}{dt}\right|_{t=0} = \left.\frac{1}{T}\,e^{-t/T}\right|_{t=0} = \frac{1}{T} \quad — (3)$$

*→ The output would reach the final value at $t=T$, if it maintained its initial speed of response. The slope of the response curve $c(t)$ monotonically from $1/T$ at $t=0$ to zero at $t=\infty$

*→ In one time constant, the exponential response curve has gone ~~from~~ 0 to 63.2% of the final value (

*→ In two time constant, the response reaches 86% of the final value. (At $t = 2T$)

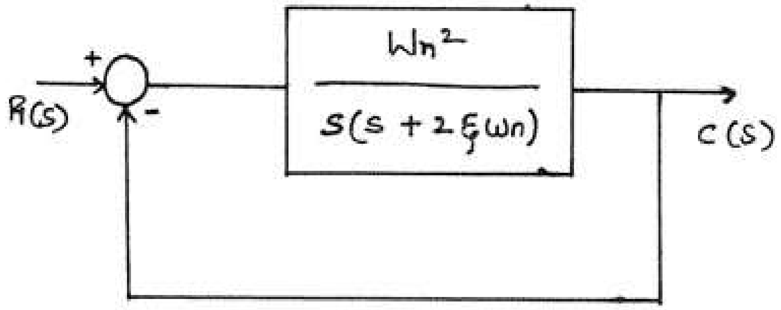
*→ At $t=3T$, $4T$ and $5T$, the response reaches 95%, 98.2% and 99.3% respectively of the final value.

*→ Thus for $t \geq 4T$, the response remains within 2% of the final value

*→ As seen from Equation (2) the steady state is reached mathematically only after an infinite time.

Time Response Of Second Order Control System

*→ The block diagram of a general second order control system is as shown below.



*→ The overall transfer function is given by.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}\cdot 1}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

*→ ω_n is known as the natural frequency.

*→ ξ is the damping ratio of the system.

*→ If $\xi = 0$, the system is undamped.

*→ If $0 < \xi < 1$, the system is under damped.

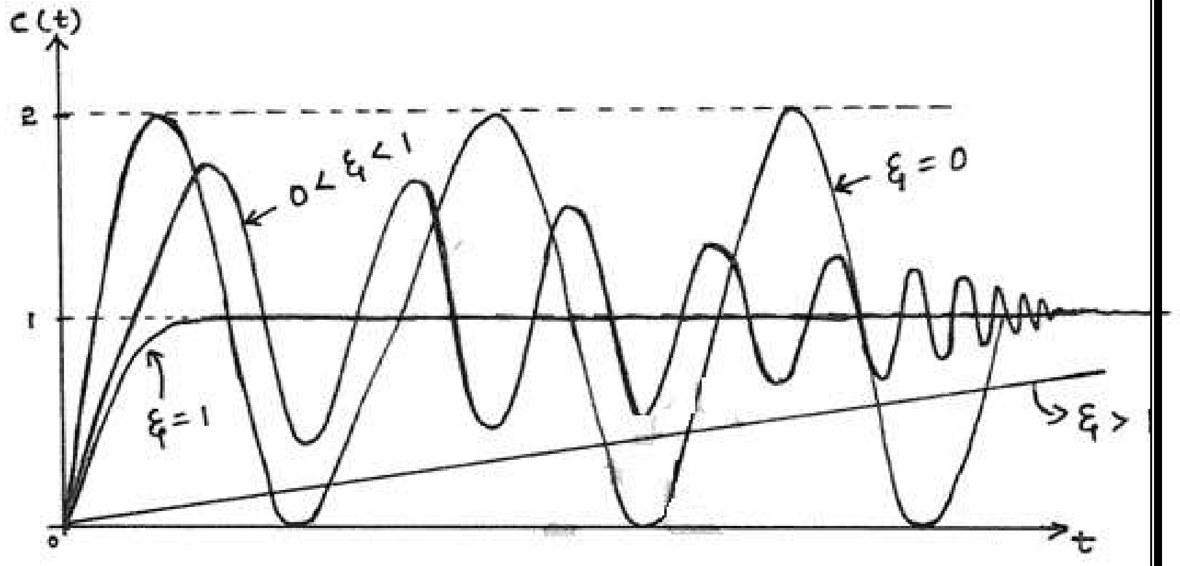
*→ If $\xi = 1$, the system is critically damped.

*→ If $\xi > 1$, the system is over damped.

*→ Denominator roots of $\frac{C(s)}{R(s)}$ are known as the poles of the closed loop transfer function and.

*→ Numerator roots of $\frac{C(s)}{R(s)}$ are known as the zero's of the closed loop transfer function.

* → The time response of a second order control system subjected to a unit step input for different values of damping ratio is as shown below.



* → The time response of any system is characterized by the roots of the denominator polynomial, which in fact are the poles of the transfer function.

* → The denominator polynomial is therefore called the characteristic polynomial and it is called the characteristic equation.

* → i.e $1+G(s)H(s)$ is known as characteristic equation.

$$1+G(s)H(s) = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} \qquad \because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= -\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}$$

$$s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

* → When $\xi=0$ poles are present on imaginary axis of plane.

* → When $\xi = 1$ Poles of the system are Real and Equal

* → When $\xi > 1$ Poles of the system are Real and UnEqual.

* → When ξ lies between 0 & 1 Poles of the system are Complex.

* → for $0 < \xi < 1$ $s = -\xi \omega_n \pm j \omega_n \sqrt{1-\xi^2}$

$$\boxed{s = -\xi \omega_n \pm j \omega_d} \qquad \text{Note:- } \boxed{\omega_d = \omega_n \sqrt{1-\xi^2}}$$

Where $\xi \omega_n$ is known as the damping factor of the System. This terms decides the rate of delay of the transcient response.

ω_d is known as damped frequency of Oscillation. It is the Angular frequency of the transcient response.

⇒ Response of an under damped System:- $(0 < \xi < 1)$

* → Response of a second Order Control system Subjected to a unit step input. The Overall transfer function of a Second Order Control system is given by.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} \cdot R(s) \quad \rightarrow (1)$$

W.K.T $R(s)$ = Unit step input.

i.e $R(s) = \frac{1}{s}$

By substituting the value of $R(s)$ in (1)

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$C(S) = \frac{\omega_n^2}{S(S^2 + 2\xi\omega_n S + \omega_n^2)}$$

Breaking R.H.S in to partial Fractions.

$$C(S) = \frac{\omega_n^2}{S(S^2+2\xi\omega_n S+\omega_n^2)} = \frac{K_1}{S} + \frac{K_2 S + K_3}{S^2 + 2\xi\omega_n S + \omega_n^2} \rightarrow ②$$

$$\omega_n^2 = K_1(S^2 + 2\xi\omega_n S + \omega_n^2) + (K_2 S + K_3)S$$

Put $S = 0$; $\omega_n^2 = K_1 \omega_n^2$; $K_1 = \frac{\omega_n^2}{\omega_n^2}$ $\boxed{K_1 = 1}$

By Comparing the co-efficients of 'S^2'

S^2 ; $0 = K_1 + K_2$ or $K_2 = -K_1$

$$\boxed{K_2 = -1}$$

By Comparing the co-efficients of 'S'

S: $0 = 2\xi\omega_n K_1 + K_3$

$K_3 = -2\xi\omega_n K_1$ $\because \boxed{K_1 = 1}$

$$\boxed{K_3 = -2\xi\omega_n}$$

By substituting the value of K_1, K_2 and K_3 in ②

$$C(S) = \frac{1}{S} + \frac{-S - 2\xi\omega_n}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$= \frac{1}{S} - \frac{S + 2\xi\omega_n}{\underbrace{S^2 + 2\xi\omega_n \cdot S + (\xi\omega_n)^2}_{(a+b)^2} - (\xi\omega_n)^2 + \omega_n^2}$$

$$C(S) = \frac{1}{S} - \frac{(S + \xi\omega_n) + \xi\omega_n}{(S + \xi\omega_n)^2 + \omega_n^2 - \xi^2\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n) + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n) + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

We know that $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\therefore \omega_d^2 = \omega_n^2(1-\xi^2)$$

$$C(s) = \frac{1}{s} - \left[\frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2 + \omega_d^2} + \frac{\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}\right]$$

By multiplying ω_d for both numerator and denominator of 3rd term

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n \cdot \omega_d}{\omega_d(s+\xi\omega_n)^2+\omega_d^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \left(\frac{\xi\omega_n}{\omega_d}\right)\left(\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}\right)$$

By taking inverse laplace transforms.

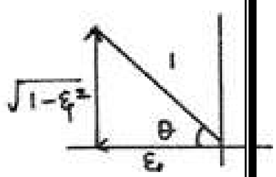
$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$$

By taking $\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$ common

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\cos\omega_d t\sqrt{1-\xi^2} + \xi\sin\omega_d t\right]$$

Put $\sin\theta = \sqrt{1-\xi^2}$, $\therefore \cos\theta = \xi$; Note:



By Substituting we get

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\theta \cos\omega_d t + \cos\theta \sin\omega_d t\right]$$

$$\boxed{C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \theta)\right]} \quad \text{for } t \geq 0 \quad \rightarrow (3)$$

*→ If the System to a step input of strength 'A' units, then the time response is given by.

$$\boxed{C(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \theta)\right]\right]}$$

*→ Since $\omega_d = \omega_n\sqrt{1-\xi^2}$ and $\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

*→ By Substituting in Equation (3), Equation is rewritte as

$$\boxed{C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]}$$

*→ The Error signal for the system is the difference between the input and the output and it is given by

$$e(t) = r(t) - C(t) \qquad \text{W.K.T } r(t) = 1$$

$$e(t) = 1 - \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]\right]$$

or

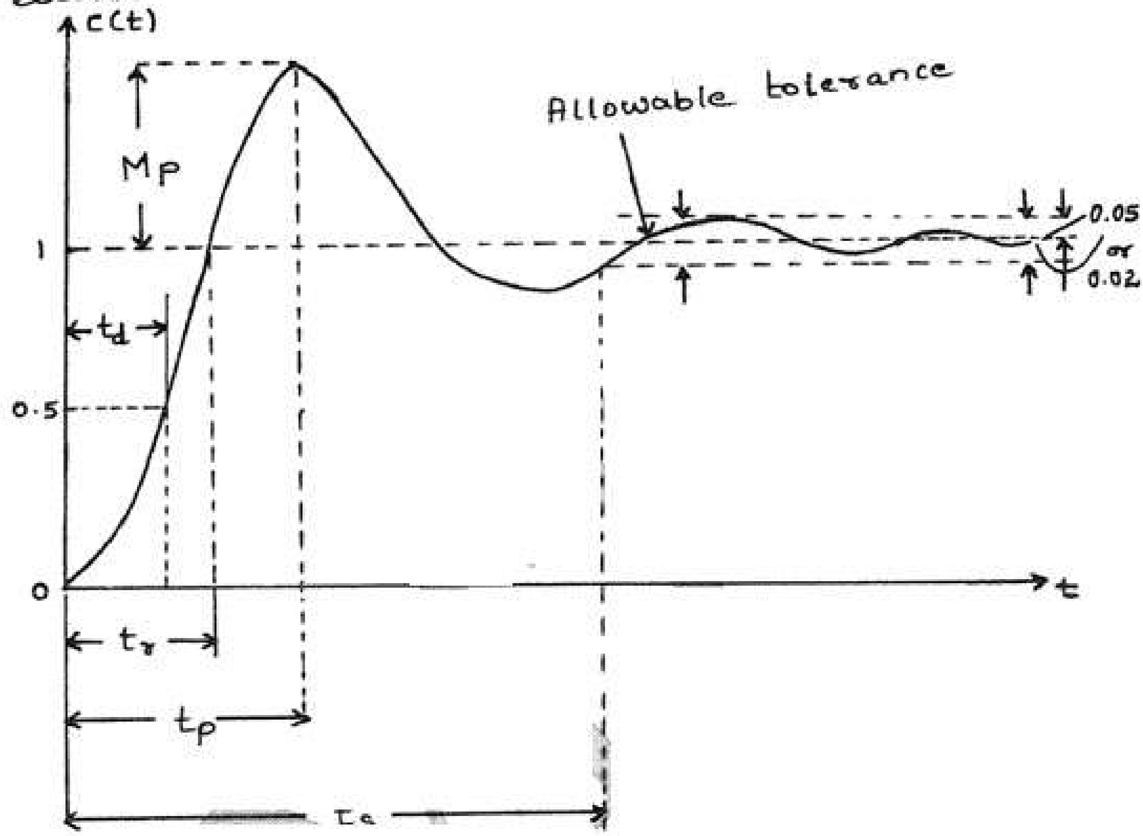
$$e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

The steady state Error :-

$$e_{ss} = \lim_{t\to\infty} \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

Time Response Specifications

* → The time response of a second order Underdamped Control System subjected to a Unit step input, is as shown below.



* → The time response specifications are

i) Rise time, (t_r)

ii) Delay time, (t_d)

iii) Peak time, (t_p)

iv) Maximum peak Overshoot, (M_p)

v) Settling time, (t_s)

i) Rise time : t_r

* Rise time is the time required for the response to rise from 0% to 100% of its final value at the very first instant.

2) Delay time : t_d

* The delay time is the time required for the response to reach 50% of the final value at the Very first time.

3) Peak time : t_p

* The peak time is the time required for the response to reach the first peak of the overshoot.

4) Maximum Overshoot : M_p

* The maximum peak overshoot is the maximum peak value of the response curve measured from unity.

* If the final steady-state value of the response differs from unity, then it is common to use to. maximum percent overshoot, It is defined by

Peak Over shoot : $\boxed{M_p = c(t_p) - c(\infty)}$

Maximum Percent Overshoot : $\boxed{\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100}$

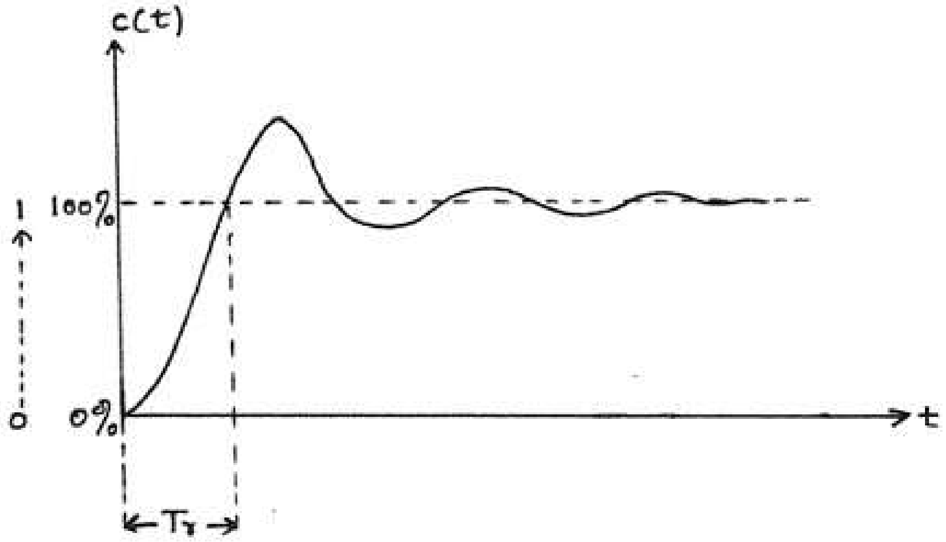
* The amount of the maximum (percent) Overshoot directly indicates the relative stability of the system.

5). Settling time : t_s

* The Settling time is the time required for the response curve to reach and stay within specified range of its final value (Within tolerance band, usually 2% or 5%).

Expression for Rise Time:

We know that, the Rise time is the time required to reach from 0% to 100% of it's final value.



* → The response of a second order control system subjected to a unit step input is given by

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \rightarrow ①$$

* → From the figure above At the risetime tr.

$$c(t) = c(t_r) = 1$$

* → By substituting in Equation ① we get

$$1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) \rightarrow ② \quad \text{W.K.T } t = t_r$$

* → Equation ② becomes

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0 \longrightarrow ③$$

* → Since in Equation ③

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \neq 0 \quad \because \text{ the time 'tr' is finite.}$$

* → $\sin(\omega_d t_r + \theta)$ must be Equal to zero.

$$\therefore \sin(\omega_d t_r + \theta) = 0$$

$$\omega_d t_r + \theta = n\pi \; ; n = 1, 2, 3 \ldots$$

$$\omega_d t_r = n\pi - \theta$$

For the Ist time $n = 1$

$$\omega_d t_r = \pi - \theta$$

Therefore, the rise time "t_r" is given by

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}} \rightarrow (4)$$

W.K.T $\theta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$

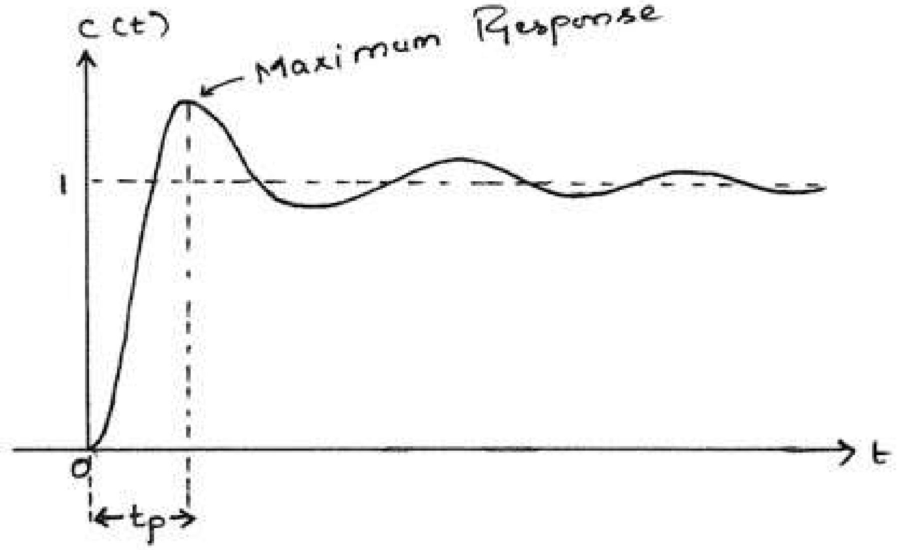
$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

* → By substituting in Equation (4) we get.

$$\boxed{t_r = \frac{\pi - \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n \sqrt{1-\xi^2}}}$$

Expression For Peak Time:-

* → We know that, Peak Time is the time required for the response to reach the first peak of the Overshoot.



* → The Output response of a Second Order Control System Subjected to a unit step input is given by.

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow ①$$

* → From the above figure, at the time $t = t_p$, the slope of c(t) must be zero. There fore. $\frac{d\,c(t)}{dt} = 0$

$$\left.\frac{d\,c(t)}{dt}\right|_{t=t_p} = 0 - \left[\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}(-\xi\omega_n)\sin(\omega_d t_p + \theta) + \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cos(\omega_d t_p + \theta)\,\omega_d\right] = 0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[(\xi\omega_n)\sin(\omega_d t_p + \theta) + (\omega_d)\cos(\omega_d t_p + \theta)\right] = 0$$

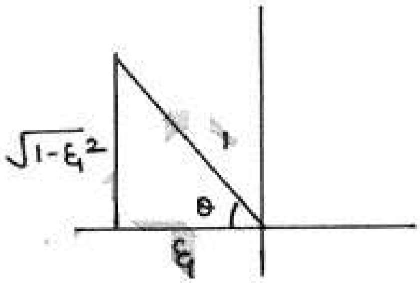
W.K.T $\quad W_d = W_n\sqrt{1-\xi^2}$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\left[(\xi\omega_n)\sin(\omega_d t_p+\theta) - \omega_n\sqrt{1-\xi^2}\,\cos(\omega_d t_p+\theta)\right] = 0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cdot\omega_n\left[\xi\,\sin(\omega_d t_p+\theta) - \sqrt{1-\xi^2}\,\cos(\omega_d t_p+\theta)\right] = 0$$

But $\cos\theta = \xi$

$\sin\theta = \sqrt{1-\xi^2}$



$\therefore$ By substituting we get

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cdot\omega_n\left[\underbrace{\underbrace{\cos\theta}_{\cos B}\ \underbrace{\sin(\omega_d t_p+\theta)}_{\sin A} - \underbrace{\sin\theta}_{\sin B}\ \underbrace{\cos(\omega_d t_p+\theta)}_{\cos A}}_{\sin(A-B)}\right] = 0$$

$$\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cdot\omega_n\ \sin(\omega_d t_p+\cancel{\theta}-\cancel{\theta}) = 0$$

$$\therefore\ \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\cdot\omega_n\ \sin(\omega_d t_p) = 0$$

$$e^{-\xi\omega_n t_p}\cdot\omega_n\sin(\omega_d t_p) = 0$$

W.K.T $\quad e^{-\xi\omega_n t_p} \neq 0 \quad \because$ 't_p' is finite

$$\therefore\ \sin(\omega_d t_p) = 0$$

$$\omega_d t_p = n\pi\ ;\ n = 1, 2, 3 \ldots\ldots$$

For the first peak Over shoot $n=1$

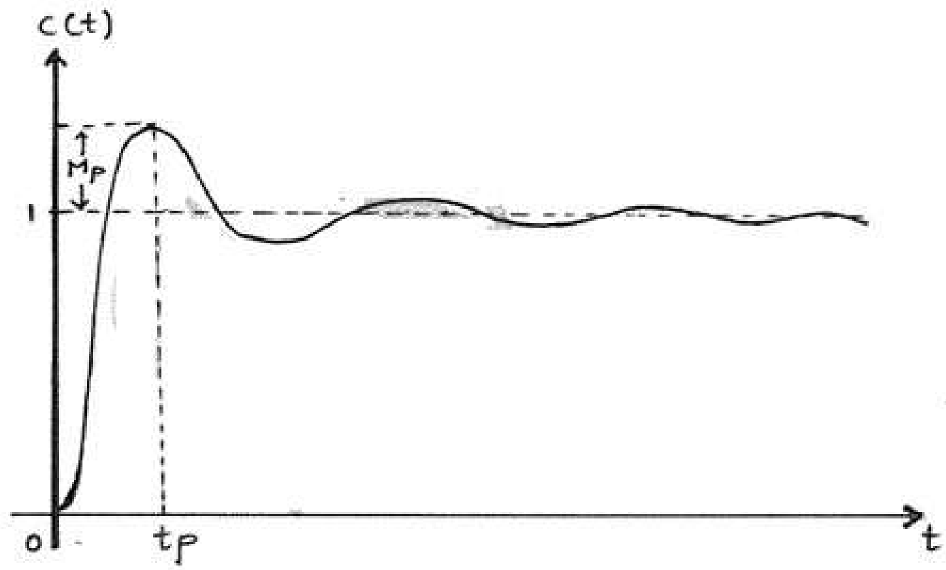
$$\therefore \omega_d t_p = \pi$$

$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

* To find time Corresponding to first under shoot, Substitu $n=2$ i.e $t_p = \frac{2\pi}{\omega_d}$.

* To fime time Corresponding to second Over shoot, Substit $n=3$ i.e $t_p = \frac{3\pi}{\omega_d}$.

* Expression for Maximum Peak Overshoot (MP):



* → We know that, the peak overshoot is the difference between the peak value and the final value.

* → The output response of a second order Control System Subjected to a Unit step input is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow (1)$$

*→ By substituting $t = t_p$ in Equation ①, we get the peak response or maximum response.

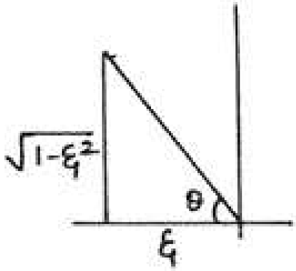
$$\therefore \text{Maximum response } c(t_p) = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$$c(t_p) = 1 - \frac{e^{-\xi \cancel{\omega_n} \frac{\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\cancel{\omega_d} \cdot \frac{\pi}{\cancel{\omega_d}} + \theta\right) \quad \because t_p = \frac{\pi}{\omega_d} \text{ \& } \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad c(t_p) = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sin(\pi + \theta)$$

$$\sin(\pi+\theta) = -\sin\theta$$

$$\therefore \quad c(t_p) = 1 - \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\cancel{\sqrt{1-\xi^2}}} \cdot (-\cancel{\sin\theta}) \quad \because \sin\theta = \sqrt{1-\xi^2}$$



$$\text{i.e} \quad \boxed{c(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

*→ We know that

Peak Over shoot is given by

$$M_p = c(t_p) - c(\infty)$$

$$M_p = \cancel{1} + e^{-\xi\pi/\sqrt{1-\xi^2}} - \cancel{1} \quad \because c(\infty) = 1 \text{ (final value)}$$

$$\boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

*→ % peak Over shoot is given by

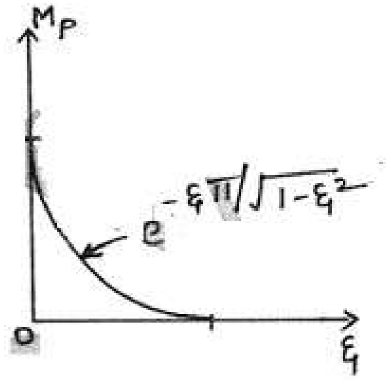
$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

As, in this case the $c(\infty) = 1$ % peak Over shoot is given by

$$\% M_p = c(t_p) * 100$$

$$\therefore \boxed{\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100}$$

* The variation of peak Over shoot w.r.t damping ratio is as shown below.



* Note:- If the system is subjected to a step input of strength 'A' unit then the peak response is given by $c(t_p) = A\left(1+e^{-\xi\pi/\sqrt{1-\xi^2}}\right)$

* Peak Over shoot $M_p = c(t_p) - c(\infty)$

$$M_p = A\left(1+e^{-\xi\pi/\sqrt{1-\xi^2}}\right) - A$$

$$M_p = \cancel{A} + Ae^{-\xi\pi/\sqrt{1-\xi^2}} - \cancel{A}$$

$$\boxed{M_p = Ae^{-\xi\pi/\sqrt{1-\xi^2}}}$$

% peak Overshoot:

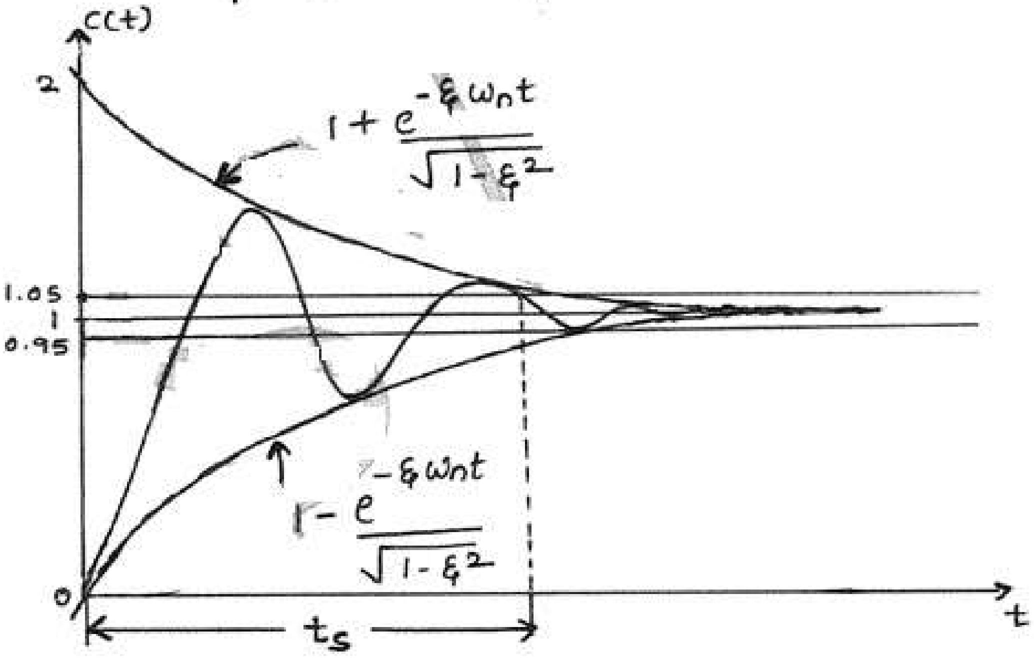
$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} * 100$$

$$= \frac{\cancel{A} e^{-\xi\pi/\sqrt{1-\xi^2}}}{\cancel{A}} * 100$$

$$\boxed{\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100}$$

Expression for Settling time:

* → we know that, the settling time is the time required for the response curve to reach and stay within specified range of it's final value.



* The Output response of a second Order System Subjected to a unit step input is given by.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \longrightarrow (1)$$

* → The Envelop of time response is as shown above and the Envelop time response is given by

Upper bound
$$1 \pm \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}}$$
Lower bound

* → From the response the Upper bound is given by

$$1 + \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} = 1.05$$

* → From the response the lower bound is given by

$$1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} = 0.95$$

* → At the settling time t_s for 5% tollerance (In Both Cases we get,

$$\frac{e^{-\xi \omega_n t_s}}{\sqrt{1-\xi^2}} = 0.05$$

$$e^{-\xi \omega_n t_s} = 0.05 \cdot \sqrt{1-\xi^2}$$

* → By taking natural log on both sides.

$$\ln\{e^{-\xi \omega_n t_s}\} = \ln\left\{0.05 \cdot \sqrt{1-\xi^2}\right\}$$

Note: $\ln\{x^y\} = y \cdot \ln(x)$

$\ln\{x \cdot y\} = \ln(x) + \ln(y)$

$$-\xi \omega_n t_s = \left\{\ln 0.05 + \ln\sqrt{1-\xi^2}\right\}$$

$$t_s = \frac{-1}{\ldots}\left\{-2.99 + \ln\sqrt{1-\xi^2}\right\}$$

$$t_s = \frac{-1}{\xi\omega_n}\{-2.99\} \qquad \because \ln\sqrt{\leq 0} = \infty$$

* → Approximate 5% settling time.

$$\boxed{t_s = \frac{3}{\xi\omega_n}}$$

* → Similarly for 2% Settling time.

$$t_s = \frac{1}{\xi\omega_n}\left\{\ln 0.02 + \ln\sqrt{1-\xi^2}\right\}$$

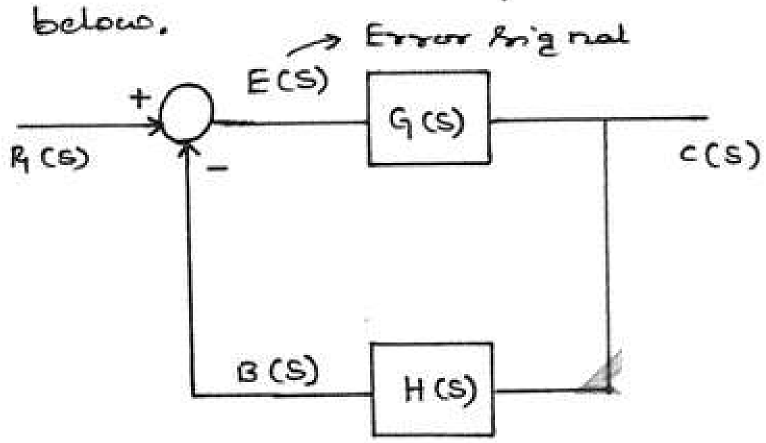
$$t_s = \frac{1}{\xi\omega_n}\{\ln 0.02\}$$

$$\boxed{t_s = \frac{4}{\xi\omega_n}}$$

Steady State Errors and Error Constants

=> Steady State Errors

*→ Consider a closed loop system as shown in the figure below.



*→ Steady state Error is the difference between the actual output and the desired output.

*→ In Time domain : The steady state Error is the value of the Error Signal as $t \to \infty$

$$\therefore \text{Steady State Error (SSE)} = \lim_{t \to \infty} e(t) \to ①$$

Using final value theorem Equation① can be written as

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(s)$$

$$\therefore \boxed{SSE = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(s)} \to ②$$

*→ From the block diagram the Error signal E(S) is given by.

$$E(S) = R(S) - B(S) \to ③$$

where, E(S) = Error signal.

R(S) = Input signal.

B(S) = Feedback signal.

$*\rightarrow$ From the definition of transfer function W.K.T

$$H(S) = \frac{B(S)}{C(S)} \quad \text{or} \quad B(S) = C(S)\,H(S)$$

$*\rightarrow$ Substituting for $B(S)$ in ③ we get

$$E(S) = R(S) - C(S) \cdot H(S)$$

From Fig ; $\quad C(S) = E(S)\,G(S)$

$$\therefore \; E(S) = R(S) - E(S)\,G(S)\,H(S)$$

$$E(S) + E(S)\,G(S)\,H(S) = R(S)$$

$$E(S)\left[1 + G(S)\,H(S)\right] = R(S)$$

$$\therefore \; E(S) = \frac{R(S)}{1 + G(S)\,H(S)} \rightarrow ④$$

If the system is unity feedback

$$E(S) = \frac{R(S)}{1 + G(S)} \rightarrow ⑤$$

Error T.F is given by

$$\frac{E(S)}{R(S)} = \frac{1}{1 + G(S)\,H(S)}$$

Steady state Error is given by $= \lim\limits_{S \to 0} S.\,E(S)$

$*\rightarrow$ By substituting Equation ④ for $E(S)$, we get

$$\boxed{SSE = \lim_{S \to 0} = S.\,\frac{R(S)}{1 + G(S)\,H(S)}}$$

Error Constants

*→ There are two types of Error Constants they are.

a) Static Error Constants
b) Dynamic Errors Constants.

a) Static Error Constants are classified in to 3 types

* Position Error Constant. (K_p)
* Velocity Error Constant. (K_v)
* Acceleration Error Constant. (K_a)

*→ Position Error Constant :- It is defined when the System is Subjected to a step input.

W.K.T, the Steady State Error is given by

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

$$\text{But } E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s)H(s)}$$

But $R(s)$ is a unit step input $\boxed{R(s) = \frac{1}{s}}$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{1}{\cancel{s}}}{1 + G(s)H(s)}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \to 0} G(s)H(s)}$$

By defining position Error Constant as

$$K_p = \lim_{s \to 0} G(s)\,H(s)$$

$$\therefore \text{SSE} \quad \boxed{e_{ss} = \frac{1}{1+K_p}}$$

* → If the system is subjected to a step i/p of Strength 'A' units.

i.e $r(t) = A\,u(t)$ then $R(s) = \frac{A}{s}$

$$\therefore \text{SSE}, \boxed{e_{ss} = \frac{A}{1+K_p}}$$

* Velocity Error Constant :- It is defined when the system is subjected to ramp input or Velocity input.

The Steady state Error is given by.

$$\text{SSE}, \quad e_{ss} = \lim_{s \to 0} s.\,E(s)$$

$$e_{ss} = \lim_{s \to 0} s.\,\frac{R(s)}{1+G(s)H(s)}$$

But $R(s)$ is a unit ramp input, i.e, $\boxed{R(s) = \frac{1}{s^2}}$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{\cancel{s}\cdot\frac{1}{s^{\cancel{2}}}}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s + s\,G(s)\,H(s)}$$

$$e_{ss} = \frac{1}{0 + \lim_{s \to 0} s.G(s)H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s.G(s)\,H(s)}$$

*→ By defining Velocity Error Constant as

$$K_v = \lim_{s \to 0} s \cdot G(s)\,H(s)$$

$$\therefore \text{ SSE; } \boxed{e_{ss} = \frac{1}{K_v}}$$

*→ If the system is subjected to ramp input of strength 'A' units.

i.e, $r(t) = A\,t\,u(t)$ then $R(s) = \frac{A}{s^2}$

$$\therefore \text{ SSE; } \boxed{e_{ss} = \frac{A}{K_v}}$$

*→ Acceleration Error Constant: It is defined when the system is subjected to a parabolic input.

The steady state error is given by.

$$\text{SSE, } e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)\,H(s)}$$

But $R(s)$ is a parabolic input, i.e $\boxed{R(s) = \frac{1}{s^3}}$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{1}{s^{\cancel{3}}}}{1+G(s)\,H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)\,H(s)}$$

$$e_{ss} = \frac{1}{0 + \lim_{s \to 0} s^2 G(s)\,H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 G(s)\,H(s)}$$

By defining acceleration Error Constant as

$$K_a = \lim_{s \to 0} s^2 G(S) H(S)$$

$\therefore$ SSE, $\boxed{e_{ss} = \frac{1}{K_a}}$

* If the System is subjected to a parabolic input of strength 'A' units.

i.e $r(t) = A \frac{t^2}{2}$ then $R(S) = \frac{A}{s^3}$

$\therefore$ SSE, $\boxed{e_{ss} = \frac{A}{K_a}}$

Note :-

$$K_p = \lim_{s\to 0} G(s)H(s) \; ; \; SSE = \frac{1}{1+K_p}$$

$$K_v = \lim_{s\to 0} s.G(s)H(s) \; ; \; SSE = \frac{1}{K_v}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) \; ; \; SSE = \frac{1}{K_a}$$

Effects of Change in G(s) H(s) on Steady state Error

*→ The Number of poles of G(s)H(s) at the origin of the s-plane gives the type of system. It is given by

$$G(s)H(s) = \frac{K(1+sT_{z1})(1+sT_{z2})----}{s^n(1+sT_{p1})(1+sT_{p2})---}$$

'n' is the type of the system

The above Equation is also called as Time Constant form.

*→ Control system are therefore classified in accordance with the number of integrations in the open-loop transfer function G(s) H(s), as

Type - 0 System ($n=0$, no integration, i.e. no pole of G(s) H(s) at the origin of s-plane)

Type - 1 System ($n=1$, one integration, i.e, one pole of G(s) H(s) at the origin of s-plane)

Type - 2 System ($n=2$, two integrations, i.e two poles of G(s) H(s) at the origin of s-plane) and so on

Steady State Error : Type 0 System :-

For a type '0' System

Consider

$$G(s)\,H(s) = \frac{K(1+sT_{z1})(1+sT_{z2})}{(1+sT_{p1})(1+sT_{p2})}$$ Note :- $n=0$, $s^n = 1$

* → We know that, the position Error constant is given by,

$$K_p = \lim_{s\to 0} G(s)\,H(s)$$

$$\therefore K_p = \lim_{s\to 0} \frac{K(1+sT_{z1})(1+sT_{z2})}{(1+sT_{p1})(1+sT_{p2})} = K$$

Steady state Error (position) is given by

$$e_{ss}\,(\text{position}) = \frac{1}{1+K_p} = \frac{1}{1+K} = \text{finite value}$$

* → |||ly, W.K.T the velocity Error Constant is given by

$$K_v = \lim_{s\to 0} s.\,G(s)\,H(s)$$

$$\therefore K_v = \lim_{s\to 0} s.\frac{K(1+sT_{z1})(1+sT_{z2})}{(1+sT_{p1})(1+sT_{p2})} = 0$$

Steady state Error (velocity) is given by

$$e_{ss}\,(\text{velocity}) = \frac{1}{K_v} = \frac{1}{0} = \infty$$

* III^ly W.K.T Acceleration Error Constant is given by

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$\therefore K_a = \lim_{s \to 0} s^2 \cdot \frac{K(1+sT_{z1})(1+sT_{z2})}{(1+sT_{p1})(1+sT_{p2})} = 0$$

Steady state Error (Acceleration) is given by

$$e_{ss}(\text{Acceleration}) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Steady State Error : Type 1 System

For a Type 1 System

$$G(s)H(s) = \frac{K(1+sT_{z1})(1+sT_{z2})}{s^1(1+sT_{p1})(1+sT_{p2})}$$

Note:
$n=1$
$s^n = s^1$

* → We Know that, the position Error Constant is given by,

$$K_p = \lim_{s \to 0} G(s) H(s)$$

$$\therefore K_p = \lim_{s \to 0} \frac{K(1+sT_{z1})(1+sT_{z2})}{s(1+sT_{p1})(1+sT_{p2})} = \frac{K}{0} = \infty$$

Steady state Error (position) is given by

$$e_{ss}(\text{position}) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

* → $|||^{ly}$ W.K.T the velocity Error Constant is given by.

$$K_v = \lim_{s\to 0} S.G(s)H(s)$$

$$\therefore K_v = \lim_{s\to 0} \frac{s \cdot K(1+sT_{z1})(1+sT_{z2})}{s(1+sT_{P1})(1+sT_{P2})} = K$$

Steady state Error (velocity) is given by

$$e_{ss}(\text{velocity}) = \frac{1}{K_v} = \frac{1}{K} = \text{Finite Value}.$$

* → $|||^{ly}$ W.K.T, the Acceleration Error Constant is given by,

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

$$\therefore K_a = \lim_{s\to 0} \frac{s^2 K(1+sT_{z1})(1+sT_{z2})}{s(1+sT_{P1})(1+sT_{P2})} = 0$$

Steady state Error (Acceleration) is given by

$$e_{ss}(\text{Acceleration}) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Steady state Error : Type 2 System

For a Type 2 System :-

$$G(s)H(s) = \frac{K(1+sT_{z1})(1+sT_{z2})}{s^2(1+sT_{p1})(1+sT_{p2})}$$

Note :- $n = 2$

$s^n = s^2$

* → We know that, the position Error Constant is given by

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$\therefore K_p = \lim_{s \to 0} \frac{K(1+sT_{z1})(1+sT_{z2})}{s^2(1+sT_{p1})(1+sT_{p2})} = \frac{K}{0} = \infty$$

Steady state Error (position) is given by

$$e_{ss}(\text{position}) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

* → III^{ly} we know that, Velocity Error Constant is given by

$$K_v = \lim_{s \to 0} s \cdot G(s)H(s)$$

$$\therefore K_v = \lim_{s \to 0} \frac{s \cdot K(1+sT_{z1})(1+sT_{z2})}{s^2(1+sT_{p1})(1+sT_{p2})} = \infty$$

Steady state Error (velocity) is given by

$$e_{ss}(\text{velocity}) = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

* Ill^{ly} we know that, the Acceleration Error Constant is given by.

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$\therefore \quad K_a = \lim_{s \to 0} \frac{s^2 K (1+sTz_1)(1+sTz_2)}{s^2 (1+sTp_1)(1+sTp_2)} = K$$

Steady state Error (Acceleration) is given by

$$e_{ss}(\text{Acceleration}) = \frac{1}{K_a} = \frac{1}{K} = \text{finite Value.}$$

* → Steady - state Errors for various inputs and System are summarized in table as shown below.

Type Of Input	Steady - State Error		
	Type-0 System	Type-1 System	Type-2 System
Unit - Step	$\frac{1}{1+K_P}$	0	0
Unit - ramp	∞	$\frac{1}{K_v}$	0
Unit - Parabolic	∞	∞	$\frac{1}{K_a}$
	$K_p = \lim_{s \to 0} G(s)H(s)$	$K_v = \lim_{s \to 0} sG(s)H(s)$	$K_a = \lim_{s \to 0} s^2 G(s)H(s)$

*→ Generalised Error Co-efficient Method (or Dynamic Error Co-efficients) (Generalize Error Series)

→ The Error transfer function is given by

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} \longrightarrow ①$$

→ By Expanding Eqn ① by Taylor's Series

$$\frac{1}{1+G(s)H(s)} = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + - + \cdots \longrightarrow ②$$

Where $C_0, C_1, C_2 \cdots$ so on are the generalised or dynamic Error Constant.

→ By taking Limits on both sides as $s \to 0$

$$\boxed{\lim_{s\to 0} \frac{1}{1+G(s)H(s)} = C_0}$$

→ Differentiating Equation ② w.r.t 's'

$$\frac{d}{ds}\left[\frac{1}{1+G(s)H(s)}\right] = C_1 + 2C_2 s + 3C_3 s^2 + 4C_4 s^3 + - + \cdots \rightarrow ③$$

→ By taking Limits on both sides $s \to 0$

$$\boxed{\lim_{s\to 0} \frac{d}{ds}\left[\frac{1}{1+G(s)H(s)}\right] = C_1}$$

→ Differentiating Equation ③ w.r.t 's'

$$\frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right] = 2\cdot1\cdot C_2 + 3\cdot2\cdot C_3 s + 4\cdot3\cdot C_4\cdot s^2 + - + \cdots ④$$

→ By taking Limit on both sides.

$$\lim_{s\to 0} \frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right] = 2\cdot C_2$$

$$\boxed{C_2 = \frac{1}{2!} \lim_{s \to 0} \frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right]}$$

In general

$$\boxed{C_n = \frac{1}{n!} \lim_{s \to 0} \frac{d^n}{ds^n}\left[\frac{1}{1+G(s)H(s)}\right]}$$

By substituting Eqn ② in ①

$$\frac{E(s)}{R(s)} = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + - + - - - -$$

$$E(s) = C_0 R(s) + C_1 s R(s) + C_2 s^2 R(s) + - + - - - - -$$

By taking Inverse laplace transform, we get the generalized Error series.

$$e(t) = C_0 r(t) + C_1 \frac{d}{dt} r(t) + C_2 \frac{d^2}{dt^2} r(t) + - + - - - -$$

$$\boxed{e(t) = C_0 r(t) + C_1 r'(t) + C_2 r''(t) + C_3 r'''(t) + - + - - - -}$$

$$\text{SSE}, \boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

Things to remember :-

* → Closed loop transfer function of a second Order Control System is given by,

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

* Where ω_n = Natural frequency of Oscillations

ξ = Damping ratio

$\xi\omega_n$ = Damping factor

$\omega_d = \omega_n\sqrt{1-\xi^2}$ = Damping frequency of oscillations.

* → The response of a second Order control system subjected to a unit step input is given by,

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)}$$

* → Rise time 'T_r' is given by

$$\boxed{T_r = \frac{\pi - \theta}{\omega_d}}$$

But $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$\omega_d = \omega_n \cdot \sqrt{1-\xi^2}$

$$\therefore \boxed{T_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}}$$

* → Peak time 'T_p' is given by

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}};$$

for $n = 2$, $t_p = \frac{2\pi}{\omega_d}$

for $n = 3$, $t_p = \frac{3\pi}{\omega_d}$

* → Peak Overshoot 'Mp' is given by

$$\boxed{M_p = A \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

Where 'A' is the strength of the input.

For Ex:- If the System is subjected to a step input of strength '2' unit then the Expression for 'Mp' is given by

$$\boxed{M_p = 2 \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

* → % peak Overshoot is given by.

$$\boxed{\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100}$$

* → Settling time 'Ts' is given by

$$\boxed{T_s = \frac{3}{\xi \omega_n}}$$ for 5% tollerance.

$$\boxed{T_s = \frac{4}{\xi \omega_n}}$$ for 2% tollerance.

* → Steady State Errors :-

The Error transfer function is given by.

$$\boxed{\frac{E(S)}{R(S)} = \frac{1}{1+G(S)H(S)}}$$

Steady state Error (SSE) is given by

$$\boxed{SSE = \lim_{t \to \infty} e(t) = \lim_{s \to 0} S \cdot E(S)}$$

$$\therefore \boxed{SSE = \lim_{s \to 0} = s \cdot \frac{R(s)}{1+G(s)H(s)}}$$

*→ Error Constants

1) Static Error Constants :-

a) position Error constants (K_p) is given by

$$\boxed{K_p = \lim_{s \to 0} G(s)H(s) \; ; \; SSE = C_{ss} = \frac{1}{1+K_p}}$$

$$\text{If } R(s) = \frac{A}{s}$$

$$\text{then, } \boxed{SSE = C_{ss} = \frac{A}{1+K_p}}$$

b) Velocity Error Constants (K_v) is given by

$$\boxed{K_v = \lim_{s \to 0} s \cdot G(s)H(s) \; ; \; SSE = C_{ss} = \frac{1}{K_v}}$$

$$\text{If } R(s) = \frac{A}{s^2}$$

$$\text{then, } \boxed{SSE = C_{ss} = \frac{A}{K_v}}$$

c) Acceleration Error Constant (K_a) is given by

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s)H(s) \; ; \; SSE = C_{ss} = \frac{1}{K_a}}$$

$$\text{If } R(s) = \frac{A}{s^3}$$

$$\text{then, } \boxed{SSE = C_{ss} = \frac{A}{K_a}}$$

* → Effects of change in G(s) H(s) on steady state Error

$$G(s)H(s) = \frac{K(1+sT_{z1})(1+sT_{z2})\cdots}{s^n(1+sT_{p1})(1+sT_{p2})\cdots}$$

'n' is the type of the system.

→ Type '0' system ; n=0 ; no integration ; no poles of G(s) H(s) at the origin of s-plane

→ Type '1' system ; n=1 ; one integration; one poles of G(s) H(s) at the origin of s-plane

→ Type '2' system ; n=2 ; two integrations, two poles of G(s) H(s) at the origin of s-plane.

* Table for steady state Error for various inputs and systems

Type of Input	Steady-state-Error		
	Type-0 System	Type-1 System	Type-2 System
Unit-step	$\frac{1}{1+K_p}$	0	0
Unit-ramp	∞	$\frac{1}{K_v}$	0
Unit-Parabolic	∞	∞	$\frac{1}{K_a}$
	$K_p = \lim_{s\to 0} G(s)H(s)$	$K_v = \lim_{s\to 0} s.G(s)H(s)$	$K_a = \lim_{s\to 0} s^2 G(s)H(s)$

* Generalized Error Series :-

$$\boxed{C_n = \frac{1}{n!} \lim_{s \to 0} \frac{d^n}{ds^n} \left[\frac{1}{1+G(s)H(s)} \right]}$$

$$\boxed{e(t) = C_0\, r(t) + C_1\, r'(t) + C_2\, r''(t) + C_3\, r'''(t) + - + - - -}$$

$$\boxed{SSE = e_{ss} = \lim_{t \to \infty} e(t)}$$

* Maximum Peak Response

$$\text{Max Response} = c(t_p) = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}}$$

if $c(t)$ = unit step with strength A

$$c(t_p) = A\left(1 + e^{-\xi\pi/\sqrt{1-\xi^2}}\right)$$

* Problems In Time Response of Control System:-

1) A first Order Control system is represented by a transfer function $\frac{C(s)}{R(s)} = \frac{1}{s+4}$, Determine the time constant and response of the system for a unit step input.

Solution:- Given $\frac{C(s)}{R(s)} = \frac{1}{s+4}$

$$C(s) = \frac{1}{s+4} \cdot R(s)$$

But the given input is unit step

$$\therefore \quad R(s) = \frac{1}{s} = \text{unit step}$$

$$\text{i.e} \quad C(s) = \frac{1}{s+4} \cdot \frac{1}{s}$$

$$C(s) = \frac{1}{s(s+4)}$$

Breaking R.H.S in to partial fractions

$$\therefore \frac{1}{s(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+4} \qquad - ①$$

$$1 = K_1(s+4) + K_2 s$$

Substitute '0' for 's' we get

$$1 = 4K_1 + 0$$

$$\boxed{K_1 = \frac{1}{4}}$$

Substitute '-4' for 's' we get

$$1 = K_1(-4+4) + K_2(-4)$$

$$1 = 0 - K_2 4$$

$$\therefore \boxed{K_2 = -\frac{1}{4}}$$

By substituting the value of K_1 and K_2 in ①

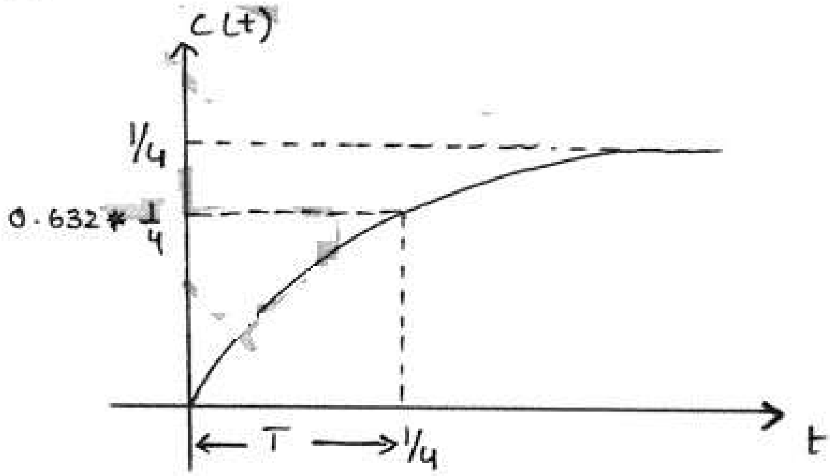
$$C(s) = \frac{1/4}{s} - \frac{1/4}{s+4}$$

By taking inverse laplace transforms

$$c(t) = \frac{1}{4}u(t) - \frac{1}{4}e^{-4t}u(t)$$

$$\therefore \text{Time response} = \boxed{c(t) = \underbrace{\frac{1}{4}}_{\text{Steady state response}} - \underbrace{\frac{1}{4}e^{-4t}}_{\text{Transient response.}}} \quad t \geq 0$$

The response is as shown below.



* Time Constant of the system is 1/4.
* For an Exponentially growing signal time Constant is defined as the time taken by the Output to attain or reach 63.2% of the final value.

Given $c(t)$ for a unit step input

i.e $r(t) = u(t) = 1$ or $R(s) = \frac{1}{s}$

$$\therefore \quad C(s) = \frac{600}{(s+10)(s+60)} \cdot R(s)$$

Closed loop transfer function is given by

$$\boxed{\frac{C(s)}{R(s)} = \frac{600}{(s+10)(s+60)}}$$

or

$$\boxed{\frac{C(s)}{R(s)} = \frac{600}{s^2 + 70s + 600}}$$

The above equation is in the form of

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

By comparing $s^2 + 70s + 600$ with $s^2 + 2\xi\omega_n s + \omega_n^2$

$$2\xi\omega_n = 70 \quad \& \quad \omega_n^2 = 600$$

$$\omega_n = \sqrt{600}$$

$$\boxed{\omega_n = 24.49} \text{ rad/sec}.$$

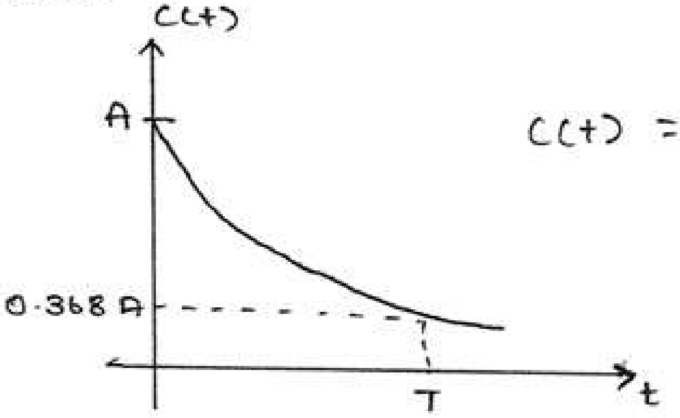
$$2\xi\omega_n = 70$$

$$\xi = \frac{70}{2\omega_n}$$

$$\xi = \frac{70}{2 \times 24.49}$$

$$\boxed{\xi = 1.429}$$

* For an exponentially decaying signal time constant is defined as the time taken to attain 36.8% of initial value.



$$C(t) = A\,e^{-4t}$$

2) The time response of second order system for a unit step input is $C(t) = 1 + 0.2\,e^{-60t} - 1.2\,e^{-10t}$ then determine the closed loop transfer function, the undamped natural frequency and damping ratio.

Solution:-

Given:- The system response

$$C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

By taking laplace transforms for the time response we get

$$C(S) = \frac{1}{S} + \frac{0.2}{S+60} - \frac{1.2}{S+10}$$

$$= \frac{(S+10)(S+60) + 0.2S(S+10) - 1.2S(S+60)}{S(S+10)(S+60)}$$

$$C(S) = \frac{\cancel{S^2} + \cancel{70S} + 600 + \cancel{0.2S^2} + \cancel{2S} - \cancel{1.2S^2} - \cancel{72S}}{S(S+10)(S+60)}$$

$$C(S) = \frac{600}{S(S+10)(S+60)}$$

3) A negative feed back system has the following transfer function $G(s) = \frac{9}{s(s+2)}$, $H(s) = 1$.

Determine its natural frequency, damping ratio, damped frequency of oscillation, damping factor, rise time, % peak overshoot, peak time and approximate 5% settling time. Assume A unit step input.

Solution: Given, $G(s) = \frac{9}{s(s+2)}$; $H(s) = 1$;

The Overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{\frac{9}{s(s+2)}}{1 + \frac{9}{s(s+2)} \cdot 1} = \frac{\frac{9}{\cancel{s^2+2s}}}{\frac{s^2+2s+9}{\cancel{s^2+2s}}}$$

$$\frac{C(s)}{R(s)} = \frac{9}{s^2+2s+9}$$

Comparing s^2+2s+9 with $s^2 + 2\xi\omega_n s + \omega_n^2$

$$2\xi\omega_n = 2 \quad + \quad \omega_n^2 = 9$$

Natural frequency; $\omega_n = \sqrt{9}$

$\boxed{\omega_n = 3}$ rad/sec

Damping factor; $\xi\omega_n = \frac{\cancel{2}}{\cancel{2}}$

$\boxed{\xi\omega_n = 1}$

Damping ratio; $\xi\omega_n = 1$

$$\xi = \frac{1}{\omega_n} = \frac{1}{3}$$

$$\boxed{\xi = \frac{1}{3}}$$

Damping frequency of Oscillations (ω_d)

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\omega_d = 3\sqrt{1-\left(\frac{1}{3}\right)^2}$$

$$\boxed{\omega_d = 2.828} \text{ rad/sec}$$

Rise time, $t_r = \dfrac{\pi - \theta}{\omega_d}$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$\xi = 1/3$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-(1/3)^2}}{1/3}\right) = 70.52^\circ$$

$$\theta = 70.52^\circ \times \frac{\pi}{180} \text{ rad}$$

$$\boxed{\theta = 1.23 \text{ rad}}$$

$$t_r = \frac{\pi - 1.23}{2.828}$$

$$\boxed{t_r = 0.67} \text{ Sec}$$

% Peak Overshoot $= \% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100$

$$\% M_p = e^{-(1/3)\pi/\sqrt{1-(1/3)^2}} * 100$$

$$\boxed{\% M_p = 32.93 \%}$$

Peak time = $t_p = \frac{\pi}{\omega_d}$

$$t_p = \frac{\pi}{2.828}$$

$$\boxed{t_p = 1.11} \text{ sec}$$

Settling time for 5% tollerance.

$$t_s = \frac{3}{\xi \omega_n}$$

$$\boxed{t_s = \frac{3}{1}} \text{ sec}$$

$$\boxed{t_s = 3}$$

4) A second order control system is represented by $\ddot{\theta}_0 + 4\dot{\theta}_0 + 25\theta_0 = 25\theta_c$. determine damping ratio, natural frequency, damping factor, damping frequency of oscillations, peak time, overshoot for a unit step input.

solution:-

Given the differential Equation of the system

$$\frac{d^2\theta_0}{dt^2} + 4\frac{d\theta_0}{dt} + 25\theta_0 = 25\theta_c$$

By taking laplace transfoms, assuming zero initial conditions. we get

$$s^2\theta_0(s) + 4s\theta_0(s) + 25\theta_0(s) = 25\theta_c(s)$$

$$\theta_0(s)\{s^2 + 4s + 25\} = 25\theta_c(s)$$

Transfer function is given by

$$\frac{\theta_o(s)}{\theta_c(s)} = \frac{25}{s^2+4s+25}$$

Comparing $s^2+4s+25$ with $s^2+2\xi\omega_n s+\omega_n^2$

→ Natural frequency (ω_n)

$$\omega_n^2 = 25$$

$$\omega_n = \sqrt{25}$$

$$\boxed{\omega_n = 5} \text{ rad/sec}$$

→ Damping factor ($\xi\omega_n$)

$$2\xi\omega_n = 4$$

$$\xi\omega_n = \frac{4}{2}$$

$$\boxed{\xi\omega_n = 2}$$

→ Damping ratio (ξ)

$$\xi\omega_n = 2$$

$$\xi = \frac{2}{\omega_n} = \frac{2}{5}$$

$$\boxed{\xi = 0.4}$$

→ Damped frequency of oscillations (ω_d)

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 5\sqrt{1-(0.4)^2}$$

$$\boxed{\omega_d = 4.582} \text{ rad/sec}$$

→ Peak time (t_p)

$$t_p = \frac{\pi}{\omega_d}$$

$$= \frac{\pi}{4.582}$$

$$\boxed{t_p = 0.685} \text{ Secs}$$

→ Max peak Overshoot (M_p)

$$M_p = A e^{-\xi\pi/\sqrt{1-\xi^2}}$$

Given $A = 1$ [unit step input]

$$M_p = e^{-0.4\pi/\sqrt{1-(0.4)^2}}$$

$$\boxed{M_p = 0.253}$$

5) A unity feedback system is characterised by open loop transfer function $G(s) = \frac{14}{s(s+10)}$ determine the gain K so that the system will have a damping ratio of 0.5 for this value of K. Determine the settling time, Peak Overshoot, and rise time for a unit step input.

Solution:-

Given,

$$G(s) = \frac{14}{s(s+10)} = \frac{14}{s^2+10s}$$

$$H(s) = 1$$

The overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{K}{s^2+10s}}{1+\frac{K}{s^2+10s}\cdot 1} = \frac{\frac{K}{\cancel{s^2+10s}}}{\frac{s^2+10s+K}{\cancel{s^2+10s}}}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+10s+K}$$

By comparing $s^2+10s+K$ with $s^2+2\xi\omega_n s+\omega_n^2$

$\omega_n^2 = K$ & $2\xi\omega_n = 10$

Given $\xi = 0.5$

$\therefore\ 2\times 0.5\ \omega_n = 10$

$\boxed{\omega_n = 10}$ rad/sec

$\therefore \boxed{K = \omega_n^2 = 100}$

*→ Settling time (t_s)

for 5% tollerance.

$$t_s = \frac{3}{\xi\omega_n}$$

$$t_s = \frac{3}{0.5\times 10} = \frac{3}{5}$$

$\boxed{t_s = 0.6}$ secs

$$t_r = \frac{\pi - \pi/3}{10\sqrt{1-(0.5)^2}}$$

$$t_r = \frac{\pi - 1.0471}{8.66}$$

$\boxed{t_r = 0.2418}$ Secs

6) For a servo mechanism system $G(s) = \frac{K_1}{s^2}$ $H(s) = 1 + K_2 s$. Determine the value of K_1 & K_2, so that the peak overshoot to unit step input is 0.25 and the peak time is 2 sec.

Solution:- Given $G(s) = \frac{K_1}{s^2}$ $H(s) = 1 + K_2 s$, $M_p = 0.25$, $t_p = 2$ secs.

→ The overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1/s^2}{1+\left(\frac{K_1}{s^2}\right)(1+K_2 s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + K_1 K_2 s + K_1}$$

By Comparing $s^2 + K_1 K_2 s + K_1$ with $s^2 + 2\xi\omega_n s + \omega_n^2$

$\boxed{K_1 = \omega_n^2}$ & $\boxed{K_1 K_2 = 2\xi\omega_n}$

Given $M_p = 0.25$ for a unit step input. & Peak time $t_p = 2s$

$M_p = A \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}$ for a unit step input $A = 1$

$M_p = 1 \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}$

By taking natural log on Both side.

$$\ln\{M_p\} = \ln\left\{e^{-\xi\pi/\sqrt{1-\xi^2}}\right\}$$

$$\ln\{0.25\} = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \ln\{e\} \qquad \because \ln[x^y] = y \ln[x]$$

$$-1.3862 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

Taking square on both sides

$$(-1.3862)^2 = \frac{(-\xi\pi)^2}{1-\xi^2}$$

$$(-1.3862)^2(1-\xi^2) = \xi^2\pi^2$$

$$(1.9215)(1-\xi^2) = \xi^2\, 9.8696$$

$$1.9215 - 1.9215\xi^2 = 9.8696\xi^2$$

$$1.9215 = 9.8696\xi^2 + 1.9215\xi^2$$

$$1.9215 = 11.7911\,\xi^2$$

$$\xi^2 = \frac{1.9215}{11.7915}$$

$$\xi^2 = 0.1629$$

$$\xi = \sqrt{0.1629}$$

$$\boxed{\xi = 0.4036}$$

Peak time ; $t_p = \frac{\pi}{W_d}$.

$$W_d = W_n\sqrt{1-\xi^2}$$

$$t_p = \frac{\pi}{W_n\sqrt{1-\xi^2}}$$

$$W_n = \frac{\pi}{t_p\sqrt{1-\xi^2}}$$

$$W_n = \frac{\pi}{2 \cdot \sqrt{1-(0.4036)^2}}$$

$\boxed{W_n = 1.7168}$ rad/sec

$$K_1 = W_n^2 = 1.7168^2$$

$$\boxed{K_1 = 2.9474}$$

$$K_1 K_2 = 2\xi W_n = 2 * (0.4036) * (1.7168)$$

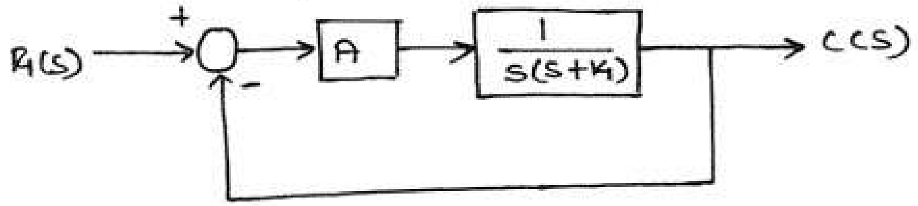
$$K_1 K_2 = 2 * (0.4036) * (1.7168)$$

$$K_2 = \frac{2 * (0.4036) * (1.7168)}{K_1}$$

$$K_2 = \frac{2 * (0.4036) * (1.7168)}{2.9474}$$

$$\boxed{K_2 = 0.4702}$$

7) A step of two is applied to the unity feed back system shown in the figure. determine the value of A & K such that damping ratio is 0.6 and damped frequency of oscillation is 8rad/sec what is the peak value of response.



Solution:- From the figure. we know that

$$G(S) = \frac{A}{s^2+Ks} \qquad H(S) = 1$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{\frac{A}{s^2+Ks}}{1+\frac{A}{s^2+Ks}\cdot 1} = \frac{\frac{A}{\cancel{s^2+Ks}}}{\frac{s^2+Ks+A}{\cancel{s^2+Ks}}}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{A}{s^2+Ks+A}}$$

By comparing s^2+Ks+A with $s^2+2\xi W_n s+W_n^2$

$$A = W_n^2 \quad ; \quad K = 2\xi W_n$$

Given $\xi = 0.6$

$W_d = 8$ rad/sec

$$W_d = W_n\sqrt{1-\xi^2}$$

$$W_n = \frac{W_d}{\sqrt{1-\xi^2}}$$

$$W_n = \frac{8}{\sqrt{1-0.6^2}}$$

$$\boxed{W_n = 10}$$

$$\therefore A = W_n^2 = 10^2$$

$$\boxed{A = 100}$$

$$K_1 = 2\xi W_n = 2 * 0.6 * 10$$

$$\boxed{K_1 = 12}$$

* -> Peak value of the response:

$$C_{max}(t) = c(t_p) = A_1\left(1 + e^{-\xi\pi/\sqrt{1-\xi^2}}\right)$$

A_1 is the strength of the unit step input

$$C_{max}(t) = c(t_p) = 2\left[1 + e^{-0.6\pi/\sqrt{1-0.6^2}}\right]$$

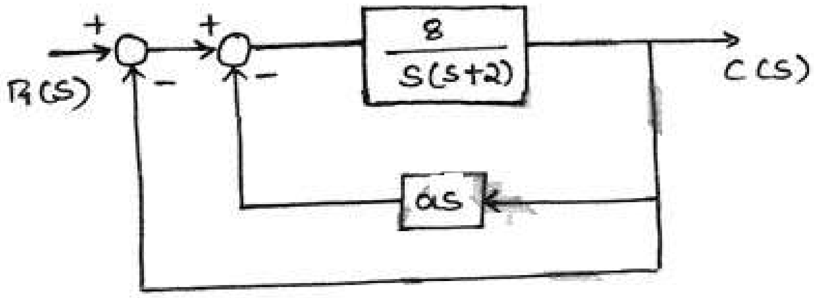
$$\boxed{C_{max}(t) = 2.189}$$

8) The System illustrated in the figure is unity feed back Control System with a minor feed back loop

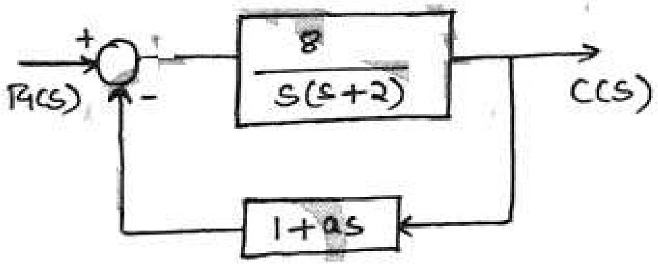
(i) in the absence of derivative feedback $(a=0)$ determine the damping ratio + Undamped natural frequency.

(ii) Determine Constant 'a' which will increase the damping ratio to 0.7.

(iii) Find the Over shoot in both the above cases.



Solution:- Above block diagram is redrawn as show below



From the figure w.k.T $G(s) = \frac{8}{s^2+2s}$; $H(s) = 1+as$

The Overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{\frac{8}{s^2+2s}}{1+\frac{8}{s^2+2s}(1+as)} = \frac{\frac{8}{\cancel{s^2+2s}}}{\frac{s^2+2s+8+8as}{\cancel{s^2+2s}}}$$

$$\frac{C(s)}{R(s)} = \frac{8}{s^2+s(2+8a)+8} \quad \text{—} \; ①$$

Case (i) when $a=0$

from Equation ① $\frac{C(s)}{R(s)} = \frac{8}{s^2+2s+8}$

By Comparing s^2+2s+8 With $s^2+2\xi W_n s+W_n^2$

$W_n^2 = 8 \quad + \quad 2\xi W_n = 2$

natural frequency:-

$\boxed{W_n = \sqrt{8} = 2\sqrt{2} = 2.828}$ rad/sec

Damping factor (ξW_n)

$$2\xi W_n = 2$$

$$\xi W_n = \cancel{\frac{2}{2}}$$

$$\boxed{\xi W_n = 1}$$

Damping ratio (ξ)

$$\xi W_n = 1$$

$$\xi = \frac{1}{W_n}$$

$$\xi = \frac{1}{2.828}$$

$$\boxed{\xi = 0.3538}$$

case (ii) when $a \neq 0$

given $\xi = 0.7$

from ① $\frac{C(s)}{R(s)} = \frac{8}{s^2+s(2+8a)+8}$

By Comparing $s^2+s(2+8a)+8$ with $s^2+2\xi W_n s+W_n^2$

$W_n^2 = 8$; $W_n = \sqrt{8} = 2\sqrt{2}$

$$\boxed{W_n = 2.828} \text{ rad/sec}$$

$2 \xi W_n = 2 + 8a$

$8a = 2 \xi W_n - 2$

$$a = \frac{2 \times 0.7 \times 2.828 - 2}{8}$$ <u>Note:</u> Given $\xi = 0.7$.

$$\boxed{a = 0.2449}$$

case (iii) To find peak Over shoot for a unit step i/p

$\therefore A = 1$

$$M_p = A \cdot e^{-\xi\pi / \sqrt{1-\xi^2}}$$

$$M_p = 1 \cdot e^{-\xi\pi / \sqrt{1-\xi^2}}$$

where $\xi = 0.3538$

$$M_p = e^{-(0.3538)\pi / \sqrt{1-(0.3538)^2}}$$

$$\boxed{M_p = 0.3047}$$

where $\xi = 0.7$

$$M_p = e^{-(0.7)\pi / \sqrt{1-(0.7)^2}}$$

$$\boxed{M_p = 0.0459}$$

9) The Open loop transfer function of a unity feedback control system is given by $G(S) = \frac{K}{S(1+ST)}$. Determine

(i) By what factor should the amplifier gain 'K' be reduced in order that the damping ratio is increased from 0.2 to 0.8

(ii) By what factor should 'K' be multiplied, so that the system overshoot for unit step excitation is reduced from 60% to 20%

Solution:- Given $G(S) = \frac{K}{S(1+ST)}$ $H(S) = 1$

The overall transfer function is given by

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)\cdot H(S)}$$

$$\frac{C(S)}{R(S)} = \frac{\frac{K}{S+S^2T}}{1+\frac{K}{S+S^2T}\cdot 1} = \frac{\frac{K}{\cancel{S+S^2T}}}{\frac{S+S^2T+K}{\cancel{S+S^2T}}}$$

$$\frac{C(S)}{R(S)} = \frac{K}{S+S^2T+K}$$

By dividing both numerator and denominator by 'T'

$$\frac{C(S)}{R(S)} = \frac{K/T}{S/T + \frac{S^2\cancel{T}}{\cancel{T}} + \frac{K}{T}} = \frac{K/T}{S^2+\frac{1}{T}S+\frac{K}{T}}$$

Comparing $s^2 + \frac{1}{T}s + \frac{K}{T}$ with $s^2 + 2\xi\omega_n s + \omega_n^2$

$$\omega_n^2 = \frac{K}{T} \quad \text{or} = \quad \omega_n = \sqrt{\frac{K}{T}}$$

$$2\xi\omega_n = \frac{1}{T}$$

$$\xi = \frac{1}{2\omega_n T} = \frac{1}{2T\sqrt{\frac{K}{T}}}$$

$$\therefore \xi = \frac{1}{2\cancel{\sqrt{T}}\cdot\sqrt{T}\cdot\frac{\sqrt{K}}{\sqrt{T}}}$$

$$\xi = \frac{1}{2\sqrt{KT}}$$

Note :- $\sqrt{T} * \sqrt{T} = T$

$\frac{\sqrt{K}}{\sqrt{T}} = \sqrt{\frac{K}{T}}$

$\sqrt{T} * \sqrt{K} = \sqrt{KT}$

Case i :- Let $\xi = \xi_1 = 0.2$ when $K = K_1$

$\xi = \xi_2 = 0.8$ when $K = K_2$

$$\xi_1 = \frac{1}{2\sqrt{K_1 T}} \quad \& \quad \xi_2 = \frac{1}{2\sqrt{K_2 T}}$$

$$\frac{\xi_1}{\xi_2} = \frac{\frac{1}{2\sqrt{K_1 T}}}{\frac{1}{2\sqrt{K_2 T}}}$$

$$= \frac{1}{\cancel{2}\sqrt{K_1}\cancel{\sqrt{T}}} * \frac{\cancel{2}\sqrt{K_2}\cancel{\sqrt{T}}}{1} = \frac{\sqrt{K_2}}{\sqrt{K_1}}$$

$$\frac{\xi_1}{\xi_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{K_2}{K_1} = \left(\frac{\xi_1}{\xi_2}\right)^2 \longrightarrow (1)$$

$$\frac{K_2}{K_1} = \left(\frac{0.2}{0.8}\right)^2$$

$$\frac{K_2}{K_1} = \frac{1}{16} = 0.0625$$

$$\boxed{K_2 = 0.0625\ K_1}$$

Conclusion:- The gain has to be reduced by $1-0.0625$

$(1-0.0625) * 100 = 93.75\%$ $\boxed{\text{Note } H(s) = 1}$

The gain has to be reduced by 93.75% to improve the damping ratio 0.2 to 0.8

Case(ii) :- Let $K = K_1$ and $\xi = \xi_1$ when

$M_p = M_{p_1} = 0.6$ and

$K = K_2$ and $\xi = \xi_2$ when

$M_p = M_{p_2} = 0.2$

→ For a unit step input

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

By taking natural log on Both sides

$$\ln\{M_p\} = \ln\left\{e^{-\xi\pi/\sqrt{1-\xi^2}}\right\}$$

$$\ln M_p = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \ln\{e\}$$

$$\ln M_p = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

$$(\ln M_p)\cdot\left(\sqrt{1-\xi^2}\right) = -\xi\pi$$

By taking Square's On Both sides

$$(\ln M_p)^2 (1-\xi^2) = \xi^2 \pi^2$$

$$(\ln M_p)^2 - (\ln M_p)^2 \xi^2 = \xi^2 \pi^2$$

$$(\ln M_p)^2 = \xi^2\pi^2 + (\ln M_p)^2 \xi^2$$

$$(\ln M_p)^2 = \xi^2\left(\pi^2 + (\ln M_p)^2\right)$$

$$\xi^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$$

$$\xi = \sqrt{\frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}} = \xi_1 = \sqrt{\frac{(\ln M_{p_1})^2}{\pi^2 + (\ln M_{p_1})^2}}$$

$$\xi_1 = \sqrt{\frac{(\ln 0.6)^2}{\pi^2 + (\ln 0.6)^2}}$$

$$\xi_1 = \sqrt{\frac{(-0.5108)^2}{\pi^2 + (-0.5108)^2}}$$

$$\boxed{\xi_1 = 0.1604}$$

$$\xi_2 = \sqrt{\frac{(\ln M_{p_2})^2}{\pi^2 + (\ln M_{p_2})^2}}$$

$$\xi_2 = \sqrt{\frac{(\ln 0.2)^2}{\pi^2 + (\ln 0.2)^2}}$$

$$\xi_2 = \sqrt{\frac{(-1.6094)^2}{\pi^2 + (-1.6094)^2}}$$

$$\boxed{\xi_2 = 0.455}$$

By Substituting the Value of ξ_1 and ξ_2 in Equation 1

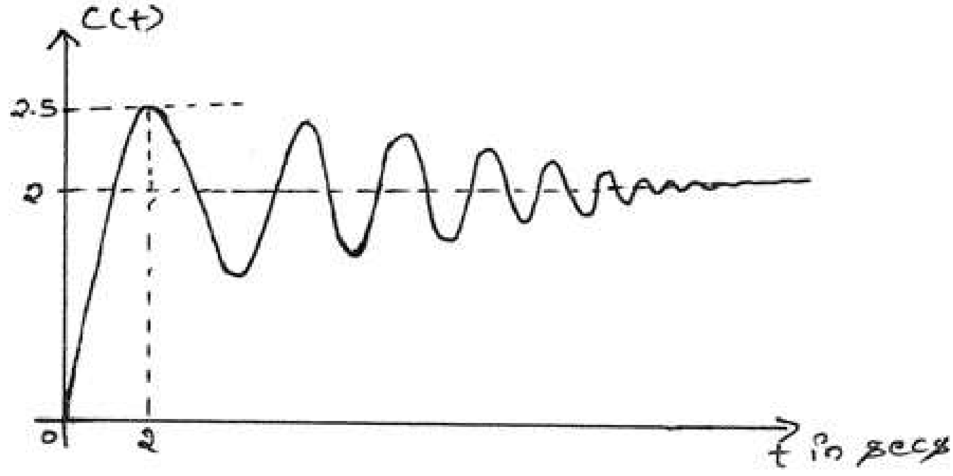
$$\frac{K_2}{K_1} = \left(\frac{0.1604}{0.455}\right)^2$$

$$\frac{K_2}{K_1} = 0.1237$$

$$K_2 = 0.1237\, K_1$$

* → Initial gain has to multiplied by 0.1237 to reduced the Over shoot from 60% to 20%

10) The step response of second Order Under damped unity feed back System shown in figure for a input of $2u(t)$ determine the Open loop and closed loop transfer function of the system.



Solution:-

From the fig :- We know that

$M_p = 0.5, \quad t_p = 2 \text{ secs}, \quad \text{input} = 2\,u(t)$

To find: $\dfrac{C(s)}{R(s)} = \dfrac{W_n^2}{s^2 + 2\xi W_n s + W_n^2} \rightarrow (1)$

$\downarrow$ A (Strength of the input)

$\therefore$ peak Overshoot $M_p = A\, e^{-\xi\pi/\sqrt{1-\xi^2}}$

Given $A = 2$

$$\therefore \quad 0.5 = 2 \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\frac{0.5}{2} = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

By taking natural log on both sides

$$\ln\left(\frac{0.5}{2}\right) = \ln\left(e^{-\xi\pi/\sqrt{1-\xi^2}}\right)$$

$$(-1.3862) = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \ln(e)$$

$$(-1.3862)\sqrt{1-\xi^2} = -\xi\pi \cdot 1$$

By taking squares on both sides

$$(-1.3862)^2 \left(\cancel{\sqrt{1-\xi^2}}\right)^{\cancel{2}} = (-\xi\pi)^2$$

$$(-1.3862)^2\ 1-\xi^2 = \xi^2\pi^2$$

$$(-1.3862)^2 - (-1.3862)^2\xi^2 = \xi^2\pi^2$$

$$1.9215 - 1.9215\,\xi^2 = \xi^2\, 9.8696$$

$$1.9215 = \xi^2\, 9.8696 + 1.9215\,\xi^2$$

$$1.9215 = 11.7911\,\xi^2$$

$$\frac{1.9215}{11.7911} = \xi^2$$

$$\xi^2 = \frac{1.9215}{11.7911}$$

$$\xi^2 = 0.16296$$

$$\xi = \sqrt{0.16296}$$

$$\boxed{\xi = 0.40368}$$

Peak time, $$t_p = \frac{\pi}{W_d}$$

$$t_p = \frac{\pi}{W_n\sqrt{1-\xi^2}}$$

$$2 = \frac{\pi}{W_n\sqrt{1-(0.40368)^2}}$$

$$2 = \frac{\pi}{W_n * 0.9149}$$

$$2 * W_n * 0.9149 = \pi$$

$$1.8298\, W_n = \pi$$

$$W_n = \frac{\pi}{1.8298}$$

$$\boxed{W_n = 1.717} \text{ rad/sec.}$$

By substituting the value of 'ξ' and 'W_n' in Equation (1) we get

Closed loop transfer function $= \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)\,H(S)}$

$$\frac{C(S)}{R(S)} = \frac{W_n^2}{S^2+2\xi W_n S + W_n^2}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{(1.717)^2}{s^2 + (2 * 0.4037 * 1.717)s + (1.717)^2}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{2.948}{s^2 + 1.3863s + 2.948}}$$ Closed loop transfer function

To find Open loop transfer function

$$\frac{C(s)}{R(s)} = \frac{2.948}{s^2 + 1.3863s\left[1 + \frac{2.948}{s^2 + 1.3863s}\right]}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{2.948}{s(s+1.3863)}}{1 + \frac{2.948}{s(s+1.3863)} \cdot 1} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

Open loop transfer function = $\boxed{G(s) = \frac{2.948}{s^2 + 1.3863s}}$

11) The closed loop poles of a system are at $-2+j3$ and $-2-j3$, Compute the value of damping ratio and the damped frequency of oscillation of system. and what is the % Overshoot of the system for a unit step input

Solution:- The Overall transfer function of a second Order Control system is given by

$$\frac{C(s)}{R(s)} = \frac{W_n^2}{s^2 + 2\xi W_n s + W_n^2}$$

The poles of a system is given by $s^2 + 2\xi W_n s + W_n^2 = 0$

$$s = \frac{-2\xi W_n \pm \sqrt{4\xi^2 W_n^2 - 4W_n^2}}{2}$$

Note:- $ax^2 + bx - c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s = \frac{-2\xi\omega_n \pm \sqrt{4\omega_n^2(\xi^2 - 1)}}{2}$$

$$s = \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2 - 1}}{2}$$

$$s = \frac{\cancel{2}(-\xi W_n \pm W_n\sqrt{\xi^2 - 1}}{\cancel{2}} = -\xi W_n \pm W_n\sqrt{\xi^2 - 1}$$

if $0 < \xi < 1$

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}$$

$$s = -\xi W_n \pm j\omega_d$$

Given : $s = -2 \pm j3$

$\therefore \xi W_n = 2 \quad \& \quad W_d = 3$

Damping factor $= \xi W_n = 2$

Damping frequency of Oscillations $= \boxed{W_d = 3}$ rad/sec

$$W_d = W_n\sqrt{1-\xi^2}$$

By taking squares on Both sides

$$W_d^2 = (W_n\sqrt{1-\xi^2})^2$$

$$W_d^2 = W_n^2(\sqrt{1-\xi^2})^2$$

$$W_d^2 = W_n^2(1-\xi^2)$$

$$W_d^2 = W_n^2 - (\xi W_n)^2$$

$$W_n^2 = W_d^2 + (\xi W_n)^2$$

$$W_n^2 = 3^2 + 2^2$$

$$W_n^2 = 9+4 = 13$$

$$W_n = \sqrt{13}$$

$$\boxed{W_n = 3.6055}$$

$$\xi W_n = 2$$

$$\xi = \frac{2}{W_n}$$

$$\xi = \frac{2}{3.6055}$$

$$\boxed{\xi = 0.5547}$$

Peak overshoot for a unit step input $\therefore A = 1$

$$M_p = 1 * e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$M_p = e^{-0.5547\pi/\sqrt{1-(0.5547)^2}}$$

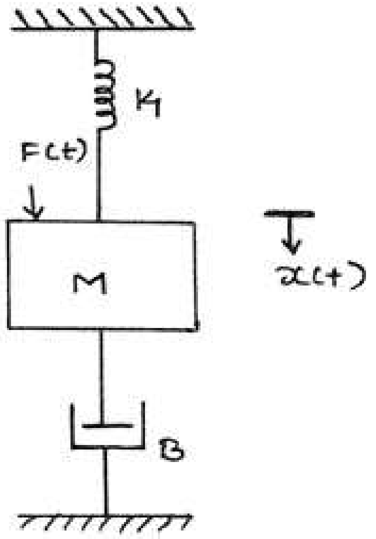
$$\boxed{M_p = 0.1236}$$

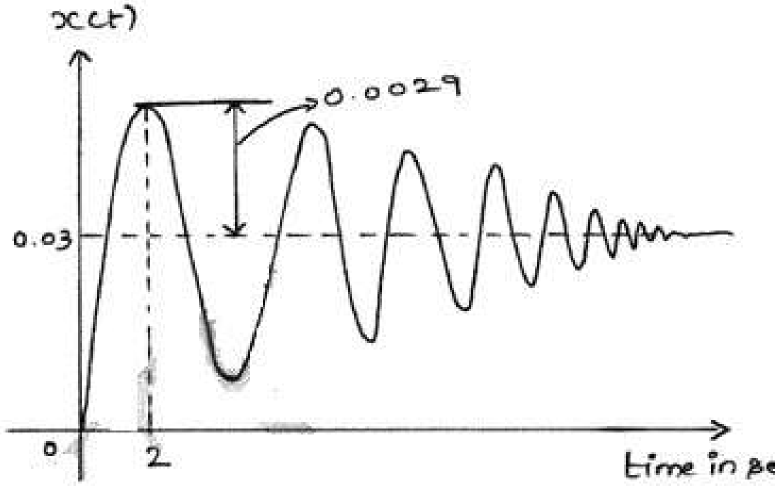
% Peak overshoot $= \% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100\%$

$$\% M_p = e^{-0.5547\pi/\sqrt{1-(0.5547)^2}}$$

$$\boxed{\% M_p = 12.36\%}$$

12) Figure shows a Mechanical Vibrating System, when a force of 8.9 newtons is applied to the system the mass oscillates as shown in figure 2. Determine the Values of M, B and K





Solution:-

The Equilibrium Equation the Mechanical system shown above, is given by

$$F(t) = M\frac{d^2x(t)}{dt^2} + Kx(t) + B\frac{d\,x(t)}{dt}$$

By taking laplace transforms assuming zero initial conditi

$$F(S) = MS^2X(S) + KX(S) + BSX(S)$$

$$= (MS^2 + BS + K)\,X(S)$$

$$\frac{X(S)}{F(S)} = \frac{1}{MS^2 + BS + K} \quad \text{--- (1)}$$

By dividing 'M' for both, Numerator and denominator

$$\frac{X(S)}{F(S)} = \frac{1/M}{\frac{MS^2}{M} + \frac{B}{m}S + \frac{K}{M}} = \frac{1/M}{S^2 + \frac{B}{M}S + \frac{K}{M}}$$

By Comparing $s^2+\frac{B}{M}s+\frac{K}{M}$ with $s^2+2\xi W_n s+W_n^2$ we get

$$\boxed{W_n^2=\frac{K}{M}} \quad \text{— } (2)$$

$$\boxed{2\xi W_n=\frac{B}{M}} \quad \text{— } (3)$$

Given:- Peak time (t_p) = 2 sec

Peak Overshoot = M_p = 0.0029

$$M_p = A e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$0.0029 = 0.03 \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\frac{0.0029}{0.03} = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

By taking natural log on both sides

$$\ln\left[\frac{0.0029}{0.03}\right] = \ln\left[e^{-\xi\pi/\sqrt{1-\xi^2}}\right]$$

$$-2.3365 = \frac{-\xi\pi}{\sqrt{1-\xi^2}} \ln[e]$$

$$(-2.3365)(\sqrt{1-\xi^2}) = -\xi\pi \cdot 1$$

By taking squares on both sides

$$(-2.3365)^2(\sqrt{1-\xi^2})^{\cancel{2}} = (-\xi\pi)^2$$

$$5.45923(1-\xi^2) = \xi^2\pi^2$$

$$5.45923 - 5.45923\xi^2 = \xi^2\, 9.8696$$

$$5.45923 = \xi^2\, 9.8696 + 5.45923\xi^2$$

$$5.45923 = 15.32883\xi^2$$

$$\frac{5.45923}{15.32883} = \xi^2$$

$$\xi = \sqrt{\frac{5.45923}{15.32883}}$$

$$\boxed{\xi = 0.59677}$$

W.K.T peak time $t_p = \frac{\pi}{W_d} = \frac{\pi}{W_n\sqrt{1-\xi^2}}$

$$t_p = \frac{\pi}{W_n\sqrt{1-\xi^2}}$$

$$W_n = \frac{\pi}{t_p\sqrt{1-\xi^2}}$$

$$W_n = \frac{\pi}{2\sqrt{1-(0.59677)^2}}$$

$$\boxed{W_n = 1.9575} \text{ rad/sec}$$

From the Equation (1)

$$X(S) = \frac{1}{MS^2 + BS + K} \cdot F(S)$$

Given $F(S) = \frac{8.9}{S}$ Note:- $F(t) = 8.9N \therefore F(S) = \frac{8.9}{S}$

and $x(t)$ at $t \to \infty = 0.03m$ (from fig)

From the final value theorem

$$x(\infty) = \lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s)$$

$$0.03 = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{Ms^2 + Bs + K} \left(\frac{8.9}{\cancel{s}}\right)$$

$$0.03 = \frac{1}{K}(8.9)$$

$$0.03 = \frac{8.9}{K}$$

$$K = \frac{8.9}{0.03}$$

$$\boxed{K = 296.67} \text{ N/M}$$

From (2)

$$\omega_n^2 = \frac{K}{M}$$

$$M = \frac{K}{\omega_n^2}$$

$$M = \frac{296.67}{(1.9575)^2}$$

$$\boxed{M = 77.42} \text{ N-Sec}^2/\text{M}$$

From (3)

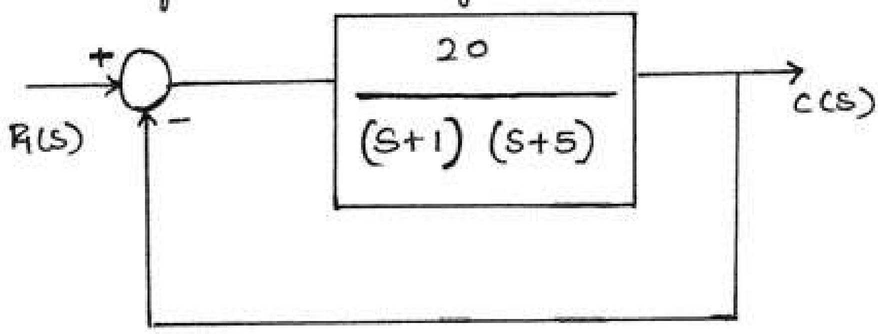
$$2\xi\omega_n = \frac{B}{M}$$

$$B = 2\xi\omega_n M$$

$$B = 2 * 0.5967 * 1.9575 * 77.42$$

$$\boxed{B = 180.86} \text{ N-Sec/M}$$

13) The block diagram of a unity feedback Control System is shown in figure. Determine the Characteristic Equation of the System, Peak time, peak Overshoot, Time at which the first undershoot occurs, the time Period of Oscillations and number of Cycle Completed before reaching the steady state.



Solution:- Given System is unity feed back System

$$G(s) = \frac{20}{(s+1)(s+5)} \qquad H(s) = 1$$

The Overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{\frac{20}{(s+1)(s+5)}}{1+\frac{20}{(s+1)(s+5)}\cdot 1} = \frac{\frac{20}{\cancel{s^2+6s+5}}}{\frac{s^2+6s+5+20}{\cancel{s^2+6s+5}}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{20}{s^2+6s+25}}$$

By comparing $s^2+6s+25$ with $s^2+2\xi\omega_n s+\omega_n^2$

$$\omega_n^2 = 25 \quad , \quad 2\xi\omega_n = 6$$

$$\omega_n = \sqrt{25}$$

$$\omega_n = 5$$

$\rightarrow \quad 2\xi \omega_n = 6$

$$\xi \omega_n = \frac{6}{2}$$

$$\boxed{\xi \omega_n = 3}$$

$$\xi = \frac{3}{\omega_n}$$

$$\xi = \frac{3}{5}$$

$$\boxed{\xi = 0.6}$$

* → The Characteristic Equation is given by

$$1 + G(s)H(s) = 0$$

$$s^2 + 6s + 25 = 0$$

* → Peak time; $t_p = \dfrac{\pi}{\omega_d}$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$t_p = \frac{\pi}{5\sqrt{1-0.6^2}}$$

$$\boxed{t_p = 0.785} \text{ secs}$$

* → Peak Overshoot, $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$ ∵ Note:- input is unit step.

$$M_p = e^{-0.6\pi/\sqrt{1-0.6^2}}$$

$$\boxed{M_p = 0.0947}$$

* → Time to the 1st under shoot.

$$t = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi}{5\sqrt{1-0.6^2}}$$

$$\boxed{t = 1.57} \text{ secs}$$

Damped frequency of Oscillations

$$W_d = W_n \sqrt{1-\xi^2}$$

$$W_d = 5\sqrt{1-0.6^2}$$

$$\boxed{W_d = 4} \text{ rad/sec}$$

But $\quad W_d = 2\pi f_0 = 2\pi \cdot \frac{1}{T_0}$

$$W_d = \frac{2\pi}{T_0}$$

$$T_0 = \frac{2\pi}{W_d}$$

Time period of oscillations

$$T_0 = \frac{2\pi}{4} = 1.57$$

$$\boxed{T_0 = 1.57} \text{ secs}$$

*→ Assuming 5% of tollerance, settling time t_s is given by.

$$t_s = \frac{3}{\xi W_n} = \frac{3}{3} = 1$$

$$\boxed{t_s = 1} \text{ sec}$$

Number of Cycles Completed before attaining steady state

$$n \geq \frac{t_s}{t_0} = \frac{1}{1.57}$$

$$n \geq \frac{1}{1.57}$$

$$n \geq 0.6369$$

$$\therefore \boxed{n = 1}$$

v

14) Consider Unity feedback Control System whose Open loop transfer function is given by $G(s) = \frac{0.4s+1}{s(s+0.6)}$, Obtain the response of unit step input for the same, Calculate rise time, maximum peak Overshoot, peak time and Settling time.

Solution:- Given $G(s) = \frac{0.4s+1}{s(s+0.6)}$; $H(s) = 1$; $\because$ unity feedback system.

The Overall transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{0.4s+1}{s^2+0.6s}}{1+\frac{0.4s+1}{s^2+0.6s}\cdot 1} = \frac{\frac{0.4s+1}{\cancel{s^2+0.6s}}}{\frac{s^2+0.6s+0.4s+1}{\cancel{s^2+0.6s}}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{0.4s+1}{s^2+1s+1}} \longrightarrow (1)$$

The output C(s) is given by:

$$C(s) = \frac{0.4s+1}{s^2+s+1}\cdot R(s)$$

Given R(s) is a unit step input $\therefore R(s) = \frac{1}{s}$

$$C(s) = \frac{0.4s+1}{s^2+s+1}\cdot\frac{1}{s}$$

By Breaking RHS in to partial fraction.

$$C(s) = \frac{0.4s+1}{s^2+s+1}\cdot\frac{1}{s} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+s+1} \longrightarrow (2)$$

$$0.4s+1 = K_1(s^2+s+1) + (K_2 s + K_3)s \longrightarrow (3)$$

To find K_1

Put $s=0$ in Equation -3

$$0+1 = K_1(0+0+1) + (K_2 \cdot 0 + K_3) \cdot 0$$

$$1 = K_1$$

$$\boxed{K_1 = 1}$$

To find K_2 and K_3

By Comparing the Co-efficients of s^2 in Equation 3

$$s^2; \quad 0 = K_1 + K_2$$

$$\boxed{K_2 = -K_1}$$

$$\boxed{K_2 = -1}$$

By Comparing the Co-efficients of s in Equation 3

$$s; \quad 0.4 = K_1 + K_3$$

$$K_3 = 0.4 - K_1$$

$$\boxed{K_3 = -0.6}$$

By substituting the value of K_1, K_2 and K_3 in 2

$$\therefore \; C(s) = \frac{1}{s} + \frac{-s-0.6}{s^2+s+1}$$

The above equation can be re-written as

$$C(s) = \frac{1}{s} - \frac{s+0.6}{\underbrace{s^2+2\cdot\frac{1}{2}s+\left(\frac{1}{2}\right)^2}_{(a+b)^2}-\left(\frac{1}{2}\right)^2+1}$$

$$= \frac{1}{s} - \frac{s+0.6}{(s+0.5)^2+\frac{3}{4}}$$

$$= \frac{1}{s} - \frac{(s+0.5)+0.1}{(s+0.5)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

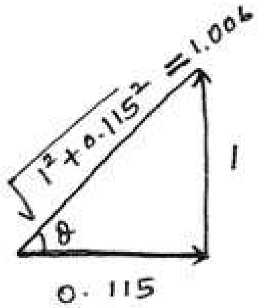
$$= \frac{1}{s} - \frac{(s+0.5)}{(s+0.5)^2 + (0.866)^2} - \frac{0.1}{0.866} * \frac{0.866}{(s+0.5)^2 + (0.866)^2}$$

By taking Inverse laplace transforms

Time response.

$$\boxed{c(t) = 1 - e^{-0.5t}\cos(0.866t) - 0.115\, e^{-0.5t}\sin(0.866t)} \rightarrow (4)$$

Note :-



$$\theta = \tan^{-1}\left(\frac{1}{0.115}\right)$$

$$\theta = 83.43^0$$

By Multiplying and dividing Equation (4) by $\sqrt{1^2+0.115^2}$

$$\therefore c(t) = 1 - e^{-0.5t}\sqrt{1+0.115^2}\left[\frac{1}{\sqrt{1+0.115^2}}\cos(0.866t) + \frac{0.115}{\sqrt{1+0.115^2}}\sin(0.866t)\right]$$

$$c(t) = 1 - e^{-0.5t}\sqrt{1+0.115^2}\left[\sin\theta \cdot \cos(0.866t) + \cos\theta \cdot \sin(0.866t)\right]$$

$$\therefore \boxed{c(t) = 1 - e^{-0.5t}(1.006) \cdot \sin(0.866t + \theta)} \quad -(5)$$

Where $\theta = 83.43^0$

At the rise time t_r, $c(t) = 1$ substitute in (5)

$$1 = 1 - 1.006\, e^{-0.5t_r}\sin(0.866\, t_r + \theta)$$

$$1.006\, e^{-0.5t_r}\sin(0.866 t_r + \theta) = \cancel{1} - \cancel{1}$$

$$1.006\, e^{-0.5 t_r} \sin(0.866 t_r + \theta) = 0$$

$$\sin(0.866 t_r + \theta) = 0$$

$$0.866 t_r + \theta = n\pi$$

For the I^{st} time, $n = 1$

$$t_r = \frac{\pi - \theta}{0.866}$$

$$t_r = \frac{\pi - 83.43 * \frac{\pi}{180}}{0.866}$$

$$\therefore \boxed{t_r = 1.9462} \text{ secs}$$

When the response is maximum at $t = t_p$,

$$\frac{d\,c(t)}{dt} = 0.$$

By Differentiating Equation ① w.r.t 't'

$$\frac{d\,c(t)}{dt} = 0 - 1.006 \left[-0.5 e^{-0.5t} \sin(0.866t + \theta) + e^{-0.5t}\, 0.866 \cos(0.866t + \theta) \right]$$

$$1.006\, e^{-0.5t} \left[0.5 \sin(0.866t + \theta) - 0.866 \cos(0.866t + \theta) \right] = 0$$

$$0.5 \sin(0.866t + \theta) - 0.866 \cos(0.866t + \theta) = 0$$

$$\tan(0.866t + \theta) = \frac{0.866}{0.5} = 1.732$$

$$\tan(0.866t + \theta) = 1.732$$

$$0.866t + \theta = \tan^{-1}(1.732)$$

$$0.866t + \theta = 60 + 180 \quad \because \text{ to avoid sign add } 180^\circ$$

$$0.866t = 180 + 60 - 83.43^\circ$$

$$0.866t = 156.57^\circ$$

$$t = \frac{156.57 * \frac{\pi}{180}}{0.866}$$

Peak time; $\boxed{t = 3.155}$ secs

By substituting $t = t_p$ in Equation ⑤ we get the maximum response.

Peak response, $C_{max}(t) = C(t_p)$

$$c(t_p) = 1 - 1.006\, e^{-0.5 * 3.155} * \sin\Big(\underbrace{0.866 * 3.155 + 83.43}_{\text{Radian Mode.}} * \frac{\pi}{180}\Big)$$

$$\boxed{c(t_p) = 1.1799}$$

Peak Over shoot, $M_p = c(t_p) - c(\infty)$

$$M_p = 1.1799 - 1$$

$$\therefore \boxed{M_p = 0.1799}$$

The Envelope of the time response is given by $= 1 \pm 1.006\, e^{-0.5t}$ at the settling time t_s,

for 5% tolerance, Envelope of the wave form is given by $= 1 \pm 0.05$

$$\therefore \quad 1 \pm 0.05 = 1 \pm 1.006\, e^{-0.5 t_s}$$

$$\cancel{1} - \cancel{1} \pm 0.05 = 1.006\, e^{-0.5 t_s}$$

$$0.05 = 1.006\, e^{-0.5 t_s}$$

$$\frac{0.05}{1.006} = e^{-0.5 t_s}$$

By taking natural log on both sides

$$\ln\left(\frac{0.05}{1.006}\right) = \ln\left(e^{-0.5 t_s}\right)$$

$$-3.0017 = -0.5 t_s$$

$$t_s = \frac{-3.0017}{-0.5}$$

$$\therefore \boxed{t_s = 6.003} \text{ secs}$$

15) Calculate the static Error Constants for the System, if transfer function $G(s) = \dfrac{10(s+2)}{s(s+3)(s+4)}$

Solution:-

Given: $G(s) = \dfrac{10(s+2)}{s(s+3)(s+4)}$

W.K.T the position error constant $(K_p) = \lim_{s \to 0} G(s)H(s)$

$$K_p = \lim_{s \to 0} \frac{10(s+2)}{s(s+3)(s+4)} \cdot 1 \quad \text{Note:- } H(s) = 1$$

$\boxed{K_p = \infty}$ for $s \to 0$

W.K.T the Velocity Error Constant $(K_v) = \lim_{s \to 0} s \cdot G(s) \cdot H(s)$

$$K_v = \lim_{s \to 0} \frac{\cancel{s} \cdot 10(s+2)}{\cancel{s}(s+3)(s+4)}$$

$$K_v = \frac{10 \times 2}{3 \times 4}$$

$\boxed{K_v = \frac{5}{3}}$ for $s \to 0$

W.K.T the Acceleration Error Constant (K_a) $= \lim_{s\to 0} s^2 G(s)H(s)$

$$K_a = \lim_{s\to 0} s^2 \cdot \frac{10(s+2)}{\cancel{s}(s+3)(s+4)}$$

$\boxed{K_a = 0}$ for $s \to 0$

1b) Find K_p, K_v and K_a for a system having $G(s) = \frac{s+10}{s(s^3+7s^2+12s)}$ also, Evaluate the steady state Error, when the input $r(t)$ is given by

i) $r(t) = 5u(t)$

ii) $r(t) = 2t\,u(t)$

iii) $r(t) = 4t^2 u(t)$

Solution:-

Given $G(s)H(s) = \frac{s+10}{s^2(s^2+7s+12)}$ with $H(s) = 1$

The given system is Type-2 and Order-4 System

*→ The position Error Constant is given by

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_p = \lim_{s\to 0} \frac{s+10}{s^2(s^2+7s+12)}$$

$\boxed{K_p = \infty}$ for $s \to 0$ $\quad \longrightarrow (1)$

* → The Velocity Error Constant is given by

$$K_v = \lim_{s \to 0} s \cdot G(s) H(s)$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot (s+10)}{s^{\cancel{2}}(s^2+7s+12)}$$

$\boxed{K_v = \infty}$ for $s \to 0$ → ②

* The Acceleration Error Constant is given by

$$K_a = \lim_{s \to 0} s^2 \cdot G(s) H(s)$$

$$K_a = \lim_{s \to 0} \frac{\cancel{s^2}(s+10)}{\cancel{s^2}(s^2+7s+12)}$$

$\boxed{K_a = \frac{10}{12} = \frac{5}{6}}$ for $s \to 0$ → ③

* The Steady state Error for an input

$r(t) = 5u(t)$

Given input is unit step

∴ Steady state error is given by

$$e_{ss} = \frac{A}{1+K_p} \quad \text{for strength of } A$$

$A = 5$

$$\therefore e_{ss} = \frac{5}{1+K_p}$$

$\boxed{e_{ss} = \frac{5}{1+\infty} = 0}$ Note :- From Eqn ① $K_p = \infty$

* → Steady state Error for an input

$$r(t) = 2\,t\,u(t)$$

Given input is ramp input $\left\{ r(t) = A\,t\,u(t) \right\}$

∴ Steady state Error is given by $\frac{A}{K_v}$

But $A = 2$

$$e_{ss} = \frac{2}{K_v}$$

$\boxed{e_{ss} = \frac{2}{\infty} = 0}$ from Eqn ② $K_v = \infty$

* → Steady state Error ~~for an~~ input

$$r(t) = 4\,t^2 u(t)$$

Given input is parabolic input $\left\{ r(t) = A\frac{t^2}{2} \right\}$

∴ Steady state Error is given by

$e_{ss} = \frac{A}{K_a}$ ~~for~~ strength of A

$$\therefore e_{ss} = 4 * \frac{2}{\frac{5}{6}}$$

$$e_{ss} = \frac{4 * 12}{5}$$

$\boxed{e_{ss} = 9.6}$ from Eqn ③ $K_a = \frac{5}{6}$

17) Find the static Error Constants for the System represented by loop transfer function $G(S)H(S) = \frac{10}{S(S+1)}$ also determine the steady state Error of the system when the input $r(t) = 10+2t$.

Solution:-

$$\text{Given } G(S)H(S) = \frac{10}{S(S+1)} \quad \text{for } H(S) = 1$$

*→ The position Error Constant is given by

$$K_p = \lim_{S \to 0} G(S)H(S)$$

$$K_p = \lim_{S \to 0} \frac{10}{S(S+1)}$$

$$\boxed{K_p = \infty} \quad \text{for } S \to 0 \longrightarrow (1)$$

*→ The Velocity Error Constant is given by

$$K_v = \lim_{S \to 0} S\, G(S)H(S)$$

$$= \lim_{S \to 0} \cancel{S} \cdot \frac{10}{\cancel{S}(S+1)}$$

$$\boxed{K_v = 10} \quad \text{for } S \to 0 \longrightarrow (2)$$

*→ The acceleration Error Constant is given by

$$K_a = \lim_{S \to 0} S^2\, G(S)H(S)$$

$$= \lim_{S \to 0} \cancel{S^2} \cdot \frac{10}{\cancel{S}(S+1)}$$

$$\boxed{K_a = 0} \quad \text{for } S \to 0 \longrightarrow (3)$$

* → Steady state Error for an input $r(t) = 10 + 2t$ is

$$e_{ss} = \frac{10}{1+K_p} + \frac{2}{K_v}$$

$$e_{ss} = \frac{10}{1+\infty} + \frac{2}{10} \quad \text{from ① and ②.}$$

$$e_{ss} = \frac{10}{\infty} + \frac{2}{10}$$

$$e_{ss} = 0 + 0.2$$

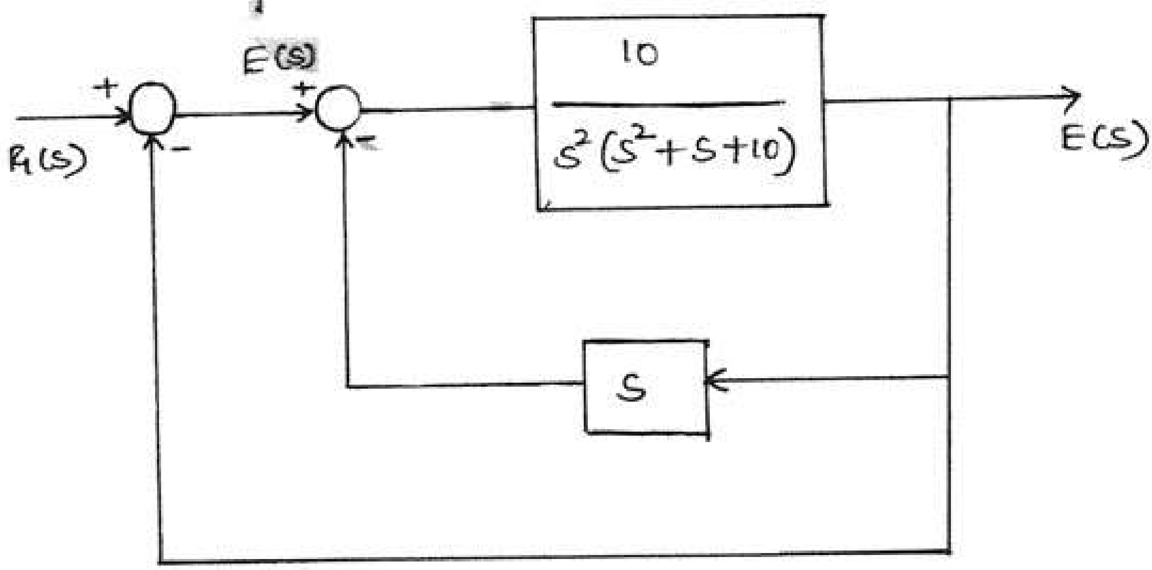
$$\boxed{e_{ss} = 0.2}$$

18) for the system shown in the figure

(i) Identify the type of $\frac{C(S)}{E(S)}$

(ii) Find the values of K_p, K_v and K_a.

(iii) If $r(t) = 10\,u(t)$ find the steady state value of the output.



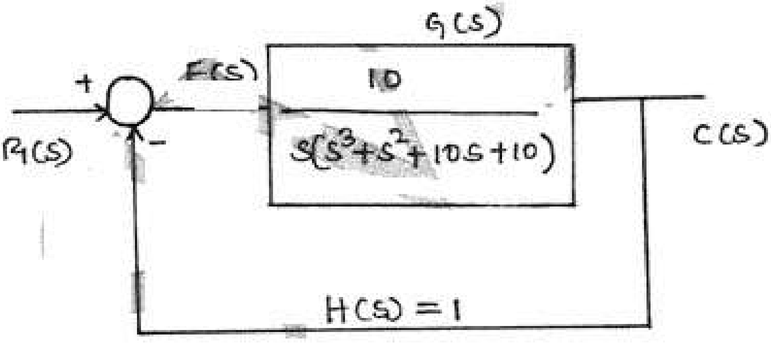
Solution:- The above block diagram is redrawn as shown below.

i) The Overall transfer $\frac{C(S)}{E(S)}$ is given by .

$$\frac{C(S)}{E(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{\frac{10}{s^2(s^2+s+10)}}{1+\frac{10}{s^2(s^2+s+10)}\cdot s}$$

$$\frac{C(S)}{E(S)} = \frac{\cancel{\frac{10}{s^4+s^3+10s^2}}}{\frac{s^4+s^3+10s^2+10s}{\cancel{s^4+s^3+10s^2}}}$$

$$\frac{C(S)}{E(S)} = \frac{10}{s^4+s^3+10s^2+10s}$$



$$\frac{C(S)}{R(S)} = \frac{10}{s(s^3+s^2+10s+10)}$$

$\frac{C(S)}{R(S)}$ Represent Type-1 System

$$G(S)\,H(S) = \frac{10}{s(s^3+s^2+10s+10)} \quad \text{for } H(S)=1$$

ii) We know that 'Kp' position Error Constant is given by.

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_p = \lim_{s\to 0} \frac{10}{s(s^3+s^2+10s+10)}$$

$\boxed{K_p = \infty}$ for $s \to 0$ $\rightarrow$ ①

*→ we know that 'Kv' velocity Error constant is given by

$$K_v = \lim_{s\to 0} s \cdot G(s)H(s)$$

$$K_v = \lim_{s\to 0} \cancel{s} \cdot \frac{10}{\cancel{s}(s^3+s^2+10s+10)}$$

$\boxed{K_v = 1}$ for $s \to 0$ $\rightarrow$ ②

*→ we know that 'Ka' Acceleration Error Constant is given by.

$$K_a = \lim_{s\to 0} s^2 \cdot G(s)H(s)$$

$$= \lim_{s\to 0} s^{\cancel{2}} \cdot \frac{10}{\cancel{s}(s^3+s^2+10s+10)}$$

$\boxed{K_a = 0}$ for $s \to 0$ $\rightarrow$ ③

iii) Steady state Error for an input $r(t) = 10u(t)$ is.

$$e_{ss} = \frac{10}{1+K_p} = \frac{10}{1+\infty}$$ from Eqn ①. $K_p = \infty$

$$\boxed{e_{ss} = \frac{10}{\infty} = 0}$$

19) A Unity feed back Control System as the forward Path transfer function $G(s) = \frac{K(2s+1)}{s(4s+1)(s+1)^2}$. the input $r(t) = 1+5t$. is Applied to the system. It is desired that the steady state value

<u>Solution:-</u> Given $G(s) = \frac{K(2s+1)}{s(4s+1)(s+1)^2}$; $r(t) = 1+5t$; $H(s)=1$

W.K.T the position Error Constant 'K_p' is given by

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$K_p = \lim_{s \to 0} \frac{K(2s+1)}{s(4s+1)(s+1)^2}$$

$$\boxed{K_p = \infty} \longrightarrow (1)$$

W.K.T the velocity Error Constant 'K_v' is given by.

$$K_v = \lim_{s \to 0} s\, G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \cancel{s} \cdot \frac{K(2s+1)}{\cancel{s}(4s+1)(s+1)^2}$$

$$= \lim_{s \to 0} \frac{K(2s+1)}{(4s+1)(s+1)^2}$$

$$\boxed{K_v = K} \longrightarrow (2)$$

* → steady state value for input $r(t) = 1+5t$

$$e_{ss} = \frac{1}{1+K_p} + \frac{5}{K_v}$$

$$e_{ss} = \frac{1}{1+\infty} + \frac{5}{K} \quad \text{from Eqn (1) and (2)}$$

$$e_{ss} = \frac{5}{\infty} + \frac{5}{K}$$

$$e_{ss} = 0 + \frac{5}{K}$$

$$\boxed{e_{ss} = \frac{5}{K}}$$

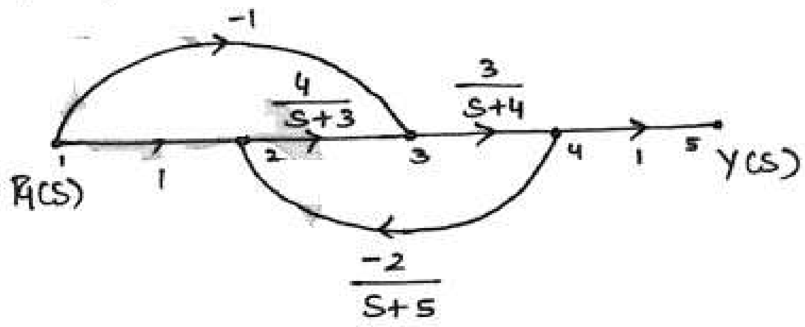
Given $e_{ss} \leq 0.1$

$$\frac{5}{K} \leq 0.1$$

$$K \geq 50$$

Minimum value of K is 50

20) For a Signal flow graph shown in the figure mention type number and Order of the System and determine the steady state Error for step and ramp input. $e(t) = r(t) - y(t)$



Solutions:-

* → Forward path gains.

$$P_1 = \frac{12}{(S+3)(S+4)} \quad (1, 2, 3, 4, 5)$$

$$P_2 = \frac{-3}{(S+4)} \quad (1, 3, 4, 5)$$

* Single loop gains.

$$P_{11} = \frac{-24}{(s+3)(s+4)(s+5)}$$

ΣP_{m_2} and on wards is '0'

* Co-factor of graph

$\Delta_1 = 1-0 = 1$

$\Delta_2 = 1-0 = 1$

$$\frac{Y(s)}{R(s)} = \frac{\sum_{k=1}^{n=2} P_k \Delta_k}{1-\sum_{m=1}^{1} P_{m_1}+0}$$

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{1-P_{11}}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{12}{(s+3)(s+4)} - \frac{3}{(s+4)}}{1+\frac{24}{(s+3)(s+4)(s+5)}}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{12-3(s+3)}{\cancel{(s+3)(s+4)}}}{\frac{(s+3)(s+4)(s+5)+24}{\cancel{(s+3)(s+4)}(s+5)}}$$

$$\frac{Y(s)}{R(s)} = \frac{12-3s+9\,(s+5)}{(s+3)(s+4)(s+5)+24}$$

$$Y(s) = \frac{12-3s+9(s+5)}{(s+3)(s+4)(s+5)+24} \cdot R(s)$$

* The Error signal is given by

$$e(t) = r(t) - y(t)$$

or

$$E(s) = R(s) - Y(s)$$

$$E(s) = R(s) - \frac{(-3s+3)(s+5)}{(s+3)(s+4)(s+5)+24} \cdot R(s)$$

$$E(s) = R(s)\left[1 + \frac{(3s-3)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

$$E(s) = R(s)\left[\frac{(s+3)(s+4)(s+5)+24+(3s-3)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

W.K.T

Steady state Error = $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

Steady state Error for a Unit step input $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0} s \cdot E(s)$$

$$\therefore e_{ss} = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}}\left[\frac{(s+3)(s+4)(s+5)+24+(3s-3)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

$$e_{ss} = \frac{(3\times4\times5)+24+(-3)(5)}{(3\times4\times5)+24} = \frac{69}{84}$$

$$\boxed{e_{ss} = \frac{68}{84}} \text{ for } s \to 0$$

Steady state Error for a unit ramp i/p $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} s.E(s)$$

$$\therefore e_{ss} = \lim_{s \to 0} \cancel{s}.\frac{1}{s^{\cancel{2}}}\left[\frac{(s+3)(s+4)(s+5)+24+(3s-3)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s}\left[\frac{(s+3)(s+4)(s+5)+24+(3s-3)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

$\boxed{e_{ss} = \infty}$ for $s \to 0$;

21) A unity feed back control system has

$$G(s) = \frac{K}{s(s+2)(s^2+2s+5)}$$

(i) For a unit ramp input, it is desired $e_{ss} \leq 0.2$, find K.

(ii) Determine e_{ss} if input $r(t) = 2+4t+\frac{t^2}{2}$

Solution :-

(i) The velocity Error Constant is given by

$$K_v = \lim_{s \to 0} s.G(s)H(s)$$

$$= \lim_{s \to 0} \cancel{s}.\frac{K}{\cancel{s}(s+2)(s^2+2s+5)}$$

$$K_v = \frac{K}{2(5)}$$

$\boxed{K_v = \frac{K}{10}}$ for $s \to 0$

iii) W.K.T

$$K_p = \lim_{s\to 0} G(s)\,H(s)$$

$$K_p = \lim_{s\to 0} \frac{K}{s(s+2)(s^2+2s+5)}$$

$$\boxed{K_p = \infty} \text{ for } s \to 0$$

iii) W.K.T

$$K_a = \lim_{s\to 0} s^2 G(s)\,H(s)$$

$$K_a = \lim_{s\to 0} s^2 \frac{K}{\cancel{s}(s+2)(s^2+2s+5)}$$

$$\boxed{K_a = 0} \text{ for } s \to 0$$

for a unit ramp input

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\frac{K}{10}} \leq 0.2$$

$$e_{ss} = \frac{10}{K} \leq 0.2$$

$$K \geq \frac{10}{0.2}$$

$$\boxed{K \geq 50}$$

ii) For a input $r(t) = 2 + 4t + \frac{t^2}{2}$

$$e_{ss} = \frac{2}{1+K_p} + \frac{4}{K_v} + \frac{1}{K_a}$$

$$= \frac{2}{\infty} + \frac{4}{K/10} + \frac{1}{0}$$

$$e_{ss} = \frac{2}{\infty} + \frac{40}{K} + \frac{1}{0}$$

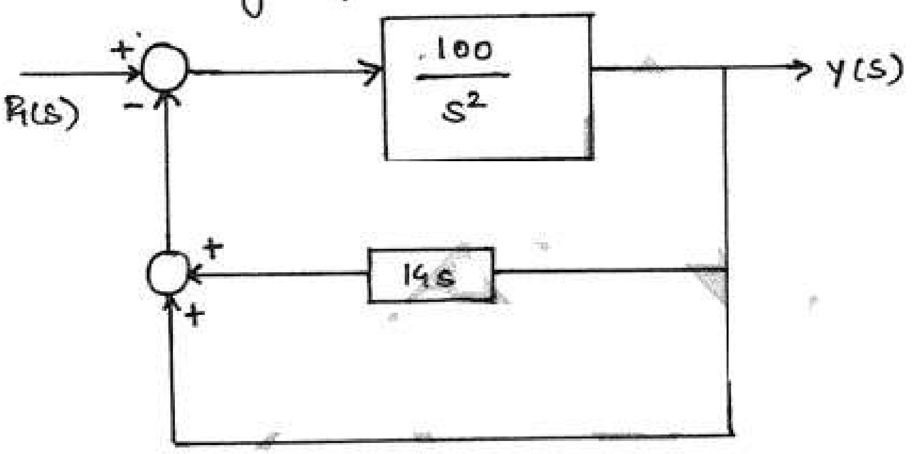
$$\boxed{e_{ss} = \infty}$$

22) A plotter may be represented by the block diagram shown in the figure below.

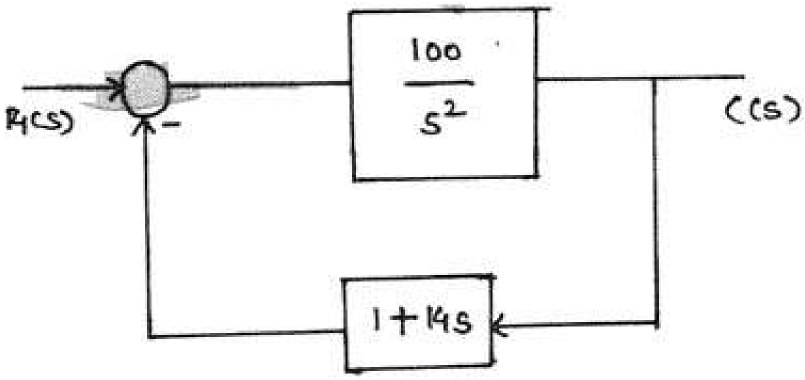
i) Determine the value of the gain 'K' that gives the Peak overshoot of 4.32%

ii) For this value of K determine the steady state error for a unit ramp input.

iii) For what range of K is the 2% settling time ≤ 1sec.



Solution:- The given block diagram is redrawn as shown below.



* The overall transfer function is given by.

$$\frac{C(s)}{R(s)} = \frac{100/\cancel{s^2}}{1 + \frac{100}{\cancel{s^2}}(1+Ks)} \rightarrow (1)$$

$$\frac{C(s)}{R(s)} = \frac{100}{s^2 + 100Ks + 100} \rightarrow (2)$$

Comparing $s^2 + 100K_1 s + 100$ with $s^2 + 2\xi\omega_n s + \omega_n^2$

$$\omega_n^2 = 100$$

$$\omega_n = \sqrt{100}$$

$$\boxed{\omega_n = 10}$$

$$2\xi\omega_n = 100K_1$$

$$K_1 = \frac{2\xi\omega_n}{100}$$

$$\boxed{K_1 = \frac{\xi\omega_n}{100}}$$

Given $\% M_p = 4.32\%$

$$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100$$

$$4.32 = e^{-\xi\pi/\sqrt{1-\xi^2}} * 100$$

$$\frac{4.32}{100} = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

taking natural log on both sides.

$$\ln\left(\frac{4.32}{100}\right) = \ln\left(e^{-\xi\pi/\sqrt{1-\xi^2}}\right)$$

$$-3.1419 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

By Taking squares on both sides.

$$(-3.1419)^2 = \left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)^2$$

$$9.8716 = \frac{\xi^2\pi^2}{1-\xi^2}$$

$$9.8716(1-\xi^2) = \xi^2\pi^2$$

$$9.8716 - 9.8716\xi^2 = \xi^2\pi^2$$

$$9.8716 - 9.8716\xi^2 = 9.8696\xi^2$$

$$9.8716 = 9.8696\xi^2 + 9.8716\xi^2$$

$$9.8716 = 19.7412\xi^2$$

$$\xi^2 = \frac{9.8716}{19.7412}$$

$$\xi = \sqrt{\frac{9.8716}{19.7412}}$$

$$\boxed{\xi = 0.707}$$

We know that

$$K = \frac{\xi W_n}{50}$$

$$K = \frac{0.707 * 10}{50}$$

$$\boxed{K = 0.1414}$$

ii) To find Velocity Error Constant

W.K.T. $K_v = \lim_{s\to 0} s\,G(s)H(s)$

From Equation ① W.K.T

$$G(s)H(s) = \frac{100}{s^2}(1+Ks)$$

$$\therefore K_v = \lim_{s\to 0} \cancel{s}\cdot\frac{100}{s^{\cancel{2}}}(1+0.1414s)$$

$\therefore \boxed{K_v = \infty}$ for $s \to 0$

Steady state Error for a unit ramp i/p is given by

$$e_{ss} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{\infty}$$

$$\boxed{e_{ss} = 0}$$

iii) Given $t_s \leq 1$ secs for 2% tolerance.

W.K.T for 2% tolerance, $t_s = \frac{4}{\xi \omega_n} \leq 1$

$$\therefore \xi \omega_n \geq 4$$

but $K = \frac{\xi \omega_n}{50}$

$$\xi \omega_n = 50K$$

$$\therefore 50K \geq 4$$

$$K \geq \frac{4}{50}$$

$$\boxed{K \geq 0.08}$$

23) The open loop transfer function of a control system with a unity feedback is $G(S) = \frac{10}{S(1+0.1S)}$. Evaluate error series for the system. Determine the steady state error of the system with the input $r(t) = 1 + 2t + t^2$

Solution:-

The error transfer function is given by.

$$\frac{E(S)}{R(S)} = \frac{1}{1+G(S)H(S)}$$

$$= \frac{1}{1+\frac{10}{S(1+0.1S)}}$$

$$= \frac{1}{\frac{S(1+0.1S)+10}{S(1+0.1S)}}$$

$$\frac{E(S)}{R(S)} = \frac{S(1+0.1S)}{S(1+0.1S)+10}$$

$$\frac{E(S)}{R(S)} = \frac{0.1S^2+S}{0.1S^2+S+10}$$

$$= \frac{\cancel{0.1}(S^2+10S)}{\cancel{0.1}(S^2+10S+100)}$$

$$\frac{E(S)}{R(S)} = \frac{S^2+10S}{S^2+10S+100} \rightarrow (1)$$

$$C_0 = \lim_{S\to 0} \frac{1}{1+G(S)H(S)} = \frac{S^2+10S}{S^2+10S+100}$$

$\boxed{C_0 = 0}$ for $S \to 0$

W.K.T

$$C_1 = \lim_{s \to 0} \frac{d}{ds}\left(\frac{1}{1+G(s)H(s)}\right)$$

$$C_2 = \frac{1}{2!} \lim_{s \to 0} \frac{d^2}{ds^2}\left(\frac{1}{1+G(s)H(s)}\right)$$

$$\Rightarrow \frac{d}{ds}\left(\frac{1}{1+G(s)H(s)}\right) = \frac{d}{ds}\left(\frac{s^2+10s}{s^2+10s+100}\right)$$

$$= \frac{(s^2+10s+100)(2s+10) - (s^2+10s)(2s+10)}{(s^2+10s+100)^2}$$

$$= \frac{(2s+10)(\cancel{s^2}+\cancel{10s}+100-\cancel{s^2}-\cancel{10s})}{(s^2+10s+100)^2}$$

$$\frac{d}{ds}\left(\frac{1}{1+G(s)H(s)}\right) = \frac{100(2s+10)}{(s^2+10s+100)^2} \longrightarrow (2).$$

W.K.T $$C_1 = \lim_{s \to 0} \frac{100(2s+10)}{(s^2+10s+100)^2}$$

$$C_1 = 100 * \frac{10}{100^2} = \frac{1}{10}$$

$$\boxed{C_1 = 0.1}$$

$$\Rightarrow \frac{d^2}{ds^2}\left(\frac{1}{1+G(s)H(s)}\right) = \frac{d}{ds}\left\{\frac{d}{ds}\left(\frac{1}{1+G(s)H(s)}\right)\right\}$$

$$= \frac{d}{ds}\left\{\frac{100(2s+10)}{(s^2+10s+100)^2}\right\}$$

$$\frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right] = 100\left\{\frac{(s^2+10s+100)^{\cancel{2}}(2) - (2s+10)\cdot 2(s^2+10s+100)(2s+10)}{(s^2+10s+100)^{\cancel{4}\,2}}\right\}$$

$$\frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right] = \frac{200\left[s^2+10s+100-(2s+10)^2\right]}{(s^2+10s+100)^2}$$

W.K.T

$$C_2 = \frac{1}{2!}\lim_{s\to 0}\frac{d^2}{ds^2}\left[\frac{1}{1+G(s)H(s)}\right]$$

$$C_2 = \frac{1}{2}\lim_{s\to 0}\frac{200\left[s^2+10s+100-4s^2-40s-100\right]}{(s^2+10s+100)^2}$$

$$C_2 = \frac{1}{2}\cdot\left\{\frac{200\,[100-100]}{(100)^2}\right\} \quad \text{for } s=0$$

$$\boxed{C_2 = 0}$$

Error series is given by:-

$$e(t) = C_0\, r(t) + C_1\, r'(t) + C_2\, r''(t) + \cdots + \cdots$$

$$r(t) = 1+2t+t^2$$

$$r'(t) = 2+2t$$

$$r''(t) = 2$$

$$r'''(t) = 0$$

Note : $C_0 = 0$

$C_1 = 0.1$

$C_2 = 0$

$$\therefore e(t) = 0 + 0.1\,(2+2t) + 0$$

$$e(t) = 0.2 + 0.2t$$

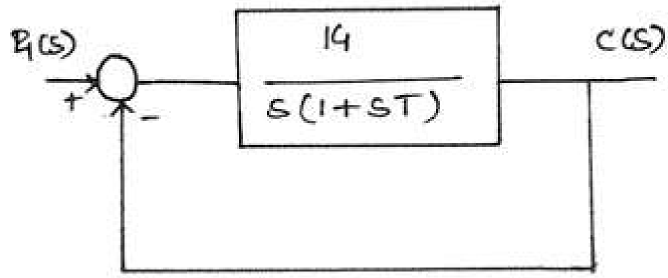
$$\boxed{e(t) = 0.2\,[1+t]}$$

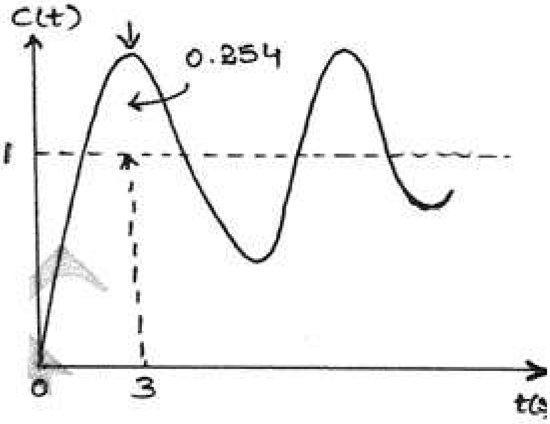
$$\therefore SSE = e_{ss} = \lim_{t\to\infty} e(t)$$

$$\therefore \boxed{e_{ss} = \infty}$$

Practise Problems:-

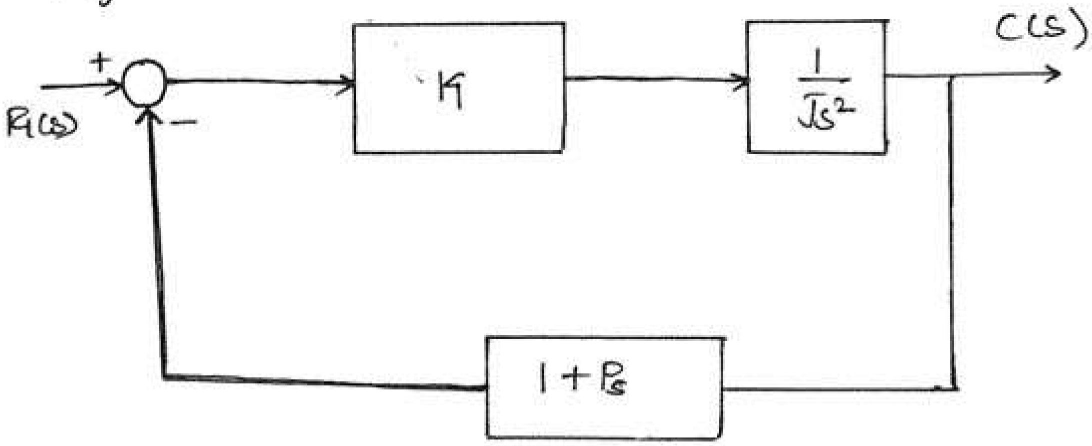
1) The System shown in the figure ⓐ when Subjected to a Unit step input gives the output response shown in the figure Ⓑ. Determine the values of K and T from the response curve.





Solution:- $T = 1.09$ & $K = 1.42$

2) Determine the value of K and P of the closed-loop system shown in the figure; so that the maximum overshoot in the unit step response is 25% and the peak time is 2 seconds. Assume that $J = 1\,kg\,m^2$



Solution:- $K = 2.95$ N-m , $P = 0.4715$

3) For the Control Systems with Open-loop transfer functions given below, Explain what type of input Signal gives rises to a constant steady-state Error and calculate there values.

(a) $G(s) = \dfrac{20}{(s+1)(s+4)}$ (b) $G(s) = \dfrac{10(s+4)}{s(s+1)(s+2)}$ (c) $G(s) = \dfrac{20}{s^2(s+1)(s+4)}$

Solution:- a) type-0 System

$$e_{ss} = \frac{1}{6}$$

b) type-1 System

$$e_{ss} = 0.05$$

c) type-2 System

$$e_{ss} = 0.2$$

4) Consider a unity feedback System with a closed transfer function $\dfrac{C(s)}{R(s)} = \dfrac{Ks+b}{s^2+as+b}$

Determine the Open-loop transfer function $G(s)$.

Show that the steady state Error with Unit ramp input is given by $\dfrac{a-K}{b}$

Solution:- $$G(s) = \frac{Ks+b}{s\left[s+(a-K)\right]}$$

Given unit ramp input. $\therefore K_v = \lim_{s\to 0} sG(s) = \dfrac{b}{a-K}$

Steady state Error for ramp input

$$e_{ss} = \frac{a-14}{b}$$

5) A Servo mechanism is represented by the Equation $\frac{d^2\theta}{dt^2} + 10\frac{d\theta}{dt} = 150E$, where $E = r - \theta$ is the actuating signal. Calculate the value of damping ratio, undamped and damped frequency of oscillations.

Solution: $\xi = 0.41$, $W_n = 12.25$ rad/sec, $W_d = 11.17$ rad/sec.

6) A certain feedback system is described by the following transfer function

$$G(S) = \frac{16}{S^2+4S+16} \qquad H(S) = 1+KS;$$

The damping factor of the system is 0.8. Determine the overshoot of the system.

Solution: $M_p = 0.015$

$\% M_p = 1.5\%$.

7) A feedback Control System is described as

$$G(S) = \frac{50}{S(S+2)(S+5)}; \quad H(S) = \frac{1}{S};$$

for a unit step input, determine the steady state Error constant

Solution: $K_p = \infty$, $K_v = \infty$, $K_a = 5$; $SSE = 0$;

8) The open loop transfer function of a feed back control system is given by

$$G(S)\,H(S) = \frac{K(S+1)}{S(1+KT)(1+2S)}$$

Determine the error co-efficients and error due to the unit positional input, unit ramp unit, and unit parabolic input, if $K=10$ and $T=4$;

Solution: $K_p = \infty$, $K_v = 10$, $K_a = 0$;

i) Unit step input $= \frac{1}{1+K_p} = 0$

ii) Unit ramp input $= \frac{1}{K_v} = 0.1$

iii) Unit parabolic $= \frac{1}{K_a} = \infty$

9) Find all the time domain specifications for a unity feedback control system whose open-loop transfer function is given by

$$G(s) = \frac{25}{S(S+6)}$$

Solution: $\omega_n = 5$ rad/sec; $\omega_d = 4$ rad/se; $t_r = 0.55$ secs,

$t_p = 0.785$ secs, $\%M_p = 9.5\%$, $M_p = 0.0947$, $t_s = 1.33$;

10) A unity feed back servo-driven instrument has an open-loop transfer function $G(S) = \frac{10}{S(S+2)}$

find,

a) The time domain response for a unit step input.

b) The natural frequency of oscillation (ω_n) and damping ratio (ξ).

(c) Maximum Overshoot and the peak time.

(d) Steady - state Error to an Input $(1+4t)$

Solution:-

(a) → $c(t) = 1 - 1.05e^{-t} \sin(3t + 71.34°)$

(b) → $\omega_n = 3.16$ rad/sec , $\xi = 0.32$

(c) → $M_p = 0.3460$; $\% M_p = 34.60\%$; $t_p = 1.04$ sec;

(d) → $e_{ss} = \lim_{s \to 0} sE(s) = 0.8$

Assignment problems :-

1) A closed-loop control system is represented by the differential Equation.

$$\frac{d^2c}{dt^2} + 4\frac{dc}{dt} = 16e$$

Where $e = r - c$ is the Error Signal. Determine the undamped natural frequency, damping ratio, and Percentage maximum overshoot for a unit step input.

2) A unity feed back system has an Open-loop transfer function $G(s) = \frac{5}{s(s+1)}$, Find the rise time, Percentage Overshoot, peak time, and Settling time for a Step input of 10 units. also determine the Peak Overshoot

3) A unity feed back system is characterised by the Open-loop transfer function.

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

Determine the steady state Errors for Unit step, unit ramp and Unit-acceleration input.

4) The Open loop transfer function of a unity feedback System is given by $G(s) = \frac{K}{s(1+sT)}$, where T and K are constants having positive values, By what factor the amplifier gain be reduced so that

(a) The peak Overshoot of unit step response of the system is reduced from 75% to 25%

(b) The damping ratio increases from 0.1 to 0.6.

5) For a unity feedback system whose open-loop transfer function is $G(s) = \frac{50}{(1+0.1s)(1+2s)}$, find the position, velocity and acceleration error constants.

6) Determine the error co-efficients and static error for unity and non unity feedback system.

$$G(s) = \frac{1}{s(s+1)(s+10)} \quad ; \quad H(s) = s+2 \; ;$$

7) A certain feedback control system is described by the following transfer function

$$G(s) = \frac{K}{s^2(s+20)(s+30)} \quad ; \quad H(s) = 1 \; ;$$

Determine steady state error co-efficients and also determine the value of 'K' to limit the error to 10 unit due to input.

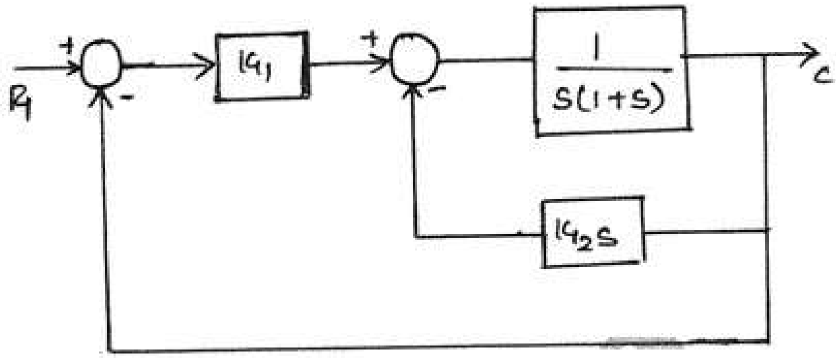
$$r(t) = 1 + 10t + 20t^2$$

8) Determine the position, velocity and acceleration error constants for a unity feedback control system whose open loop transfer function given by

$$G(s) = \frac{K}{s(s+4)(s+10)}$$

If $K = 400$, determine the steady-state error for a unit ramp input.

9) A feedback system Employing Output rate damping is shown in figure a) Find value of K_1 and K_2 so that close-loop system resembles a second Order System with damping ratio Equal to 0.5 and frequency of damped Oscillations 9.5 rad/sec.



b) With the above value of K_1 and K_2 find the percentage overshoot when input is step input

c) What is the settling time for 2 percent tolerance.

10) A second Order servo system has unity feedback and an Open loop transfer function.

$$\boxed{G(S) = \frac{500}{S(S+15)}}$$

a) Draw a block diagram for the closed loop transfer function.

b) What is the characteristic Equation of the System.

c) What is the value of natural frequency (ωn) and damping ratio (ζ).

d) Sketch the transient response for a unit step inpu

e) Obtain the value of percentage Overshoot and the Peak time.

f) What is the Settling time of the system.

9) If the system is subjected to a ramp input of 0.5 rad/s; what is the steady state error?

Introduction to PI, PD, and PID Controller

P-Controller:-

* The proportional Controller is a device that produces an output signal which is proportional to the input signal.
* The proportional Controller improves the steady state tracking accuracy, disturbance signal rejection and relative stability. It also decreases sensitivity of the system to parameter variations.
* The disadvantage of a proportional Controller is that it produces a constant steady state Error.

PI-Controller:-

* The PI-Controller is a device that produces an output signal consisting of two terms - one proportional to input signal and the other proportional to integral of input signal.
* The Effect of a PI Controller on the system performance is that it increases the Order of the system by One, which results in the reduction of the steady-state Error. But the system becomes less stable than the Original One.
* The transfer function of PI Controller is given by

$$\boxed{G_c(s) = K_p + \frac{K_i}{s}}$$

PD- Controller:-

* The PD Controller is a device that produces an Output signal consisting of two terms - One proportional to input signal and the Other proportional to derivative of the input signal.

* The PD - Controller increases the damping of the system which results in reducing the peak Overshoot.

* The Effect of PD Controller On the System Performance is to increase the damping ratio of the system and so the peak Overshoot is reduced.

* The transfer function of PD Controller is given by

$$\boxed{G_c(s) = K_p + K_d S}$$

PID - Controller:

* The PID Controller is a device which produces an Output signal consisting of three terms - one Proportional to input signal, another One proportional to integral of input signal and the third One proportional to derivative of input signal.

* The PID - Controller Stabilizes the gain, reduces the steady state Error and peak Overshoot of the system. The transfer function for PID Controller is given by

$$\boxed{G_c(S) = K_p + K_i/_S + K_d S}$$

Theory questions:-

1) With a usual notations. Derive an Expression for a unit step response of an Underdamped second Order System.

2) Considering the response of a Second Order Underdamped System to a step input, derive the following.

i) peak time ii) Rise time iii) Maximum peak Overshoot iv) Settling time.

3) Draw the time response curve and Define the time domain Specifications for a Second Order Control System for a unit step input

4) Define the following terms

(a) Transient response

(b) Steady state response.

5) Derive the Output response for First Order System with a relevant figures.

6) write a short note on

a) PI Controller.

b) PD Controller.

c) PID Controller.